HYDRODYNAMICS OF ACCRETION COLUMNS

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ABSTRACT. A detailed understanding of how the infalling matter in accretion columns is decelerated is essential for the calculation of the emitted radiation. On neutron stars, the deceleration takes place mainly by the interaction of the plasma with radiation, at least for the high-luminosity sources. We report on our two-dimensional calculations of the hydrodynamic flow in such accretion columns. The radiation transport is treated in the diffusion approximation, and we are looking for a stationary solution for the velocity field. The dependence of the results on physical parameters, especially on the accretion rate is discussed. Due to the non-linearity of the problem it turns out that only in certain parameter ranges stationary solutions seem to exist. For accretion rates higher than a critical value there are no stationary accretion flows. This leads us to the conclusion that a time-dependent picture for the accretion is unavoidable.

1. INTRODUCTION AND BASIC EQUATIONS

The infalling matter in the accretion columns of magnetized neutron stars is decelerated mainly by the interaction with the produced X-ray radiation, at least for the high-luminosity sources (cf. Wang and Frank 1981). The velocity profile forming through this interaction is essential for an understanding of the radiation transport and the emission characteristics and pulse shapes of the observed X-rays. We report on our calculations of the position and structure of the possible stationary deceleration zone and discuss the dependence of the results on physical parameters.

The basic assumptions which we have employed are that the accretion column is axially symmetric with radius R_{col} (we use cylindrical coordinates r,z),that the homogeneous magnetic field points in z-direction, $\underline{B} = B\underline{e}_z$, and that the flow velocity is parallel to the magnetic field lines, $\underline{v} = -v(r,z)\underline{e}_z$. Then the equations which describe a stationary accretion flow are as follows:

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The continuity equation

$$div(\rho \mathbf{v}) = 0 \qquad (1)$$

can immediately be integrated with the result that the mass flow $s = \rho v$ is independent of z. The quantity s may be a function of r, $s = s(r)$, depending on the processes by which the magnetic field lines are loaded with plasma (this is a question of the interaction of the accretion disk with the magnetosphere etc.). In our calculations it is always assumed for simplicity that there is a transversely uniform mass flow, i.e. s = const.

From the momentum equation

$$\rho(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla(P_{gas} + P_{rad}) - \rho g \mathbf{e}_z \qquad (2)$$

which contains the gas pressure P_{gas}, the radiation pressure P_{rad} and the gravitational force ρg, only the z-component is used, since the transverse momentum balance is made by the (enormous) magnetic field pressure.

The energy equation

$$div\,\mathbf{F}_{tot} = 0 \qquad (3)$$

contains the total energy flux

$$\mathbf{F}_{tot} = \mathbf{F}_{rad} + (\frac{1}{2}\rho v^2 + u_{gas} + P_{gas} + \rho\phi_{grav} + u_{rad} + P_{rad})\mathbf{v} \qquad (4)$$

where $\underline{F}_{rad}$ is the radiation flux, u_{gas} and u_{rad} are the internal energy density of the gas and of the radiation respectively, and Φ_{grav} is the gravitational potential.

2. APPROXIMATIONS

The equations (1) - (4) can be simplified with the help of the following approximations: first, for high-luminosity accretion, which we consider here, the gas pressure - and the internal energy of the gas, too - is negligible compared with the radiation pressure. Additionally, the radiative transfer is described by the diffusion approximation so that the radiation flux and the radiation pressure are given by

$$\mathbf{F}_{rad} = -\frac{c}{3\kappa\rho}\nabla u_{rad} \quad ; \quad P_{rad} = \frac{1}{3}u_{rad} \qquad (5)$$

For the opacity κ we use an frequency averaged value of the order of the Thomson opacity; at this stage any magnetic modifications are not taken into account.

As a further approximation we neglect the gravitational acceleration in the lower part of the accretion column so that the matter would fall with constant free-fall velocity $v_\infty = (2GM/R)^{1/2}$, unless the radiation pressure would decelerate it. The quality of this approximation is not very good if the velocity becomes small, and one has to check afterwards where that assumption breaks down. An advantage of the neglect of gravitation is that the momentum equation parallel to the magnetic field

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can be integrated and leads to $P_{rad} = u_{rad}/3 = s(v_\infty - v)$. This result can be inserted into the energy equation and thus the problem is reduced to one non-linear elliptic partial differential equation for the quantity $Q = (v/v_\infty)^2$, which reads

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right) Q = R_0 \frac{\partial}{\partial z}\left(8\sqrt{Q} - 7Q\right) \qquad (6)$$

The coordinates r and z are measured in units of R_{col} and the dimensionless parameter R_0 , which is proportional to the accretion rate, is given by

$$R_0 = \frac{\kappa s R_{col}}{c} = 1.14 \left(\frac{L_x}{10^{37}\mathrm{erg/s}}\right)\left(\frac{M}{1.4 M_\odot}\right)^{-1}\left(\frac{R}{10\,\mathrm{km}}\right)\left(\frac{R_{col}}{1\,\mathrm{km}}\right)^{-1}\left(\frac{\kappa}{\kappa_{Th}}\right) \qquad (7)$$

To solve equation (6) boundary conditions have to be specified: at the top of the column we take for v the free-fall velocity v_∞, at the mantle of the column (r = 1) we use the Marshak boundary condition $(F_{rad})_r = \frac{1}{2} c u_{rad}$, at the bottom the energy flux penetrating into the star is prescribed by means of the parameter δ (= energy flux entering the star/incoming kinetic energy flux from free fall).

3. NUMERICAL RESULTS

Solutions of equation (6) have been obtained by standard discretization methods and a modified Newton scheme adapted for the continuation over limit and bifurcation points (Keller 1977). The results show that, on the one hand, the width of the deceleration region depends on R_0, i.e. the accretion rate, and, on the other hand, the position of the

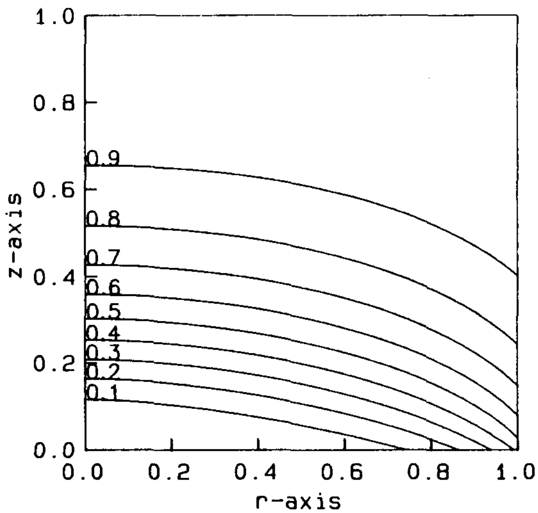

Fig.1: Contour lines of the field $Q = (v/v_\infty)^2$ for $R_0 = 1.229$

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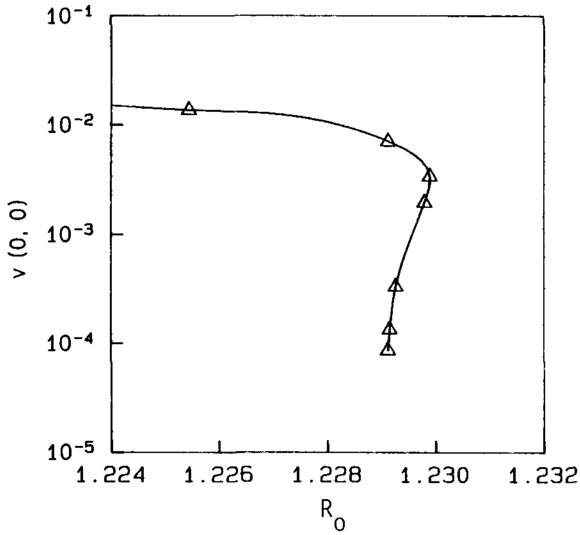

Fig.2: Velocity v/v_∞ at the bottom in the middle of the column as function of R_0

"radiative shock" is very sensitive to the parameter δ. A typical example for the structure of the velocity profile is shown in Fig.1, where we have used $\delta = 0$, because, from an estimate of the heat and radiation transport properties of the neutron star surface, it is probable that the energy flux into the star can be neglected.

The investigation of solutions for the velocity field as function of R_0 has revealed interesting effects: beyond a certain value of the accretion rate there do not exist any solutions for equation (6) and - related to this fact - just below this critical value the solutions are no longer unique. This can be seen from Fig.2, where the velocity at the bottom in the middle of the column is plotted as (double-valued) function of R_0. The test of these results with alternative discretization schemes has confirmed, although the details of Fig.2 somewhat depend on the numerical method applied, that the feature of a limit point for the accretion rate definitely remains. No stationary solutions seem to exist above this critical value. Our conclusion from this observation is that non-stationary accretion flows are unavoidable in this parameter range. In the future time-dependent calculations are required to confirm this suggestion.

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