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PRE-NEIGHBOURHOOD Λ-SEMI-LATTICES, NEIGHBOURHOOD LATTICES, AND THEIR ASSOCIATED CONTINUOUS FUNCTIONS

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This thesis is concerned with defining a topology-like structure on Λ-semi-lattices and examining the properties of the associated continuous functions.

A general pre-neighbourhood structure, which satisfies the poset analogues of the neighbourhood systems of points in a topological space, is introduced on an Λ-semi-lattice, and is used to define open elements. However, without the complete atomic Boolean structure of $P(X)$, the supremum of pre-neighbourhood open elements need not be open. Neighbourhood structures are introduced on arbitrary lattices to remove this deficiency.

The duals of pre-neighbourhood and neighbourhood systems are used to generate closed elements in an arbitrary lattice, independently of closure operators or complementation.

Pre-neighbourhood and neighbourhood continuity is defined and is used to prove the central result of the thesis, which effectively states that a function f mapping a topological space X to a topological space Y is continuous if and only if the induced lattice function $f_* : P(X) \to P(Y)$, given by $f_*(A) = \{f(a) : a \in A\}$, is continuous in the lattice sense in an appropriate neighbourhood system.

It is further shown that neighbourhood lattices have the appropriate

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structure for generalising the concept of continuity from functions to relations and multi-functions. It is proved that there are essentially three non-equivalent ways of defining continuous multi-functions, and that many of the relations encountered at the Calculus level are continuous in all three senses. Finally, the relationship between the continuity of an equivalence relation R on a topological space X and the topological properties of the projection into the set X/R is discussed.

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