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A NOTE ON THE CIRCLE ACTIONS ON EINSTEIN MANIFOLDS

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A fundamental result in the theory of black holes due to Hawking asserts that the event horizon of a black hole in the stationary space-time is a 2-sphere topologically. In this article we prove the Riemannian analogue of Hawking's result. In other words, we prove that each bolt of a 4-dimensional complete noncompact Einstein manifold of zero scalar curvature admitting a semifree isometric circle action is a 2-sphere topologically. We also study the structure of the orbit space of an Einstein manifold admitting a free isometric circle action.

I. INTRODUCTION

Let (M^4, g) be a complete 4-dimensional Einstein manifold admitting a semifree isometric S^1 group action. A metric g on a manifold M^4 is called Einstein if the Ricci curvature is proportional to the metric g; in other words,

$$r_g = \lambda g$$

for some constant λ. A group action is called semifree if it is free away from its fixed-point set. Let $\check{M}^3$ be a 3-dimensional space of non-trivial orbits of the isometric action. Then we have a Riemannian submersion $\pi : M^4 \to \check{M}^3$, and the metric g can be written as

$$g = \pi^* \check{g} + \theta^2$$

where $\check{g}$ is the metric on $\check{M}^3$ and θ is a connection 1-form on the S^1-bundle. The manifold M^4 can roughly be considered as a principal S^1-bundle over $\check{M}^3$.

Let $F = \bigcup_i F_i$ be a decomposition of the set of fixed points of an isometry into its connected components. Kobayashi [8] and Gibbons and Hawking [4] showed that each F_i is either a 2-dimensional surface B_i, called a *bolt*, or an isolated point N_i, called a *nut*. Since the action is semifree, it is easy to see that $\check{M}^3$ is a manifold with boundary $\partial \check{M}^3 = \bigcup_i B_i = F \setminus \bigcup_i N_i$ without orbifold singularities. A simple example of a bolt is the horizon 2-sphere of the Riemannian Schwarzschild manifold with the isometric S^1 group

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action, the periodic group of imaginary time translations. There are spaces with both bolts and nuts. The simplest example is CP^2. It has a natural S^1 group action, with a bolt consisting of a CP^1 and a nut at the cut locus of the bolt.

If our metric g is of zero scalar curvature, in other words, $\lambda = s/4 = 0$ with scalar curvature s, it becomes the Riemannian version of the Lorentzian *stationary* metric. A fundamental result in the theory of black holes due to Hawking asserts that, in the steady state limit, the spatial boundary of a black hole in the stationary space-time is a 2-sphere topologically [5]. The purpose of the present paper is to show that the Riemannian analogue of Hawking's result holds. Our main result states that, playing the role of the spatial boundary of black holes, each bolt of (M^4, g) of zero scalar curvature has a 2-sphere topology. More precisely, we prove the following result:

MAIN THEOREM. *Let M^4 be a 4-dimensional complete noncompact Einstein manifold of zero scalar curvature admitting a semifree isometric circle action. Suppose that the set F of fixed points is compact. Let $\Omega \subset \check{M}^3$ be a neighbourhood of F such that $\partial\Omega = \left(\bigcup_i B_i\right) \cup \partial\Omega^E$, where $\partial\Omega^E$ is mean convex, oriented, and has at least one 2-sphere component. Then $\partial\Omega^E$ is connected, each bolt B_i is a 2-sphere topologically, and Ω is diffeomorphic to a closed 3-ball minus k open 3-balls, where k is the number of B_i.*

For a 4-dimensional compact Einstein manifold admitting a semifree isometric circle action, see [4]. As an application of the Main Theorem, we have the following Theorem:

THEOREM 1. *Let M^4 be an oriented 4-dimensional compact Einstein manifold of positive scalar curvature admitting a free isometric circle action. Suppose that the orbit space $\check{M}^3$ is oriented. Then $H_2(\check{M}^3, \mathbb{Z}) = 0$. In particular, the first Betti number of $\check{M}^3$ is zero.*

This is comparable with Bochner's result which asserts that if $r > 0$, then the first Betti number of M^n must be zero. On the other hand, since it is easy to see that the above construction can be generalised to an n-dimensional Einstein manifold, we have the following Theorem:

THEOREM 2. *Let M^n, $n \geqslant 3$, be an n-dimensional compact Einstein manifold of positive scalar curvature admitting a free isometric circle action. Then an embedded oriented stable minimal hypersurface Σ in $\check{M}^{n-1}$ is totally geodesic.*

REMARK 1. (1) In our Main Theorem, the assumption that our circle action is semifree is necessary in order to avoid the orbifold singularities occurring in the orbit space. Otherwise, there might be orbits with non-trivial finite cyclic isotropy, and $\check{M}^3$ may have orbifold singularities in the interior. For example, one can construct an S^1 group action on a manifold M^4 with the multi-Eguchi-Hanson metric such that the orbit space $\check{M}^3$ is an orbifold and is singular along a sequence of line segments. In the present paper, we restrict our considerations to semifree actions only.

(2) There is difficulty in the adaptation of Hawking's proof to our case, since his proof

depends on physical consideration. Our Main Theorem is a generalisation of Galloway's result for *static* Lorentzian metric [2]. A static metric is a special case of a stationary metric. Galloway mentioned that he would pursue the study of Lorentzian stationary metrics. Our Main Theorem is concerned with the Riemannian version of such metrics. In fact, it is closely related to the *Riemannian Kerr metric uniqueness conjecture*, which states that if our metric, which is Einstein and of zero scalar curvature admitting an isometric circle action, is asymptotically flat, it is isometric to a Riemannian Kerr metric. A resolution of this conjecture will help us to understand a characterisation of higher dimensional Einstein manifolds of zero scalar curvature. Our Main Theorem is an affirmation of the conjecture by showing that such a metric has a simple topological structure under a natural boundary condition. In the spirit of the uniqueness theorem [6], we would like to pursue the study of this conjecture in the future.

II. The Proof of Main Theorem

This section is devoted to the proof of two lemmas first and then to the proof of our Main Theorem. Let A be a $(2,1)$ tensor field on M whose values on vector fields E_1, E_2 are given by

$$A_{E_1}E_2 = \mathcal{H}D_{\mathcal{H}E_1}\mathcal{V}E_2 + \mathcal{V}D_{\mathcal{H}E_1}\mathcal{H}E_2.$$

Here $\mathcal{H}$ is the horizontal distribution of the Riemannian submersion π and $\mathcal{V}$ the vertical distribution of π. The tensor field A is related to the obstruction to integrability of the horizontal distribution of the submersion π. If the tensor field A vanishes identically on M^n, the base space $\check{M}^{n-1}$ is, at least locally, a submanifold of M^n.

In the present paper, we denote by r the Ricci curvature of M^n, $\check{r}$ the Ricci curvature of $\check{M}^{n-1}$, s the scalar curvature of M^n, and $\check{s}$ the scalar curvature of $\check{M}^{n-1}$. By definition, $F = h^{-1}(0)$. The following lemma describes the behaviour of the norm h of the Killing vector field.

Lemma 1. *On the space $\check{M}^{n-1} \setminus F$, we have the following equations:*

$$\Delta_{\check{g}}h - \left(|A|^2 - r(U,U)\right)h = 0 \tag{1}$$

$$\check{r}(X,Y) = r(X,Y) + 2(A_XU, A_YU) + \frac{Ddh(X,Y)}{h} \tag{2}$$

$$\check{s}\cdot\pi = 3|A|^2 + s - 2r(U,U) \tag{3}$$

where U is a unit vertical vector field.

Remark 2. In Lemma 1, we only assume that M^n admits an isometric S^1 group action, and do not impose any conditions on the Ricci curvature. Compare the equations (1)–(3) to the stationary metric equations of [1, (1.3), (1.4) and (1.13)].

PROOF: By definition, a unit vector field U is in the vertical subspace $\mathcal{V}_x$ at x in M. We denote by X and Y horizontal vector fields. Let X_i for $i = 1, \dots, n-1$ be a (local) orthonormal basis of the horizontal subspace $\mathcal{H}_x$ at x in M, such that $D_{X_i}X_i = 0$. The basic formula for the Riemannian submersion gives

$$\check{s} \cdot \pi = s + |A|^2 + |T|^2 + |N|^2 + 2\check{\delta}N \tag{4}$$

where T is a $(2,1)$ tensor field on M^n and N a mean curvature vector field. Equation (3) may be proved by substituting the following two equations into (4):

$$\check{\delta}N = -(\widetilde{\delta}T)(U,U) = |A|^2 - |N|^2 - r(U,U) \tag{5}$$

and

$$|T|^2 - |N|^2 = \sum_i \langle \mathcal{V}D_U X_i, T_U X_i\rangle - \sum_i \langle X_i, \mathcal{H}D_U U\rangle^2 = 0. \tag{6}$$

Since the Killing vector field K for the isometric S^1 group action is vertical by definition, we have

$$N = \mathcal{H}D_U U = h^{-2}\sum_{i=1}^{n-1}\langle D_K, K, X_i\rangle X_i = -\frac{1}{h}\sum_{i=1}^{n-1} X_i(h)X_i = -\frac{dh}{h}. \tag{7}$$

Substituting (7) into the definition of $\check{\delta}N$, we have

$$\check{\delta}N = -\sum_i \langle D_{X_i}N, X_i\rangle = \frac{\Delta_{\check{g}}h}{h} - \frac{|dh|^2}{h^2}. \tag{8}$$

Equation (1) may be obtained by substituting (7) into (5) and then equating (5) and (8). Finally, (7) gives

$$D_X N = \frac{dh(X)}{h^2}dh - \frac{D_X dh}{h} \text{ and } T_U X = -\langle N, X\rangle U = \frac{dh(X)}{h}U. \tag{9}$$

Equation (2) follows from (9) and the following equation for the Riemannian submersion

$$\begin{aligned}\check{r}(X,Y) = r(X,Y) &+ 2\langle A_X U, A_Y U\rangle \\ &- \frac{1}{2}\big(\langle D_X N, Y\rangle + \langle D_Y N, X\rangle\big) + \langle TX, TY\rangle.\end{aligned}$$

$\square$

As a corollary to Lemma 1, we have the following result, see [6]:

COROLLARY 1. *Let M^n be an n-dimensional complete noncompact Einstein manifold of zero scalar curvature admitting a free isometric circle action. Suppose* *that there exists a compact set* $K \subset \check{M}^{n-1}$ *such that there is a* C^∞ *diffeomorphism* $\Phi : \check{M}^{n-1} \setminus K \to \mathbb{R}^{n-1} \setminus B_1(0)$ *which satisfies*

$$\Phi_* \check{g} - \delta_{\alpha\beta} \in C^{1,\alpha}_{-p}$$

where $\delta_{\alpha\beta}$ *is the standard metric on* $\mathbb{R}^{n-1}$ *and*

$$1 - h \in C^{1,\alpha}_{-(n-3)}(\check{M}^{n-1} \setminus K).$$

Here, $C^{k,\alpha}_\beta$ *is defined as usual, see* [3]. *Then* M^n *is isometric to* $\check{M}^{n-1} \times S^1$.

The following Lemma 2 is needed in the proof of the Main Theorem. This lemma determines the structure of a neighbourhood of a compact minimal hypersurface Σ of least area locally in $\check{M}^3$. It will be presented in an n-dimensional version.

LEMMA 2. *Let* M^n *be an* n*-dimensional Einstein manifold of non-negative scalar curvature admitting a semifree isometric circle action. Suppose that* $\partial\check{M}^{n-1}$ *is compact. If there is a compact oriented minimal hypersurface* Σ *in* $\check{M}^{n-1} \setminus \partial\check{M}^{n-1}$ *which is of least area locally, and if the norm* h *of the Killing vector field is strictly positive on* Σ, *then* Σ *is totally geodesic and* A *vanishes identically in a small neighbourhood of* Σ.

PROOF: Let ν be a smooth unit normal along Σ. Using the conformally related metric $\widetilde{g}^2 = \phi^{-2} g$ for any positive function ϕ, we define a variation $u \to \Sigma_u$ of $\Sigma = \Sigma_0$ in $\check{X}^{n-1}$. Let $\widetilde{E} : (-\varepsilon, \varepsilon) \times \Sigma \to \check{X}^{n-1}$ be the normal exponential map of Σ with respect to the metric $\widetilde{g}$. In other words, $\widetilde{E}(u,q) = \gamma_q(u)$, where γ_q is the $\widetilde{g}$-geodesic satisfying $\gamma_q(0) = q$ and $\gamma_q'(0) = \phi(q)\nu_q$. Choose ε sufficiently small so that $\widetilde{E}$ is a diffeomorphism onto $U = \widetilde{E}\big((-\varepsilon,\varepsilon) \times \Sigma\big)$ and U does not contain N_i. Then for each $u \in (-\varepsilon, \varepsilon)$, define the normal geodesic to $\Sigma_u = \widetilde{E}(u, \Sigma) = \big\{\widetilde{E}(u,q) : q \in \Sigma\big\}$. Then Σ_u is the hypersurface obtained by pushing out along the normal geodesics to $\Sigma_0 = \Sigma$ in the metric. We denote by II the second fundamental form and m the mean curvature of Σ_u, having the positive sign on a sphere in $\mathbb{R}^n$ for the outward normal vector field. In our metric $\widetilde{g} = \phi^{-2} g$, we have, for an orthonormal basis $\{\widetilde{e}_1, \dots, \widetilde{e}_{n-1}\}$ with $\widetilde{D}_{\widetilde{\nu}} \widetilde{e}_i = 0$,

$$-\widetilde{\nu}\big(\widetilde{II}(\widetilde{e}_i, \widetilde{e}_j)\big) = \widetilde{R}(\widetilde{e}_i, \widetilde{\nu}, \widetilde{e}_j, \widetilde{\nu}) + \widetilde{g}(\widetilde{D}_{\widetilde{e}_i} \widetilde{\nu}, \widetilde{D}_{\widetilde{e}_j} \widetilde{\nu}).$$

Hence, using

$$\widetilde{R} = \phi^{-2}\left(R + g \circ \left(\frac{Dd\phi}{\phi} - \frac{1}{2}\frac{|d\phi|^2}{\phi^2} g\right)\right).$$

we have in the original induced metric

$$-\frac{\partial}{\partial u}\big(II(e_i, e_j)\big) = \phi\big\langle R(e_i, \nu)e_j, \nu\big\rangle + \phi\langle D_{e_i}\nu, D_{e_j}\nu\rangle + e_i e_j(\phi) \tag{10}$$

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where $\diamond$ is the Kulkarni-Nomizu product with $\widetilde{\nu} = \phi\nu$, $\widetilde{e}_i = \phi e_i$ for $i = 1, \dots, n-1$, and $\dfrac{\partial}{\partial u} = \widetilde{\nu} = \phi\nu$. Taking the trace of both sides of (10),

$$-\frac{\partial m}{\partial u} = \phi\tilde{r}(\nu,\nu) + \phi|II|^2 + \Delta_{\Sigma_u}\phi. \tag{11}$$

On the other hand, equation (2) gives

$$\tilde{r}(\nu,\nu) = \frac{s}{n} + 2|A_\nu U|^2 + \frac{Ddh(\nu,\nu)}{h}. \tag{12}$$

From (11) and (12) with $\phi = h > 0$, we have

$$-\frac{\partial m}{\partial u} = \frac{s}{n}h + 2h|A_\nu U|^2 + Ddh(\nu,\nu) + h|II|^2 + \Delta_{\Sigma_u}h. \tag{13}$$

Since the Laplacians $\Delta_{\Sigma_u}h$ and Δh are related by

$$\Delta_{\Sigma_u}h = \Delta h - Ddh(\nu,\nu) - \frac{m}{h}\frac{\partial h}{\partial u} \tag{14}$$

and $\Delta h = |A|^2 - (s/n)h$ in virtue of (1), equation (13) with (14) becomes

$$-\frac{\partial}{\partial u}\left(\frac{m}{h}\right) = 2|A_\nu U|^2 + \frac{1}{h}|A|^2 + |II|^2 \geqslant 0. \tag{15}$$

Since m/h is non-increasing by (15) and $m = 0$ at $u = 0$ by the minimality of $\Sigma_0 = \Sigma$, we have $m/h \leqslant 0$ for $u \in [0,\varepsilon)$. Hence, from the first area variation formula we have

$$S'(u) = \int_{\Sigma_u} \operatorname{div}_{\Sigma_u}(h\nu)\, dA = \int_{\Sigma_u} hm\, dA = \int_{\Sigma_u} h^2\frac{m}{h}\ dA \leqslant 0 \tag{16}$$

where $S(u)$ is the area of Σ_u. If $S'(u_0) < 0$ for some $u_0 \in [0,\varepsilon)$, then $S(u_0) < S(0)$, contradicting that $\Sigma_0 = \Sigma$ is of least area locally. Hence $S'(u) \equiv 0$ for all u in $[0,\varepsilon)$. Therefore we have $m/h \equiv 0$ for the same u. In virtue of (16), this implies that for a neighbourhood U of Σ the following equation holds:

$$A = 0 \ \text{ and } \ II = 0, \text{ on } U \cap \{0 \leqslant u < \varepsilon\}.$$

Applying the same method to the normal $-\nu$, we finally conclude that A and II vanish in all of U. $\square$

Now, we are ready to prove our Main Theorem. Let $\Omega \subset \tilde{M}^3$ be a neighbourhood of F such that $\partial\Omega = \left(\bigcup_i B_i\right) \cup \partial\Omega^E$, where $\partial\Omega^E$ is mean convex, oriented, and has at least one 2-sphere component by the assumption. Since F is compact, the numbers of B_i and N_i are finite. In virtue of the topological Lemma of [2], Ω satisfies exactly one of the following two statements:

(i) there is a compact minimal 2-sphere (or projective plane) Σ contained in $\Omega \setminus \partial\Omega$ which is of least area locally, or

(ii) Ω is diffeomorphic to a closed 3-ball minus k open 3-balls.

Therefore, since the statement (ii) is the conclusion of our Main Theorem, the proof of our Main Theorem is reduced to showing the non-existence of such a surface Σ in $\Omega \setminus \partial\Omega$. In the present paper, we prove it by assuming the existence of such a surface and then indicating a contradiction.

Now, assume that such a surface Σ exists in $\Omega \setminus \partial\Omega$. The proof may be divided into the following two cases:

CASE I. Σ does not contain N_i for any i.

CASE II. Σ contains N_i for some i.

PROOF OF CASE I: Since $F \cap \Sigma \neq 0$ by the assumption, we have $h > 0$ on Σ. Being a special case $n = 4$ of Lemma 2, $II = 0$ in a small neighbourhood of Σ in Ω. Thus in virtue of (10) we have

$$h^{-1} Ddh_{\Sigma}(X, Y) = -\langle R(X, \nu)Y, \nu\rangle \tag{17}$$

for tangent vectors X, Y to Σ. Using (17) as a key equation, it can be shown that

$$Ddh_{\Sigma} = \frac{1}{2} hK g_{\Sigma} \tag{18}$$

and $\operatorname{grad}_{\Sigma}(h^3 K) = 0$, or $h^3 K = c$ for some constant c, where K is the intrinsic sectional curvature of Σ, see [2]. Substituting $K = ch^{-3}$ into (18) and contracting, we have $\Delta_{\Sigma} h = ch^{-2}$. Integration of this equation over Σ gives $c = 0$, implying that $K \equiv 0$. This contradicts the fact that Σ is a 2-sphere topologically. Therefore there is no such surface Σ in $\Omega \setminus \partial\Omega$, proving our Main Theorem in this case. ∎

PROOF OF CASE II: Let $V_{r_0} = \Sigma \setminus \bigcup_i B_{r_0}(N_i)$, where r_0 is taken sufficiently small so that for all points N_i, the geodesic balls $B_{r_0}(N_i)$ of radius r_0 are disjoint. Since Σ is of least area locally, so is V_{r_0}. It is clear that $h > 0$ on V_{r_0} by the definition. Therefore we may apply the same argument as in the proof of Lemma 2 to V_{r_0} to obtain (10)-(15). However, the first area variation formula for $V_u = V_{r_0,u}$, a manifold with boundary, is modified to the following equations in this case:

$$S'(u) = \int_{V_u} \operatorname{div}_{V_u}(h\nu) = -\int_{\partial V_u} \langle h\nu, \eta\rangle + \int_{V_u} hm\, dA \tag{19}$$

where η is the co-normal of ∂V_u. Furthermore, since the first term of the last equality of (16) vanishes, we may conclude that II and A also vanish on $\widetilde{E}\big((-\varepsilon, \varepsilon) \times V_{r_0}\big)$, and $h^3 K = c$ on V_{r_0} for some constant c. Letting r_0 approach to zero, h tends to 0, and hence we obtain $c = 0$. This implies that $K \equiv 0$ on Σ, which is again a contradiction to the fact that Σ is topologically a 2-sphere. Therefore, there is no such surface Σ in $\Omega \setminus \partial\Omega$, proving our Main Theorem in this case, too. ∎

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III. The Proof of two Theorems

In this section we prove Theorems 1 and 2 introduced in Section I.

Proof of Theorem 1. It is well known that, for $n \leqslant 7$, each element in $H_{n-1}(\check{M}^{n-1}, \mathbb{Z})$ can be represented by sums of embedded, compact, oriented, and stable hypersurfaces of least area locally [9, p.51]. Let Σ be an area-minimising surface in $\check{M}^3$ representing a non-trivial element of $H_2(\check{M}^3, \mathbb{Z})$. Then, by Lemma 2, tensors II and A vanishes in a small neighbourhood of Σ. As shown in the proof of our Main Theorem, such a surface Σ does not exist. Hence, the proof of Theorem 1 is completed. $\square$

Remark 3. In Theorem 1 we assumed that $s > 0$. If $s = 0$, it becomes a trivial case and $\check{M}^3$ is flat. In general, if M^n is an n-dimensional compact Einstein manifold of zero scalar curvature admitting a free isometric circle action, the Ricci tensor $\check{r}$ of $\check{M}^{n-1}$ vanishes identically.

For the special case that Σ is of least area locally, it is clear that the proof of Theorem 2 follows immediately from Lemma 2. Note that every locally area-minimising minimal hypersurface is stable. The following proof is a modification of the proof of [7, Main Theorem].

Proof of Theorem 2: Since Σ is clearly compact, the stability condition gives

$$\int_\Sigma h^2\big(r(\nu,\nu) + |II|^2\big) \leqslant \int_\Sigma |\nabla h|^2 = \int_\Sigma -h\Delta_\Sigma h \tag{20}$$

where $|\nabla h|$ is a norm by the induced metric on Σ. On the other hand, from (12), (14), and (1), we have

$$\int_\Sigma h^2 r(\nu,\nu) = \int_\Sigma 2|A_\nu U|^2 h^2 + |A|^2 h - h\Delta_\Sigma h. \tag{21}$$

Substitution of (21) into (20) gives

$$\int_\Sigma 2|A_\nu U|^2 h^2 + |A|^2 h + h^2|II|^2 \leqslant 0. \tag{22}$$

Since $h > 0$ on Σ, (22) implies that A and II vanish on Σ. $\square$

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