On an approximate construction for a regular polygon

By S. A. Scott.

My attention was drawn by an Art teacher to the following approximate construction[1] for inscribing a regular polygon of n sides in a given circle, having diameter AB, centre O. Find C in AB so that $AC : AB = 2 : n$, and construct the equilateral triangle ABD. If DC produced meets the circle in E, then AE is approximately a side of the required polygon.

I have found[2] that the value of tan AOE, as given by this construction, is $(\sqrt{3n^2 + 48n - 96} - \sqrt{3n^2}) / (2n - 8)$, from which the values tabulated below were calculated. For large n, the expansion of this expression begins with the terms $\sqrt{3}(4n^{-1} - 8n^{-2} + \ldots)$ as compared with $\tan 2\pi/n = 2\pi n^{-1} + \frac{8}{3}\pi^3 n^{-3} - \ldots$. Evidently the construction becomes increasingly erroneous as n increases: the limiting percentage error is about 10·3°/ₒ. Yet when n is not large it is surprisingly accurate, and it happens to be precise for $n = 2, 3, 4$ or 6.

n:	3	4	5	6	7	8	10	20	60
$360°/n$:	120°	90°	72°	60°	51°26′	45°	36°	18°	6°
AOE:	120°	90°	71°57′	60°	51°31′	45°11′	36°21′	18°38′	6°26′
%-error:	0	0	−0·07	0	0·17	0·41	0·97	3·5	7·2,

[1] I. H. Morris, *Geometrical Drawing for Art Students*, p. 40; Longmans Green & Co.

[2] The author has supplied two proofs, using coordinate and trigonometrical methods. Readers, or their pupils, may find this an interesting exercise.—*Editor*.

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