The criterion afforded by the sign of C is sufficient for every case; and as it is simple and complete, and arises naturally in the process of solution, it ought, I think, to take the first place in teaching. The only other point to note is that if $b\sin A/a > 1$ there is of course no angle B for which $\sin B = b\sin A/a$, and therefore no solution. This point also arises in the course of the actual calculations required.

Of course when the subject is being taught for the first time the usual geom rical discussion is quite in place. But I think it should be made clear that the ordinary process of calculation furnishes naturally all the information required.

R. F. MUIRHEAD.

A method of calculating logarithms, using merely the ordinary laws of indices.—*To determine logarithms to the base* 1·1.—If $N = a^x$, x is the logarithm of N to the base a. Hence, if $N = 1{\cdot}1^x$, x is the logarithm of N to the base 1·1, and, by plotting different values of x with the corresponding values of N, a logarithmic curve may be drawn. This curve will furnish an approximate logarithm of any number to the base 1·1.

To plot the curve: $y = 1{\cdot}1^x$.—The method of determining values of y to satisfy this equation is exceedingly simple, resolving itself into a series of multiplications by 11. This may be readily done by putting down the last figure and taking the sums of the consecutive figures in pairs. This process may be carried to any required degree of accuracy.

y.	x.
11	1
121	2
1331	3
14641	4
161051	5
177156	6

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194872	7
214359	8
235795	9
259375	10
285313	11
313844	12
345228	13
379751	14
417726	15
459499	16
505449	17
555994	18
611593	19
672752	20
740027	21
814030	22
895433	23
984976	24
1083474	25

From this graph, $\log_{1\cdot1}10$ is found to be approximately 24·16. This may be roughly verified by applying the method of differences to the last 2 values of y given above. In this way a value of 24·153 is obtained.

Instead of applying the method of differences, a much more accurate $\log_{1\cdot1}10$ may be obtained by making use of $\log_{1\cdot1}1{\cdot}01$. Thus, if this be n (where n lies between 0 and 1), $9{\cdot}84976 \times 1{\cdot}01$ is the number whose logarithm is $24+n$. In this way the error will be considerably diminished.

To determine $log_{1\cdot1}1{\cdot}01$.—This is the reciprocal of $\log_{1\cdot01}1{\cdot}1$, which may be easily calculated by a method practically similar to that employed above for finding $\log_{1\cdot1}10$, namely, by putting down the last 2 figures, and taking the sums of alternate figures in pairs.

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101	1
10201	2
1030301	3
104060401	4
105101005	5
106152015	6
107213535	7
108285670	8
109368527	9
110462212	10

Taking differences between the last 2 values,

110462212	110000000
− 109368527	− 109368527
= 1093685	= 631473

```
          577381
1,0,9,3,6,8,5/631473
           84630
            8072
             416
              88
               1
```

∴ $\log_{1\cdot 01} 1\cdot 1 = 9\cdot 577381.$

Taking the reciprocal,

```
            0·1044127
9·577381/1·
              0422619
                39524
                 1215
                  257
                   65
```

$\log_{1\cdot 1} 1\cdot 01 = 0\cdot 1044127$ (approximately).

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This value may be made still more accurate by using $\log_{1\cdot01} 1\cdot001$. This is obtained by inverting $\log_{1\cdot001} 1\cdot01$, which is calculated by precisely similar methods to the above.

1004006004	4
1005010010	5
1006015020	6
1007021035	7
1008028056	8
1009036084	9
1010045120	10

1010045120	1010000000
− 1009036084	− 1009036084
= 1009036	= 963916

```
               955284
1,0,0,9,0,3,6/963916
                55784
                 5332
                  287
                   85
                    4
```

$\therefore \qquad \log_{1\cdot001} 1\cdot01 = 9\cdot955284.$

Inverting, we obtain

$$\log_{1\cdot01} 1\cdot001 = 0\cdot1004492.$$

This may be used as follows for determining $\log_{1\cdot1} 1\cdot01$

109368527	9
110462212	10
109477896	9·1004492
109587374	9·2008984
109696961	9·3013476
109806658	9·4017968

{ 109916465	9·5022460
{ 110026381	9·6026952
109916	
83535	
75999	9·5022460
1,0,9,9,1,6/83535	·075999 × 1·004492
6594	
1098	
109	$\log_{1\cdot01}1\cdot1 = 9\cdot578585$
10	

Inverting, we obtain

$$0\cdot1043996 = \log_{1\cdot1}1\cdot01.$$

From $\log_{1\cdot01}1\cdot001$ may also be determined $\log_{1\cdot1}1\cdot001$,

for $$\log_{1\cdot1}1\cdot001 = \frac{\log_{1\cdot01}1\cdot001}{\log_{1\cdot01}1\cdot1}.$$

$$= 0\cdot1004492 \div 9\cdot578585,$$

substituting values already obtained.

Hence $\log_{1\cdot1}1\cdot001 = 0\cdot0104869$

We may assume that $\log_{1\cdot1}1\cdot0001 = \frac{1}{10}\log_{1\cdot1}1\cdot001$

$= 0\cdot0010487$

$\log_{1\cdot1}1\cdot00001 = \frac{1}{10}\log_{1\cdot1}1\cdot0001$

$= 0\cdot0001049$, etc.

With these data we are enabled to determine $\log_{1\cdot1}10$ with accuracy. The calculation is given in full below.

{ 984976	24·
{ 1083474	25·
{ 994826	24·1043996
{ 1004774	24·2087992
995821	24·1148865
996817	24·1253734
997814	24·1358603
998812	24·1463472

{999811	24·1568341
{1000811	24·1673210
{999911	24·1578828
{1000011	24·1589315
999921	24·1579877
999931	24·1580926
999941	24·1581975
999951	24·1583024
999961	24·1584073
999971	24·1585122
999981	24·1586171
999991	24·1587220
999992	24·1587325
999993	24·1587430
999994	24·1587535
999995	24·1587640
999996	24·1587745
999997	24·1587850
999998	24·1587955
999999	24-1588060
1000000	24·1588165

Hence the accurate $\log_{1\cdot 1} 10 = 24\cdot1588165$

By a precisely similar method we may find $\log_{1\cdot 1} n$.

e.g., to determine log.$_{1\cdot 1}$ 5·467321.

{505449	17·
{555994	18·
510503	17·1043996
515608	17·2087992
520764	17·3131988
525972	17·4175984
531232	17·5219980
536544	17·6263976
{541909	17·7307972
{547328	17·8351968

542451	17·7412841
542993	17·7517710
543536	17·7622579
544080	17·7727448
544624	17·7832317
545169	17·7937186
545714	17·8042055
{546260	17 8146924
{546806	17.8251793
546315	17·8157411
546370	17·8167898
546425	17·8178385
546480	17·8188872
546535	17·8199359
546590	17·8209846
546645	17·8220333
{546700	17·8230820
{546755	17·8241307
546705	17·8231869
546710	17-8232918
546715	17·8233967
546720	17·8235016
546725	17·8236065
{546730	17·8237114
{546735	17·8238163
5467305	17·8237219
5467310	17·8237324
5467315	17·8237429
5467320	17·8237534

Thus $\log_{1\cdot 1} 5{\cdot}46732 = 17{\cdot}8237534.$

Now dividing by $\log_{1\cdot 1} 10$, we get

$$\log_{10} 5{\cdot}46732 = 0{\cdot}7377743.$$

Accurately (using tables), we have

$$\log_{10} 5{\cdot}46732 = 0{\cdot}7377745.$$

Four-figure tables can be easily and expeditiously calculated in this way, the necessary apparatus consisting of $\log_{1\cdot1} 1{\cdot}01$, $\log_{1\cdot1} 1{\cdot}001$, $\log_{1\cdot1} 1{\cdot}0001$, $\log_{1\cdot1} 10$.

For nearer approximations it would have been necessary to take more figures in the original block which gave the powers of 1·1.

The method of differences might have been used at various stages to shorten the work further without great loss of accuracy. Graphical interpolation after the $\log_{1\cdot1} 1{\cdot}01$ stage would render this method more suitable for pedagogical purposes.

Finally, consider again the function $y = a^x$.

For an increment ·001 in x, y has to be multiplied by

$$\frac{a^{\cdot001} - 1}{\cdot001} \times \cdot001$$

to get the necessary increment in the dependent variable.

It would be convenient therefore if $\dfrac{a^{\cdot001} - 1}{\cdot001}$ were equal to unity.

By logarithms we find the necessary value of a is 2·718.

This is therefore a convenient base for calculation of 4-fig. logarithms. Moreover, for higher accuracy we might take as base $Lt(1+h)^{\frac{1}{h}}$ when $h = 0$: for increment δx (very small) of x, y increases by $y\delta x$ approx.

Thus the connection between the foregoing method and the ordinary is exhibited; while the essential property of the exponential function, viz., that its rate of increase is equal to its own value, for any value of the independent variable, is seen to be intimately associated with both methods.

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W. VAUGHAN JOHNSTON.

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