A UNIFIED TREATMENT OF THE FILAMENT AND FLARE INSTABILITIES*

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ABSTRACT

Filaments and flares occur in sheared magnetic structures as a result of radiative cooling and resistive reconnection, respectively. A new integrated theory of these two unstable processes is described, which includes the relevant effects of magnetohydrodynamics and energy transport. The normally dissociated thermal and tearing phenomena are coupled together by a temperature-dependent Coulomb resistivity. As a result, the filamentation and flaring instabilities of a sheared field may coexist, as is familiar from the solar example.

The growth rates and spatial structures of these two modes are detailed here. The much faster radiative instability is shown to provide significant magnetic reconnection, particularly at shorter wavelengths. The long-wavelength reconnection mode is found to be abetted by the resistivity increase caused by the dominance of cooling at the X point, in contrast to its nonradiative behavior. Implications of these results for the development of coronal activity are described.

INTRODUCTION

It is well known, as exemplified by the development of solar activity, that increasing magnetic-field shear or stress gives rise to the formation of filaments (Chiuderi and Van Hoven, 1979) and to flares (Van Hoven, 1979). Theories of the former mechanism (Field, 1965), which is driven by a radiation ouput that decreases with temperature, have usually ignored the resistive magneto- hydrodynamic effects of the resulting, very collisional, relatively low-temperature plasma. Treatments of the latter

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reconnection instability (Furth, Killeen, and Rosenbluth, 1963), which is catalyzed by finite conductivity, have usually ignored the energy-flux consequences of the resulting magnetic energy release. Since, in astrophysical situations, radiation is a strong effect and the relevant resistivity is the temperature-dependent Coulomb value, these two dynamic processes should be treated in a unified way (Van Hoven, Steinolfson, and Tachi, 1983). This paper describes the outcome of such a coupled-instability study, in which the relevant linearized equations are solved numerically over a range of parameters, and interprets the most important results.

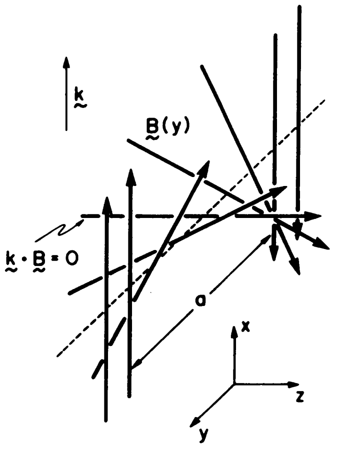

Figure 1. Sheared force-free field.

FORMULATION

We model a current-carrying volume of plasma by the sheared-field form

$$\mathbf{B}_o = B_o[\mathbf{e}_x \tanh(y/a) + \mathbf{e}_z \operatorname{sech}(y/a)] \quad , \tag{1}$$

shown in Fig. 1, which is force-free and thus consistent with an initially uniform temperature and density. We describe the temporal development of this system by using the resistive MHD equations, relating flow velocity to magnetic induction,

$$\rho\, d\mathbf{v}/dt = \mathbf{J} \times \mathbf{B} \qquad (\mu_o \mathbf{J} = \nabla \times \mathbf{B}) \tag{2}$$

$$d\mathbf{B}/dt = (\mathbf{B}\cdot\nabla)\mathbf{v} + \underline{(\eta(T)/\mu_o)\nabla^2\mathbf{B}} \quad , \tag{3}$$

along with the incompressible energy-transport equation

$$K'\rho dT/dt = \nabla\cdot[\mathbf{B}\kappa_{\parallel}B^{-2}(\mathbf{B}\cdot\nabla)T] - R\rho^2 T^r + \underline{\eta(T)J}^2 \quad . \tag{4}$$

The notation here is the same as that in Van Hoven, Steinolfson and Tachi (1983), except that K' is Boltzmann's constant per unit mass, divided by $\gamma-1$. These equations are solved by assuming small-amplitude perturbations of the form (dictated by symmetry) $T_1(\underset{\sim}{r},t) = T_1(y,t)\exp(ikx)$, for example, where $T_1 << T_o$, the equilibrium value.

If we temporarily ignore the underlined resistive terms in (2) - (4), we obtain two uncoupled dynamic systems. The first, from (2) and (3), describes the frozen-in-field condition, and variations on the hydromagnetic time scale $\tau_{hm} = a(\mu_o\rho_o)^{1/2}/B_o$. The second, from (4), provides unstable, incompressible, radiative cooling at

$$\tau_{ra} \cong \Omega_\rho^{-1} = [- rR\rho_o T_o^{r-1}/K']^{-1}, \quad \text{when } r < 0 \text{ so that the}$$

radiation falls with temperature (Hildner 1974). This unstable cooling only occurs, however, in the center of the magnetic-shear layer $y = 0$, where $\underset{\sim}{B}\cdot\underset{\sim}{\nabla} = ikB_x = ikB_o\tanh(y/a) = 0$, so that the much stronger parallel (to $\underset{\sim}{B}$) thermal conduction is ineffective (Chiuderi and Van Hoven, 1979).

Let us now restore the underlined resistive terms to (2) - (4), using the Coulomb value $\eta(T) = \bar{\eta}T^{-3/2}$ (Spitzer, 1962) as the relevant form for astrophysical applications. This added level of fidelity has two important coupling effects, beyond the superposition of (slow) resistive diffusion on the time scale $\tau_{re} \sim \mu_o a^2/\eta$. First, as is well known (Furth, Killeen, and Rosenbluth, 1963; Van Hoven, 1979), Eqs. (2) and (3) now exhibit a new reconnection, or magnetic-tearing, excitation which grows on the hybrid time scale $\tau_{te} \sim \tau_{hm}^{\nu}\,\tau_{re}^{1-\nu}$, where $2/5 \leq \nu \leq 2/3$ (Steinolfson and Van Hoven, 1983). The tearing mode, as with the radiative-cooling perturbation of (4), exists primarily in the layer $y \approx 0$, where the usually dominant first term on the right of (3) is ineffective. This local frustration of the normal frozen-in-field constraint allows the magnetic field to reconnect into a topology from which it can release its magnetic energy (Van Hoven, 1979). [For completeness, the resistive term has no significant influence (for astrophysical conditions) on the radiative phenomena described by (4) alone.]

The second primary effect of a temperature dependent resistivity is the coupling together of the magnetodynamic system (2,3) and the energy transport of (4) through the underlined Ohmic diffusion/heating terms. The consequences of this new interaction form the principal topic of this paper.

RESULTS

The coupled dynamic system (2) - (4) exhibits two, growing, small-amplitude excitations, a (mainly) radiative mode and a tearing-like mode. We solve the linearized equations by using a finite-difference scheme (Killeen, 1970), with the addition of variable grid spacing (Steinolfson and Van Hoven, 1983) to resolve the sharp central gradients which arise in this problem. The growth rate ω is measured from $\partial T_1/T_1\partial t$, when this quantity becomes uniform throughout the x-y grid, at which time various eigenfunction profiles and topographic plots are produced.

The input parameters and output quantities are expressed in normalized terms. These include the wavenumber $\alpha = ka$, the magnetic Reynolds number τ_{re}/τ_{hm}, and the ratio of the equilibrium radiation to Ohmic heating

$\varepsilon = R\rho_o^2 T_o^r/\eta_o J_o^2 = -\beta\Omega_\rho\tau_{re}/3r$ (with $\gamma = 5/3$ and β the ratio of plasma to magnetic pressures).

Growth rates ω are given either in terms of the hydromagnetic time, $\bar{p} = \omega\tau_{hm}$, or of the resistive time, $p = \omega\tau_{re}$, convertible by a factor of S. The magnetic Reynolds number S is artificially scaled (Van Hoven, Steinolfson, and Tachi, 1983) by the coefficient $\bar{\eta}$ of the Coulomb resistivity, with the equilibrium temperature T_o held constant. Thus, $S = S_c\bar{\eta}_c/\bar{\eta}$, where the subscript c denotes the classical (or correct) value. Since the parameter $\varepsilon \propto \bar{\eta}^{-1}$, it also varies as $\varepsilon_c S/S_c$.

An illustrative run of growth rates $\bar{p}$ vs S, for $\alpha = 10^{-1}$ is displayed in Fig. 2. Under conditions of large S ($S_s = S_c$ is the classical solar coronal value), the radiative filamentation) mode grows ~100x more quickly than the tearing (flare) mode, and the character of the eigenfunctions ($\underset{\sim}{B}, T, \underset{\sim}{v}$) is quite different, as will be described in what follows. As the value of S drops, the ratio of radiation loss to Ohmic heating falls to unity with ε, and the mode profiles become quite similar. Below $S \approx 10^{7.3}$, where the curves meet, the growth rates are complex.

A more general plot of the growth rate, normalized to the resistive-diffusion time, is shown in Fig. 3, where the α (wavenumber) dependence is emphasized. [The physical conditions here, which can affect the normalized magnitudes, are $T = 10^6$K, $n = 10^{10}$, $B = 83$G and $a = 10^7$ cm.] The solid curves specify the growth rates of the tearing mode, and the dashed line segments those of the radiative

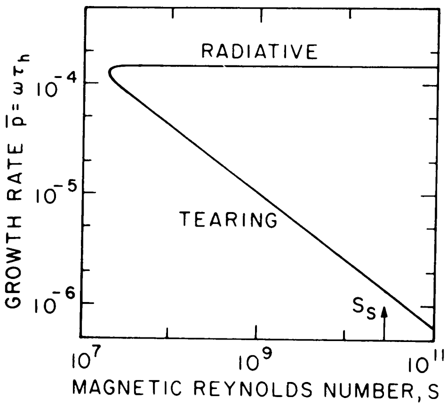

Figure 2. Growth rates at $\alpha = 0.1$.

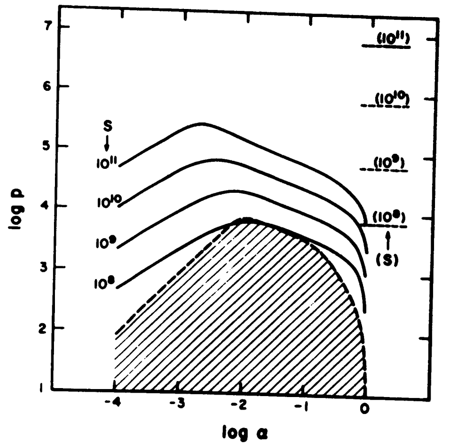

Figure 3. Growth rates vs wavenumber.

mode (Van Hoven, Tachi and Steinolfson, 1983). The growth of the latter mode is dispersionless over this range of (relatively long) wave-lengths and so is not shown in detail. [For solar coronal parameters, the growth rate is independent of parallel (to $\underset{\sim}{B}$) heat flow and does not drop until $\alpha \sim 10^{-2}$ where perpendicular thermal conduction becomes important.] The cross-hatched area is the regime of strong radiative-to-tearing mode coupling (as in the $S < 10^{7.3}$ range of Fig. 2), in which the growth rate becomes complex.

For S values above 10^9, the tearing mode growth is unchanged from the normal (non-radiative) case. That is, $p \sim S^{-\nu}$, where $\nu = 2/3$ for (fixed) α to the left of the peak, $\nu = 1/2$ at the peak values of α, and $\nu = 2/5$ to the right (Steinolfson and Van Hoven, 1983). The growth rate of the radiative mode can be expressed as $p \approx -2\ (\gamma-1)\ r\ \varepsilon/\beta$, which is equivalent to the well-known result $\omega \approx \Omega_\rho$ (Chiuderi and Van Hoven, 1979).

Some of the more interesting aspects of this coupled reconnection/radiation system arise when one looks at the structure of the eigenfunctions (Steinolfson and Van Hoven, 1984). Figure 4 shows a situation in which each of the two modes (at S_s) has individually (orthogonally) grown to a level in which nonlinearity would just start to become important. [The sum of the nonlinear terms, computed but not used in the dynamic equations, is a fixed fraction of the largest linear term; this measure of equivalency is approximately the same as that of equal linear energy content.] The plots of Fig. 4 refer to the coordinate system of Fig. 1, and show the x-y halfplane (on its side) as seen from the z direction. The horizontal (x) scale is linear and shows one wavelength, and the vertical scale ($y/a = 10^{-2}$ at the top) is expanded near the origin to better show the details in this narrow layer.

The top plot (a) of Fig. 4 shows the magnetic flux function, or B_x-B_y field lines, artificially amplified by a factor of 10^2 so that the fine structure can be appreciated. On the left is the well-known X-point field pattern of the tearing mode (Furth, Killeen, and Rosenbluth, 1963). A new result is shown at the right. At an equivalent energy level, the fast radiative mode provides 30% as much magnetic reconnection as the tearing mode, with a similar potential for magnetic energy release. Part (b) displays the z-directed current density which, when multiplied by η_o, provides the parallel (to $\underset{\sim}{B}$) electric field intrinsic to the reconnection process. The peak levels shown (with respect to J_o) are small, at this amplitude, but the radiative-mode $E_\parallel$ field is relatively 35x larger.

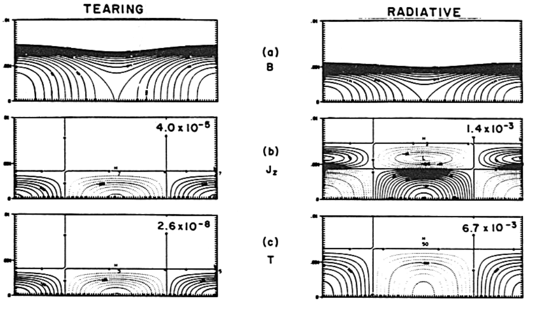

Figure 4. Mode structures at equivalent levels.

Part (c) of Fig. 4 shows the isotherms of the perturbations, with the dotted curves indicating negative values. The strong temperature reduction in the center of the radiative mode was expected, but the moderate cooling of the X point of the tearing mode is significant. This behavior is opposite to that in the absence of radiation (Tachi, Steinolfson, and Van Hoven, 1983), when the locally concentrated Ohmic dissipation heats the plasma and can reduce the level of Coulomb resistivity which catalyzes the reconnection.

DISCUSSION

This paper has described the first unified treatment of the resistive filamentation/radiation and flare/reconnection instabilities. The growth rates and excitation structures of the coupled linear modes in a sheared magnetic field (Van Hoven, Steinolfson, and Tachi, 1983), as determined from numerical computations, have been detailed. [An alternative, approximate, analytic calculation of the growth rates has been given by Steinolfson (1983)].

The growth-rate results show a strong coupling of the modes below a minimum value of the magnetic Reynolds number. However, for solar coronal $\underline{S}$ conditions, the growth rates are similar to their uncoupled values, which are separated by a factor of order 10^2, with the radiative mode being the faster. For $T = 10^6$ K, $B = 10^2$ G, $a = 10^7$ cm and $n = 10^9 - 10^{10}$, the growth time of the radiative mode is 3 - 15 minutes (Van Hoven, Tachi, and Steinolfson, 1983).

Although the growth behavior is not much modified under high-$\underline{S}$, sheared-field, astrophysical conditions, the individual mode structures are significantly changed. The tearing-like mode, as shown in Fig. 4, exhibits a negative temperature perturbation at the X point. This is the result of the strong radiation dominance at high $\underline{S}$, and indicates that the ηJ^2 heating accompanying tearing cannot lower the local Coulomb resistivity. Thus, the magnetic reconnection can proceed to higher levels, and further energy release, without the self-quenching or slowing-down that a thermally reduced resistivity would provide.

What is more important is the fact that the radiative mode has a previously unknown magnetic-reconnection component (Steinolfson and Van Hoven, 1984). This cooling mode is 30x times faster than, and provides 30% of the flux reconnection of, the normal tearing mode. This radiative excitation, therefore, has the potential of providing magnetic energy release on a much shorter time scale than previously believed possible.

The results of this paper show that there is a more fundamental connection between filaments and flares than the fact that they occur at the same magnetic-field site in the central ($\underset{\sim}{k}\cdot\underset{\sim}{B} = 0$) layer of Fig. 1. Radiative filament formation can lead directly to magnetic energy release and thus to flares. Normal resistive flare reconnection is also aided by radiative cooling, so that it may go beyond the limitations due to self heating. The final characterization of the connection between these two instabilities must come from nonlinear computations, which are under development.

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DISCUSSION

Mahajan: In the presence of tearing modes, sharp gradients can build up. In that case can one justify using the simple Ohm's law $E = \eta J$? Anomalous viscosity could be a very important term, and is shown to be so in Tokamaks. In your range of parameters, what are the effects of viscosity?

Van Hoven: We have included viscosity in other calculations. The effects on the growth of the modes are small (factors of 3) and only occur at the shorter wavelengths.

Sturrock: What solar phenomena do you attribute to this radiatively enhanced reconnection process?

Van Hoven: The answer to this question will come from the nonlinear behavior. Radiative reconnection may initially favor short wavelengths (compressibility effects indicate this) and thereby merely provide the smaller-scale, transverse, magnetic fields observed in prominences. Such excitations would saturate early, as is known from the tearing mode, because of the free energy invested in bending the nearby frozen fields. Then, after a time of continuing shear increase, a long-wavelength radiative reconnection mode would take off, which would have much better access to the stored magnetic energy (again, by analogy with normal tearing) and thus provide the energy release of a flare. These possibilities require, I must emphasize, a nonlinear leap of the imagination.

Priest: If the electric field is 3 times bigger in the radiative mode than in the tearing mode, how do the magnetic energy releases compare?

Van Hoven: The parallel electric field in the radiative mode is 250 times larger (the value given below Fig. 4 did not include the resistivity perturbation). The reconnected fluxes (or island widths) differ by a factor of 3 which, in this linear calculation, is the ratio of the magnetic-energy perturbations. A proper evaluation of the energy release requires a nonlinear computation.

Priest: You are modelling an arcade or loop magnetic configuration by a field that varies with one variable alone and are taking boundary conditions along a second direction. What are the boundary conditions in the third direction? Would not thermal conduction along the field in that direction be important?

Van Hoven: For the formation of a filament, conduction must be suppressed so that one requires the distance (ℓ) to the boundary in the z direction to satisfy $\ell^2 >> R\ell^2 T^{r-1}/\kappa_{\parallel}$.

Migliuolo: What is the role played by compressibility in radiative and tearing modes?

Van Hoven: The compressibility influence on tearing modes is negligible; the radiative growth is increased by a factor of 2 (for coronal parameters) for ka > 1, before it falls at ka > 10^2 because of perpendicular thermal conduction.

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