

research nor the practical ice services can now forgo the combined use of synoptic weather maps and synoptic oceanographical charts in conjunction with the use of ice charts.

The difference between the older working methods and the new course followed by sea ice research now becomes clear. The causality of phenomena gradually gains in importance. In consequence of the new course new problems and new methods are appearing. Specific laws governing the formation of ice are established, and a specific terminology will necessarily follow. With greater knowledge of the internal relationship between ice formation and ice metamorphosis, a uniform terminology will be evolved and scientifically based ice forecasts will provide additional security for navigation. With the establishment of specific laws and concepts, sea ice research will attain equality of status with glacier research as a special branch of science.

REFERENCES

1. Blüthgen, J. Probleme der Vereisung von Meeresräumen. *Die Naturwissenschaften*. Bd. 26, 1938, p. 618-24.
2. Büdel, J. *Atlas der Eisverhältnisse des Nordatlantischen Ozeans und Übersichtskarten der Eisverhältnisse des Nord- und Südpolargebietes*. Hamburg, Deutsches Hydrographisches Institut, 1950.
3. Zukriegel, J. *Cryologia Maris*, Travaux Géographiques Tchèques, Institut Géographique de l'Université Charles IV, Prague, 1935, 177 p.
4. Granqvist, G. *Über das Studium der Eisverhältnisse der Ostsee*. Riga, Baltische Hydrologische Konferenzen, 1937.
5. Hydrographic dictionary compiled by the International Hydrographic Bureau. Second edition. *International Hydrographic Bureau. Special Publication*, No. 32, 1951, [vi, 178, lxxvii] p.
6. World Meteorological Organization. Commission for Maritime Meteorology. *Abridged final report of the first session, London, 14th-29th July, 1952*. Geneva, Secretariat of the World Meteorological Organization, [1952?].

OBSERVATIONS IN A COLD ICE CAP

By R. HAEFELI and F. BRENTANI

(Continued from Vol. 2, No. 18, 1955, p. 571-81)

PART II

IV. RELATIONSHIPS OF STRESS AND TEMPERATURE

The stress, movement, temperature and viscosity conditions in cold ice affect each other very closely.

1. *Stress Relationships*

In the purely schematic illustration in Fig. 11 (p. 625) some of the prominent features of stress and temperature variations in the ice cap are indicated in general terms in connection with the progress of movement. On examining the normal vertical and horizontal stresses σ_y and σ_z , a distinction must be made between states of stress before and after the formation of longitudinal crevasses. Before the formation of cracks, the distribution of horizontal stresses σ_y along a vertical in the region of the division of movement should be qualitatively similar to Diagram 4 (Fig. 11). The horizontal tensile stresses in the centre of the cap are influenced both by the specific boundary conditions and by the variations of viscosity in a vertical direction.

The concentration of tensile stresses in the central zone of the cross-section leads to the observed formation of longitudinal crevasses, whereupon the picture of stress is radically altered; the horizontal tensile stress is in places reduced to zero, so that in certain zones a bi-axial state of compression arises (σ_x and σ_z) with σ_z as the overburden pressure and σ_x the compressive stress perpendicular to the plane of the figure. The gradual filling up of the cracks with water gives a lateral water pressure, which produces yet another radical alteration in the state of stress and deformation of the cold ice, and also gives periodic fluctuations, depending upon the height of the water level.

Technically the possibility now arises of draining the cracks, as was done during the construction of the cross-tunnels, and thus of influencing the state of stress and movement in the cap in a particular manner and of slowing down the horizontal movement to both north and south.

Except for a short section of method (b), we here limit ourselves to an explanation of methods (a) and (c), using the measurements of strain made in cross-tunnel Q₁₂₀.

(a) If one examines a prismatic ice lamella (Figs. 11 and 12) in the region of the division of movement,* bounded on both sides by longitudinal cracks, then the horizontal principal stress $\sigma_3 = \sigma_y$ in the plane of the picture can be approximately put equal to zero, so that in this plane (y-z), the vertical pressure from above alone acts as principal stress $\sigma_1 = \sigma_z$. If one ignores the insufficiently well known, yet relatively small alterations in length perpendicular to the plane of the picture, this corresponds to the supposition that the so-called rest pressure is effective in the x-direction (neutral zone).

With these assumptions, the apparent viscosity of the ice can be calculated as follows (Fig. 12):

$$\sigma_3 \sim 0; \mu_m = \frac{\tau}{\omega} = \frac{\sigma_1}{2\omega}; \quad \omega = \frac{dy}{dt} \sim \frac{\Delta l}{l} \cdot \frac{1}{\Delta t} = \epsilon_y \quad (3)$$

The angular rate of change ω is thus identical with the rate of strain ϵ_y of the cross-tunnel. If we substitute the value of ϵ_y which was taken during the second period of measurement (see Part I, p. 578), while σ_1 is estimated as the overburden pressure of 28 m. of ice and firn at an estimated mean density of $\rho_i = 850 \text{ kg./m.}^3$, then it follows that:

$$\begin{aligned} \sigma_1 &= 850 \times 28 = 24 \times 10^3 \text{ kg./m.}^2 \\ \epsilon_y &= 0.0087\% \text{ per day} \sim 1.0 \times 10^{-9} \text{ sec.}^{-1} \\ \mu_m &= \frac{\sigma_1}{2\epsilon_y} = \frac{24 \times 10^3}{2 \times 10^{-9}} = 12 \times 10^{12} \text{ kg. sec./m.}^2 \\ &= 11.8 \times 10^{14} \text{ poise} \quad (4) \end{aligned}$$

(b) The measurements in circular profile K₂ (~2.6 m. diameter) do not cover a sufficiently long period of time to provide reliable results. An initial measurement period of 90 days showed that the deformed circular profile resembled an ellipse, from which it follows—contrary to our expectations—that the state of stress in the plane considered is not hydrostatic. The orientation of the two axes of the ellipse indicates the direction of the standard principal stresses, while the ratio of the principal stresses can be deduced from the change in length of these axes. A rough application of the relations given under (c) showed in the main that the apparent viscosity of the ice may here reach 6×10^{14} poise.

For the sake of comparison, it should be mentioned that in the circular profile of the Z'Mutt tunnel, i.e. in temperate ice, the following values for the apparent viscosity were discovered (see Ref. 1, Part I, p. 580):

$$\begin{aligned} \text{Pressure from above } p &= 39 \text{ t./m.}^2, \mu = 2.5 \times 10^{14} \text{ poise} \\ \text{,, ,, ,, } p &= 22 \text{ t./m.}^2, \mu = 7.2 \times 10^{14} \text{ poise.} \end{aligned}$$

The dependence of the μ value upon the pressure from above, or the shear stress, is conditioned by the exponential character of the stress-strain curve (cf. Fig. 12)^{10, 11, 24}.

(c) In the deformation of the circular profile K₁ (2.9 m. dia.) observed in the longitudinal tunnel, the horizontal measurement increased somewhat to begin with, but then shrank again approximately to its original value. From this, a certain time variation of horizontal side pressure is indicated (cf. Fig. 8, p. 579 of Part I).

The general calculation of deformation in a circular profile under the influence of a vertical pressure from above p_1 and a horizontal pressure p_2 gives the following radial alterations in length for an elastic material¹³:

$$\text{Vertical:} \quad \Delta r_1 = \left[\frac{2}{3} p_1 - \frac{1}{6} p_2 \right] \cdot \frac{r}{G}, \quad (5)$$

where G is the shear modulus.

* Absolute zero point of horizontal displacements.

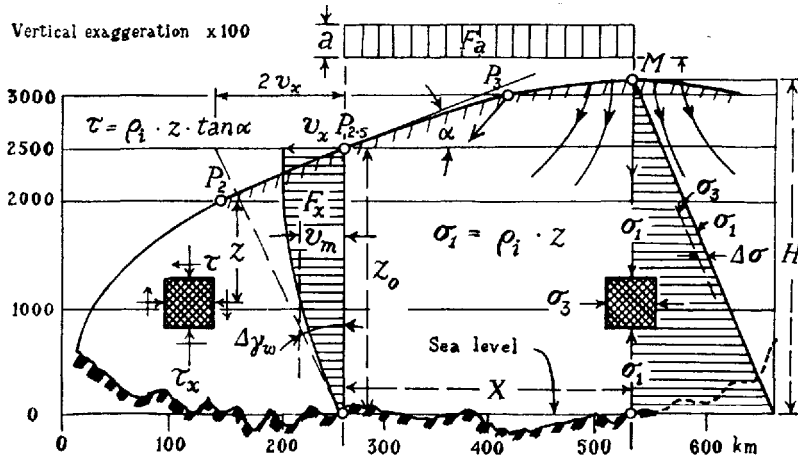


Fig. 13. Diagrammatic representation of the relations in the Greenland ice sheet. (Profile after Expeditions Polaires Françaises, Missions Paul-Emile Victor. Vertical exaggeration $\times 100$).

At the division of moment $\Delta\sigma \sim 2\mu a/H$; ρ_i = Mean density of ice.
 The balance equation is, Area $F_x = F_a$; $v_m = a \cdot x/z_0 = \alpha \cdot v_x$.

For F_x a parabola: $v_x = 3/2 \cdot \frac{ax}{z_0}$; $\mu_m = \tan \alpha \cdot \rho_i \cdot z_0^3/3a \cdot x$

deviator of about 60 kg./m.², which has no great practical significance. In the ice centre, therefore, there exists almost a hydrostatic state of stress.

On the other hand, it follows that a very small deviator $\Delta\sigma$ suffices to make possible a stationary condition in the ice centre.

Whereas the horizontal shear stresses at the division of movement are $\tau_{xz} = 0$, this is not the case in any general vertical section (P) through the cap because of the tilting of the surface. Here τ_{xz} is the more decisive factor for the progress of movement, and with a horizontal rock base and a small angle of slope α of the firn surface, the following equation is valid 14:

$$\tau_{xz} \cong \rho_i z \tan \alpha \quad (11)$$

If one supposes the average firn accumulation a from M to P to be known, then from considering the balance it follows in terms of the quantities defined in Fig. 13 that:

$$F_x = F_a; v_m = \frac{ax}{z_0} = \lambda \cdot v_x \quad (v_x = \text{surface velocity}) \quad (12)$$

This equation for the average cross-section velocity v_m in the stationary state of the cap is generally valid, i.e. it is wholly independent of the particular form of the velocity profile or the question of whether or not slipping on the base takes place (pure condition of continuity).

For the simple assumption of a parabolic velocity profile, which would correspond to a constant viscosity of the medium without slipping on the base ($v_u = 0$), the following relationships are valid:

$$v_x = \frac{3}{2} \frac{ax}{z_0}; \mu_m \sim \tan \alpha \cdot \rho_i \cdot \frac{z_0^3}{3ax} \quad (13)$$

Thanks to the Greenland expedition of Paul Victor we are to-day adequately informed about the contours of the rock base in Greenland (see Ref. 4, Pt. I, p. 580). The following numerical example is based on the profile contained in *Rapport Préliminaire* No. 15, *Série Scientifique* (Campagne 1950). For a hypothetically selected mean annual accumulation of $a = 0.3$ m. of ice, the average values set out in Table II work out as follows (Equations 13 and 14):

$$\dagger \omega = \frac{\Delta y}{\Delta t} \sim \frac{\tau_{xz0}}{\mu_m} = \frac{2v_x}{z_0}; v_x \text{ from Eqn. (13), } \tau_{xz0} \text{ from Eqn. (11)} \quad \mu_m \sim \frac{\tau_{xz0} \cdot z_0}{2v_x} = \tan \alpha \cdot \rho_i \cdot \frac{z_0^3}{3ax} \quad (14)$$

TABLE II. ICE SHEET MOVEMENT AND VISCOSITY RELATIONS
(for slipping velocity on the rock bed $v_u=0$)

Point	a m./yr.	z_0 m.	x km.	$\tan \alpha$ %	v_m m./yr.	$v_x=1.5v_m$ m./yr.	τ_{xz_0} t./m. ²	μ_m poise
1	0.3	2000	385	0.54	58	87	9.7	$\times 10^{14}$ 3.5
2	0.3	2500	270	0.33	33	50	7.5	6
3	0.3	3000	115	0.22	11.5	17	6.0	16

τ_{xz_0} = shear stress on the bottom of the Ice Sheet. $\rho_i=0.9$ t./m.³ (1 t./m.² ~ 0.1 bars)

With regard to magnitude, these apparent viscosities agree with the values reached for the cold ice cap on the Jungfrauoch. From this it follows that the movement in the firn area of inland ice can also be conceived as occurring without slipping on the base, *i.e.* only as a result of the plastic deformation (creep) of the cap.

We are fully aware that the comparison of the small ice cap on the Jungfrauoch with the enormous inland ice mass, whose greatest thickness is 60–70 times and breadth up to 3000 times larger, must be regarded at least as daring. But the following considerations may be borne in mind to assist such a comparison.

In the light of recent glaciological research, the plastic properties of polycrystalline glacier ice, or its apparent viscosity, seen from a crystallographical point of view, depends chiefly upon the following factors: on the one hand upon the significant shear stress and on the other upon the temperature of the ice. As far as the shear stresses are concerned, the difference between Greenland and the Jungfrauoch is of no consequence, for the greatest shear stress on the top of the ice tunnel, which, with no side pressure, corresponds to half the pressure from above, is of the same magnitude as the shear stress along the rock base of the inland ice (Table II).^{*} On the other hand, the normal stresses, or hydrostatic pressures, differ widely; yet up to the present, no resulting influence of hydrostatic pressure upon the plastic behaviour of cold ice can, according to Steinemann, be confirmed.

With respect to ice temperature, we are in both cases concerned with cold ice, although the mean temperature of the ice on the Jungfrauoch is appreciably higher than in central Greenland. The relatively high value of viscosity for Point 3 of the inland ice (Table II) indicates a low mean ice temperature, together with small shear stresses.

Following the seismic examination of the ice sheet by the Expéditions Polaires Françaises (Missions Paul-Émile Victor) and the relatively high speed of seismic waves recorded thereby (*ca.* 3800 m./sec.), J.-J. Holtzscherer also comes to the conclusion that the central inland ice is probably cold down to the base: that is to say, its temperature throughout lies below the point of pressure melting and frost penetrates down to the bottom (see Ref. 4, Part I, p. 580). Robin reached a similar conclusion, proving theoretically by a study of heat flow that with an annual accumulation of 30 cm. of ice, the temperature of the rock base in the central zone of the Greenland ice sheet lies well below the point of pressure melting, which under a thickness of 3000 m. of ice is about -2° C.¹⁵, while for a surface temperature of -29° C. the calculated bottom temperature according to Robin is about -12° C. From seismic shooting information Holtzscherer estimates the basal temperature near the centre of the ice sheet at around -10° C.

Quite independently of the above-mentioned seismic and thermodynamic examinations, the results gained from the ice cap on the Jungfrauoch offered the possibility of shedding light upon the Greenland problem from a third angle, namely on the basis of glacier movement. The attempted extrapolation of the small ice cap onto the Greenland ice sheet leads one to presume that in central Greenland there exists a core in which the ice does not slip on the base but is frozen hard to it, where the resultant shear stresses lie well below the shear strength. This would mean that the 0° isotherm in the core dips below the glacier bottom (Permafrost) corresponding to the above-mentioned hypotheses based on seismic and thermodynamic examination^{4, 15}.

^{*} The division of movement in central Greenland is an exception; here the shear stress ($\tau_{xz_0} \sim 0$) is very small.

The fact that three different methods lead to basically the same result should not allow one to overlook the provisional nature of the extrapolation of the results obtained from the Jungfraujoch. The deviation of the actual three-dimensional motion from the above, where a plane creep motion has been assumed, requires further clarification. The question of the influence of pressure from all sides upon the ability of the ice to deform is vital for the justification of such an extrapolation. In this connection, mention must be made of the latest experiments of S. Steinemann, where superposition on a shear stress of a hydrostatic pressure up to 90 kg./cm.² reveals practically no influence of this pressure upon the plastic behaviour of the cold ice.* On the other hand we must remember that in central Greenland the overburden pressure of the ice reaches 300 kg./cm.².

The observation and measurement of smaller and more easily accessible natural objects, such as for example the ice cap of the Jungfraujoch, offers the possibility, among other things, of acting as a link between arctic exploration on the one hand and research in the ice laboratory on the other, so as to further our knowledge of nature by direct observation¹⁶⁻²³.

Acknowledgements

Finally we would not like to forget to record our thanks to the general directorate of the Swiss Post, Telegraph and Telephone authorities (P.T.T.) and to the director of the Versuchsanstalt für Wasserbau & Erdbau, ETH, for permission to publish the above material and for their energetic support, to the engineering offices of Rothpletz, Lienhard & Cie., as well as to Herren Ing. P. Kasser, G. Amberg, H. Röthlisberger and H. Wiederkehr for their assistance, to the Hochalpinen Forschungsinstitut auf Jungfraujoch for their kind hospitality, to Mr. S. W. Martin for his translation of the original paper, and to Dr. J. W. Glen for his careful preparation of the translation for publication.

The division of the work between the authors of this paper is as follows: the first-named technical adviser to the P.T.T. took over the leadership and valuation of the glaciological examinations, while the second carried out the geodetic measurements as local leader of the tunnel construction work.

M.S. received 18 November 1954

* Publication in preparation.

REFERENCES

References 1 to 7 will be found at the end of Part I of this paper, *Journal of Glaciology*, Vol. 2, No. 18, 1955, p. 580.

8. Seligman, G. Forschungsergebnisse am Grossen Aletschgletscher. *Die Alpen*, Jahrg. 19, Ht. 12, 1943, p. 1-8.
9. Perutz, M. F. Report on problems relating to the flow of glaciers. *Journal of Glaciology*, Vol. 1, No. 2, 1947, p. 47-51.
10. Nye, J. F. The flow law of ice from measurements in glacier tunnels, laboratory experiments and the Jungfraujoch borehole experiment. *Proceedings of the Royal Society*, Series A, Vol. 219, No. 1139, 1953, p. 477-89.
11. Steinemann, S. On the plastic behaviour of polycrystalline ice. *U.G.G.I., Association Internationale d'Hydrologie Scientifique. Congres de Rome 1954.* (Manuscript, privately circulated.)
12. Haefeli, R. Creep problems in soils, snow and ice. *Proceedings of the Third International Conference on Soil Mechanics and Foundation Engineering*, Zürich, 1953, Vol. 3, 1953, p. 238-51. Also published in German as *Versuchs Anstalt für Wasserbau & Erdbau, E.T.H., Mitteilungen*, No. 30, 1954, 19 p.
13. Föppl, A. *Vorlesungen über technische Mechanik*. Leipzig & Berlin, B. G. Teubner, 1920 (5 vols.), p. 355.
14. Haefeli, R. Schnee, Lawinen, Firn und Gletscher. *Ingenieurgeologie*. (L. Bendel, ed.) Wien, Springer, 1948, Bd. 2, p. 663-731.
15. Robin, G. de Q. Ice movement and temperature distribution in glaciers and ice sheets. *Journal of Glaciology*, Vol. 2, No. 18, 1955, p. 523-32.
16. Glen, J. W., and Perutz, M. F. The growth and deformation of ice crystals. *Journal of Glaciology*, Vol. 2, No. 16, 1954, p. 395-403.
17. Nye, J. F. The mechanics of glacier flow. *Journal of Glaciology*, Vol. 2, No. 12, 1952, p. 82-93.
18. Nye, J. F. A comparison between the theoretical and measured long profile of the Unteraar Glacier. *Journal of Glaciology*, Vol. 2, No. 12, 1952, 103-07.
19. Nye, J. F. Reply to Mr. Joel E. Fisher's comments. *Journal of Glaciology*, Vol. 2, No. 11, 1952, p. 52-53.
20. Haefeli, R. Observations on the quasi-viscous behaviour of ice in a tunnel in the Z'Mutt Glacier. *Journal of Glaciology*, Vol. 2, No. 12, 1952, p. 94-99.
21. McCall, J. G. The internal structure of a cirque glacier. *Journal of Glaciology*, Vol. 2, No. 12, 1952, p. 122-31.
22. Terzaghi, K., and Richart, F. E. Stresses in rock about cavities. *Harvard Soil Mechanics Series*, No. 41, 1952.
23. Bader, H. Sorge's law of densification of snow on high polar glaciers. *Journal of Glaciology*, Vol. 2, No. 15, 1954, p. 319-23.
24. Glen, J. W. The creep of polycrystalline ice. *Proceedings of the Royal Society*, Series A, Vol. 228, No. 1175, 1955, p. 519-38.