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21-CM OBSERVATIONS OF REDSHIFTS AND THEIR IMPLICATIONS FOR THE DISTANCE SCALE

R. Brent TULLY
Institute for Astronomy, University of Hawaii
and
J. Richard FISHER
National Radio Astronomy Observatory

Résumé

Il existe une corrélation étroite entre la largeur de la raie d'hydrogène neutre à 21 cm et la luminosité des galaxies spirales ainsi qu'entre cette largeur et le diamètre des galaxies spirales. Elle existe pour un échantillon de spirales proches dont la distance est calibrée par les céphéides, ainsi que pour des spirales dans quelques amas. La comparaison entre ces deux relations nous donne la distance de l'amas et une valeur pour la constante de Hubble. Pour l'amas de Virgo la méthode nous donne une distance de 14,5Mpc, donc H_0 = 77 km/s/Mpc. Pour l'amas de la Grande Ourse, une distance de 12,6 Mpc, donc H_0 = 75 km/s/Mpc. Il faut signaler qu'on trouve ces résultats avec des galaxies vues plutôt par la tranche. Il existe une différence systématique : avec les galaxies vue de face on trouve des distances plus grandes. Pour les galaxies de champ la corrélation existe, bien que la dispersion augmente. Avec 74 galaxies des types Sc-Sd et d'inclinaison 45°-75° on trouve H_0 = 80 km/s/Mpc, avec celles situées dans l'amas supergalactique on trouve H_0 = 75, et avec celles hors de l'amas supergalactique on trouve H_0 = 90 km/s/Mpc. Cette différence est significative à 2 sigma près.

HI Profile Distances

In the chain leading to a determination of the Hubble constant perhaps the last link is the weakest: the determination of distances beyond where cepheids can be seen out to where the Hubble expansion ought to dominate. The difficulties encountered with distance indicators in this domain have fostered the "if not quality then quantity" approach by van den Bergh (1975) and reiterated by Malcom Smith in the previous lecture.

It would be premature for us to announce that a superior tool for determining distances to galaxies is at hand, but we can tentatively suggest that it might be. The method we will discuss today (Tully and Fisher, 1976, hereafter Paper I) is easily applied and useful to considerable distances (say to 100 Mpc).

The method presumes that for spiral galaxies: (1) total luminosity is related to total mass, and (2) the maximum velocity of rotation is related to total mass. We then look for an empirical correlation between the luminosity and the width of the global HI profile which is a measure of the peak rotational velocity. The method is quite similar to the total mass-luminosity relationship explored by Roberts (1969) and others (references in Paper I). However, total mass requires an additional observable, a linear dimension, which adds scatter and which is itself distance dependent.

Neutral hydrogen profiles have very characteristic shapes. We see three examples in Figure 1. With giant galaxies (Fig. 1a) the profiles have sides which are very steep and maxima can be expected near the extremities in velocity. Radical departures from this pattern are generally only seen with confused or disrupted systems. At the other extreme, with dwarf galaxies (Fig. 1c), the profile might well be approximated by a gaussian.

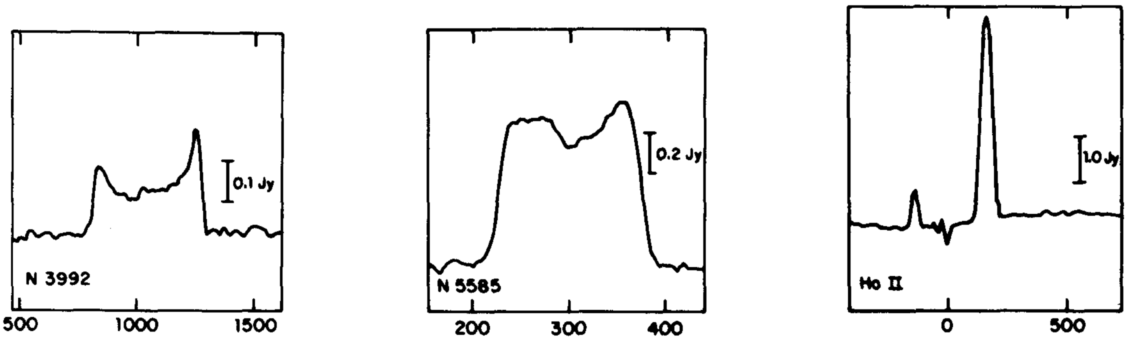

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1.—Three global HI profiles: (a) the giant spiral (SBbc I) NGC 3992 with ΔV^i = 558 km/s, (b) the small spiral (SABd IV) NGC 5585 with ΔV^i = 218 km/s, and (c) the magellanic irregular (Im IV-V) Holmberg II with ΔV^i = 126 km/s.

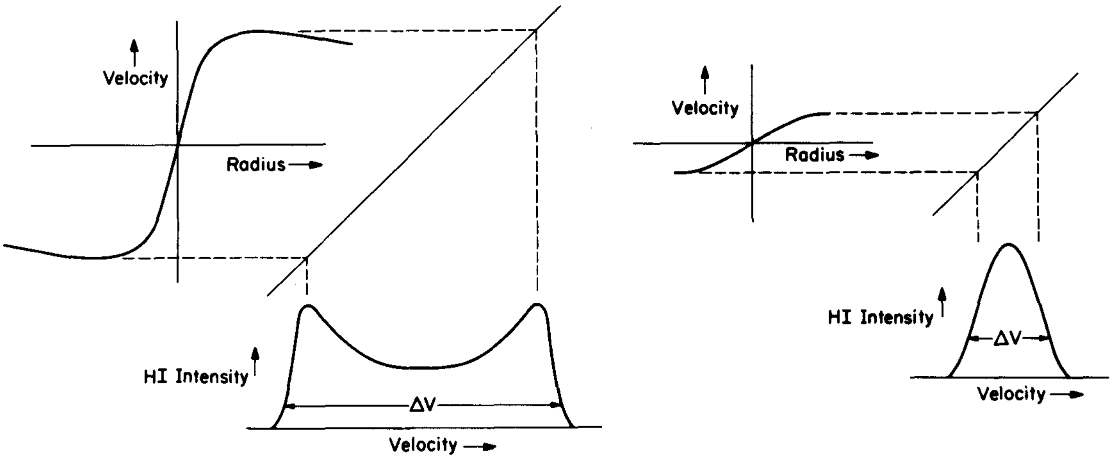

Fig. 2a

Fig. 2b

Fig. 2.—Schematic diagrams illustrating characteristic rotation curves and HI profiles for (a) a giant spiral and (b) a dwarf spiral or irregular. The profile width is defined as the full width at 20% of the maximum.

We can appreciate how these profiles arise when we look at Figure 2. A schematic rotation curve is shown for a giant galaxy in Figure 2a. The essential feature is that, after a quick rise, velocities remain near the maximum over much of the disk. There is plenty of HI toward each extreme in the global profile, and the cut-off is abrupt. By contrast, in dwarfs (Fig. 2b) the rotation curve rises much more leisurely and the maximum may even be beyond the visible disk. HI is not piled up at the extreme velocities.

In making these general statements, we reveal our intuition that galaxies with a given total mass have similar mass distributions. Clearly the relationship we will look for will be tightest if this is the case. Reason for optimism is found in the good correlation between luminosity and diameter for spirals of all types (Heidmann, 1969). Also, Freeman (1970) showed that the surface brightness of disks share a common radial dependence and a characteristic extrapolated central value.

The form of the correlation between luminosity, L, and the HI profile width corrected for projection effect, ΔV^i, can be crudely anticipated. With the notation, total mass, M_T, and radius, R, we assume:

$$M_T \sim (\Delta V^i)^2 R$$

$$M_T/L \sim \text{constant}$$

$$L \sim R^{2\cdot 8}.$$

The latter relation is given by Heidmann (1969). It follows very approximately that

$$L \sim (\Delta V^i)^3.$$

Let us now look at the relationship in several samples of galaxies. Only if distances are well known or if there are enough candidates at a common distance will it be possible to estimate the intrinsic scatter. First we will consider only those nearby spirals with distances determined by cepheids. Then we will look at the relative relations for the Virgo and Ursa Major clusters. Scaling to the local sample will give distances to these clusters. Finally, galaxies in the field will be considered. Again, the local sample will provide the absolute scale calibration.

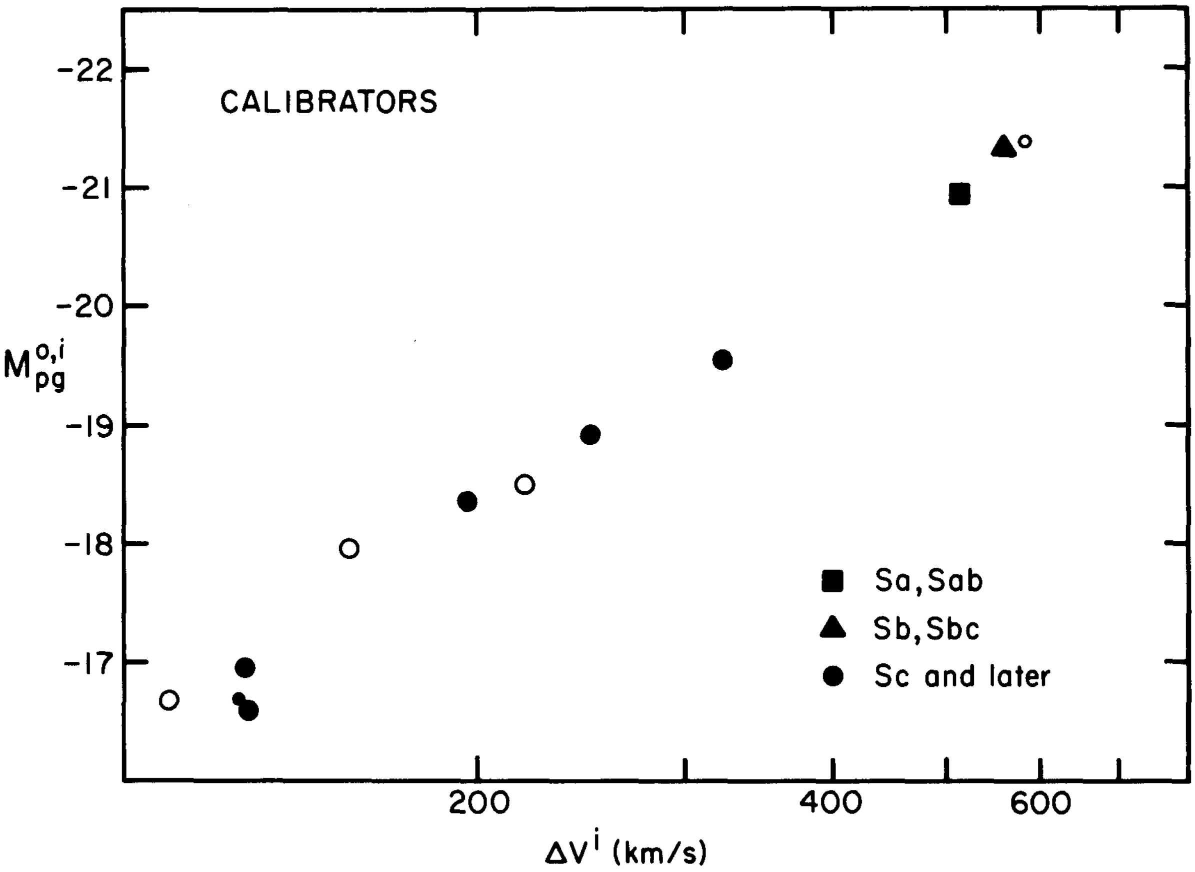

Fig. 3.—Absolute magnitude-HI profile width plot for nearby galaxies with well determined distances. Filled symbols: members of the local group and M87 group. Open symbols: members of the M101 group. Minature symbols: the face-on galaxies M101 and HoII.

The Local Calibrators

In Figure 3 we show the absolute magnitude-HI profile width diagram for nearby spirals with reliable distances. The galaxies corresponding to the filled symbols have distances tied to the cepheid scale: M31, M33, M81, NGC 2403, and smaller members of the M81 group. The galaxies represented by open symbols are members of the M101 group, for which there is a distance from a number of secondary indicators. We are accepting the scale established locally by Sandage and Tammann (Tammann and Sandage, 1968; Sandage and Tammann, 1974a, b, c, hereafter ST I-III). Apparent magnitudes and corrections for tilt and galactic absorption follow recipes by Sandage and Tammann (1976, hereafter ST VII). In fact, Figure 3 is taken from ST VII. A similar plot, differing only in a few details, is given in our Paper I.

There are too few galaxies and too much mixture of types in Figure 3 for the shape and intrinsic scatter of the correlation to be ascertained. However, we are tantalized with the possibility that the relationship may be very tight indeed.

Absolute diameters can replace magnitudes and the consequence for the local calibrators is seen in Figure 2 of Paper I.

Virgo Cluster

The shape and scatter of the luminosity-profile width correlation might be found by studying a cluster of negligible depth. Comparison with the local calibrators will provide the absolute scale and a in Paper I gave us $\mu_o = 30^{m}.6 \pm 0.2$. In ST VII, the same sample of galaxies gave $30^{m}.8$, and an augmented sample gave $31^{m}.3$. Only this latter value is compatible with the Sandage and Tammann (1974d, hereafter ST IV) Virgo modulus of $31^{m}.45$. Why do these divergent results arise?

For the sample in common to Paper I and ST VII the small difference can be attributed to (1) the choice of magnitude corrections for inclination, (2) the choice of HI profiles from the literature, and (3) systematic differences in inclination, usually small but occasionally significant. We have discussed these points in fair detail elsewhere (Fisher and Tully, 1976, hereafter Paper II). We are willing to accept the ST VII magnitude scale in the same spirit that in Paper I we accepted the ST I-VI scale. The choice of HI profiles is of minor concern (see Figure 4). Disagreements should tend to disappear as marginal signals are improved upon. However, there is a problem with inclinations.

In Figure 5 we show the relationship between apparent magnitudes and HI profile widths for the galaxies discussed in Paper I, plus one more galaxy we accept from ST VII that shared our requirement that the inclination exceed 45^{o}. The ST VII magnitudes and, in general, the ST VII inclinations and profile widths were accepted. Exceptions are noted in our Paper II. Figure 6 shows the fit to the local calibrators for the Virgo distance modulus of $30^{m}.8 \pm 0.2$ (estimated error). The rms scatter of the Virgo points about the straight line (drawn visually through the calibrators) is $\pm 0^{m}.4$.

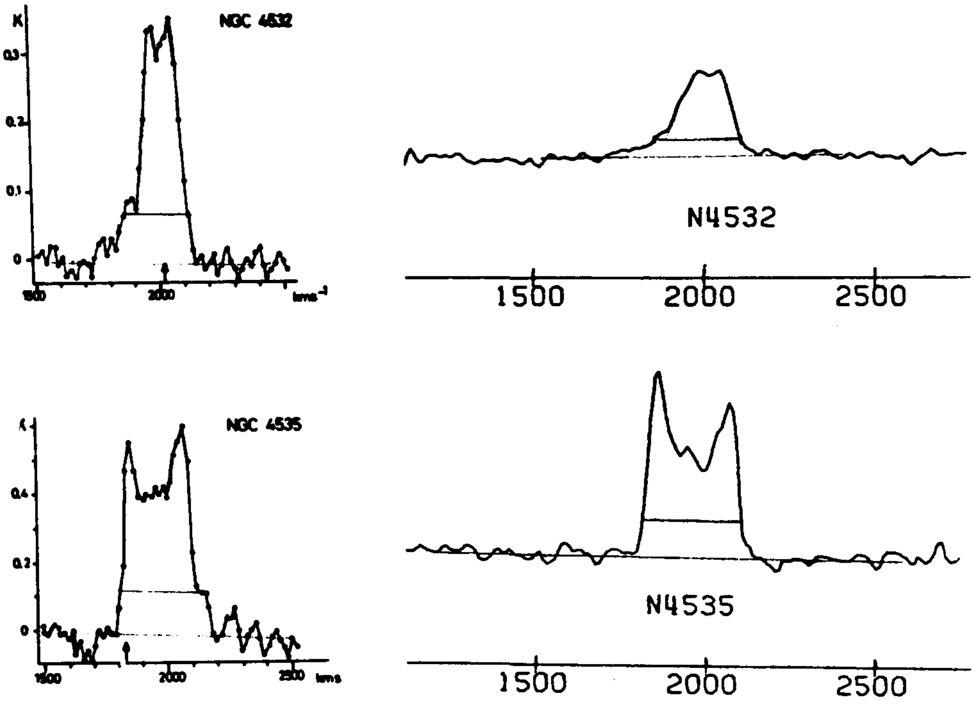

Fig. 4.—Profiles by Huchtmeier, et al (1976) and ours (right) are shown for each of two galaxies. NGC 4532 has a wing on the low velocity side which is seen in both profiles. The feature must be real, but it certainly makes a spurious contribution to the HI profile width. NGC 4535 has a wing on the high velocity side on one profile only, so the feature is probably not real.

If all the galaxies discussed in ST VII are accepted, the result is seen in Figure 7. The same straight line now represents a distance modulus of $31^{m}.3 \pm 0.4$ (estimated error), with the rms scatter now being $\pm 0^{m}.9$. We notice that the new galaxies added to the discussion, galaxies all more face-on than 45°, scatter systematically below and to the right of those shown in Figures 5 and 6. All galaxies more edge-on than 45° lie on or above the straight line while all galaxies more face-on than 30° lie below it!

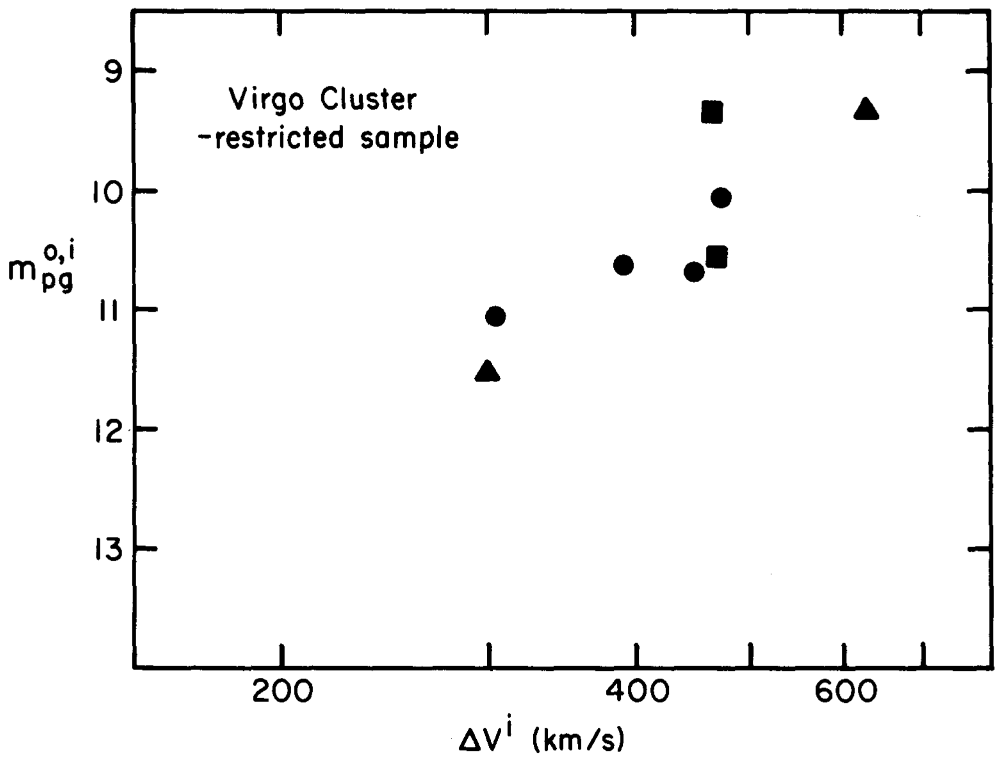

Fig. 5.—Apparent magnitude-HI profile width diagram for Virgo galaxies more edge-on than 45° (one exception; see Paper II).

The tightness of the edge-on sample argues against acute difficulties in the magnitude tilt corrections. The largest corrections, and hence differential uncertainties, lie with these edge-on galaxies.

We suggest that there are systematic errors in inclinations derived by taking the ratio of the minor to major optical axis. In a few special cases where spiral structure is well defined we have found that rectification of this spiral structure suggests inclinations to be considerably more edge-on than given by $\cos i \sim b/a$. However, we do not presently understand why such a systematic error might occur. Our temporary conclusion is that the inclinations for the face-on systems ($i < 45^\circ$) are not to be trusted. There could be systematic errors in the more edge-on sample through the choice of inclinations, but, if so, they are not likely to affect the Virgo modulus by more than $0^m.2$ or so.

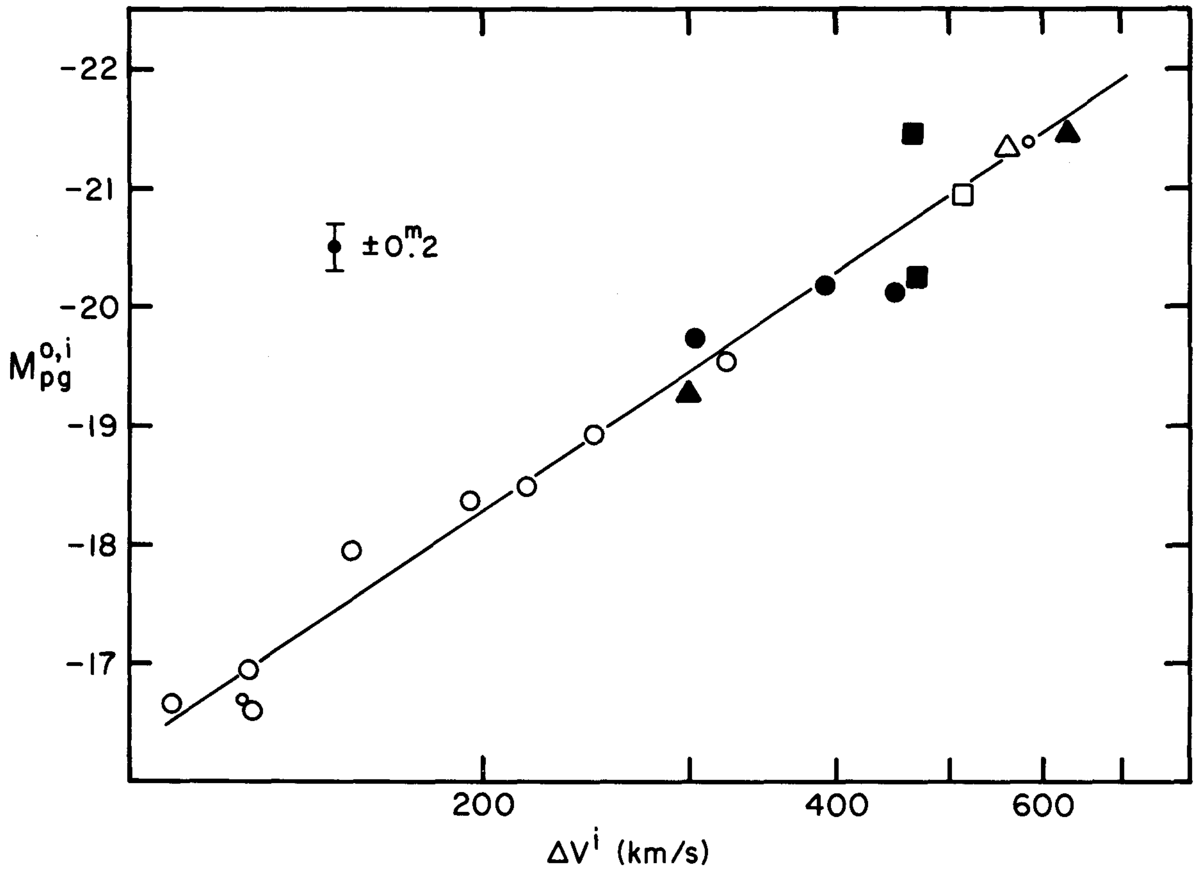

Fig. 6.—The best fit of the edge-on Virgo sample to the nearby calibrators corresponding to a Virgo distance modulus of $30^m_{\cdot}8$. Filled symbols from Figure 5; open symbols from Figure 1. The straight line is visual fit through the open symbols. It corresponds to the relation: $L \sim (\Delta V^i)^{2.7}$.

In Figure 4 of Paper I we show the apparent diameter-profile width relation for the edge-on sample in Virgo. In Figure 6 of Paper I these points are scaled to the local calibrators to derive a Virgo distance. The results are the same as with the magnitude relations, albeit with more scatter. The edge-on sample ($i > 45^o$) gives a distance modulus of $30^m_{\cdot}8 \pm 0.4$. More face-on galaxies scatter below and to the right and so tend to push the distance modulus up.

For the moment we accept a Virgo modulus of $30^m_{\cdot}8 \pm 0.2$, corresponding to a distance of 14.5 ± 1.5 Mpc. If the systemic velocity of the cluster is 1111 km/s (ST IV) then a Hubble constant of 77 km/s/Mpc is suggested.

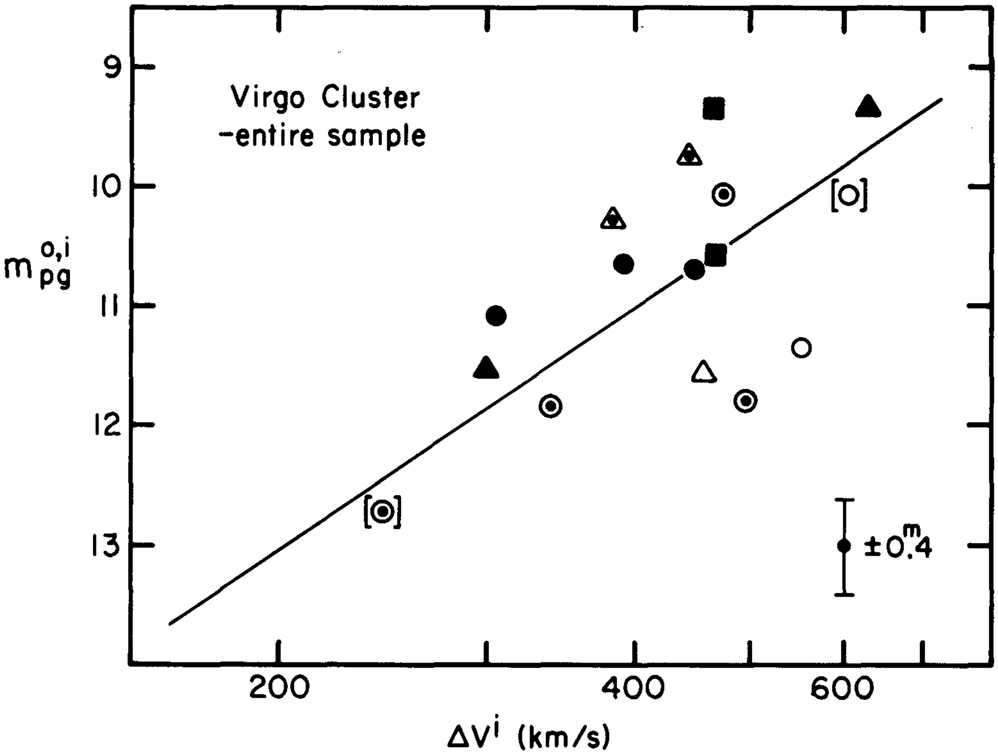

Fig. 7.—Apparent magnitude-HI profile width diagram for the entire ST VII sample of Virgo galaxies as accepted in Paper II. Solid symbols, $i > 45^o$; open symbols with dot, $30^o < i < 45^o$; open symbols, $i < 30^o$. The straight line is the same as seen through the calibrators in Figure 6, transposed to correspond to a Virgo modulus of $31^m.3$.

Ursa Major Cluster

The Ursa Major cluster is a little studied entity at roughly the same distance as the Virgo cluster and with a similar angular extent. De Vaucouleurs (1975) refers to it as two groups, U Ma N (his group N°34) and U Ma S (his group N°32), but we now have 44 redshifts and find no reason to think that there is other than a single cluster. The composition of U Ma is very different from Virgo in that it is made up predominantly of late spirals and magellanic irregulars. The dispersion in velocities is only ± 125 km/s (rms), about a fifth the Virgo value, so the cluster crossing time would compare with the age of the universe.

Consequently, we are giving this cluster a great deal of attention with regard to the application of our method. It is:

--relatively nearby and clean with respect to foreground/background confusion,

--rich in late spirals resembling those found in the field,

--sufficiently loose that encounters between galaxies are not too frequent. (It is not clear how encounters affect the observable parameters, especially diameters.)

The one outstanding problem with studying this cluster is the almost complete lack of photometric data. In Figure 7 of Paper I the diameter-profile width relation using diameters out of Nilson (1973) is presented and, in Figure 8 of Paper I, the same plot is superimposed on the local calibrators. We conclude that the distance modulus to the Ursa Major cluster is $30^{m}.5 \pm 0.35$, corresponding to a distance of 12.6 ± 2 Mpc. With a systemic velocity of 949 ± 19 km/s then H_o = 75 km/s/Mpc.

Field Galaxies

For several years we have been gathering HI data on a large sample of galaxies, observing with the 91-meter and the 43-meter telescopes at Green Bank and, to a certain extent, with the 100-meter of the Max-Planck Institute. Our sample is composed of all galaxies north of -45^{o} declination and later than about Sbc that we judge should have a distance corresponding to a redshift of less than 2000 km/s, but which had no previous redshift determination. We observe with a velocity window of -400 to +3000 km/s.

Consequently, our sample has unusual selection effects. It is a distance limited sample (although without a sharp boundary) and, on the faint side, it is not at all magnitude limited. However, it is missing a lot of the brighter, more obvious candidates because they already had known redshifts. Some brighter galaxies have satisfactory published HI profiles and others we have observed for special reasons, but the coverage is spotty. We are now endeavoring to obtain profiles for all the larger galaxies that are important for our method. This present discussion must be considered very preliminary.

Unfortunately, due to the fact that our sample is largely composed of galaxies without previously known redshifts,photometric magnitudes are generally not available. For positive declinations, Zwicky magnitudes (Zwicky, et al., 1961-68) can be used. They must be corrected for systematic errors as a function of surface area for which we follow Balkowski, et al. (1974). Corrections for galactic absorption and tilt are made as in our previous samples.

An apparent magnitude-profile width diagram can be generated taking distances from redshifts but without recourse to an assumption about the value of the Hubble constant by forming a "Hubble modulus":

$$HM = m_{pg}^{o,i} - 5 \log V_o = M_{pg}^{o,i} - 5 \log H_o + 25$$

This parameter is similar to that introduced by Rubin, et al. (1976) except we multiply through by a constant to give units in magnitudes. Then a direct comparison can be made with the calibrator diagram. For the calibrators, the $M_{pg}^{o,i}$ are known and H_o is selected to give the best fitting Hubble modulus.

The question of whether there is a type dependence in the magnitude-profile width relationship remains open. With our large sample we can afford to be restrictive and accept only types Sc, Scd and Sd. The sample is further limited by rejecting those galaxies requiring large corrections for galactic absorption ($|b| < 25^o$) and the nearest galaxies for which distances from V_o are too unreliable ($V_o < 300$ km/s). Eventually mean group velocities should be used but for the present this refinement has not been incorporated.

As an initial attempt, all 200 or so galaxies available to us fulfilling the above requirements, irrespective of inclinations, were used to compose a "Hubble modulus"-profile width diagram. As seen in Figure 8, the result is a good approximation to a scatter diagram. However, we see the familiar pattern among those galaxies more face-on than 45^o identified by crosses. It is they that cause much of the scatter and there is a strong preponderance of those face-on systems low and to the right.

We then considered the very restricted sample with $50^o < i < 70^o$ (inclination measured from face-on), a range sufficiently edge-on that rectified profile widths should be well determined and sufficiently narrow that differential uncertainties in magnitude tilt corrections should be small. Note that there are three local calibrator galaxies which satisfy the requirements for membership in this sample (Sc to Sd and $50^o < i < 70^o$); those are M33, NGC 2403, and NGC 5585.

If the inclination conditions are relaxed slightly to $45^o < i < 75^o$ there is no systematic displacement of the correlation nor is the scatter increased (excluding 2 divergent points) and the sample is augmented from

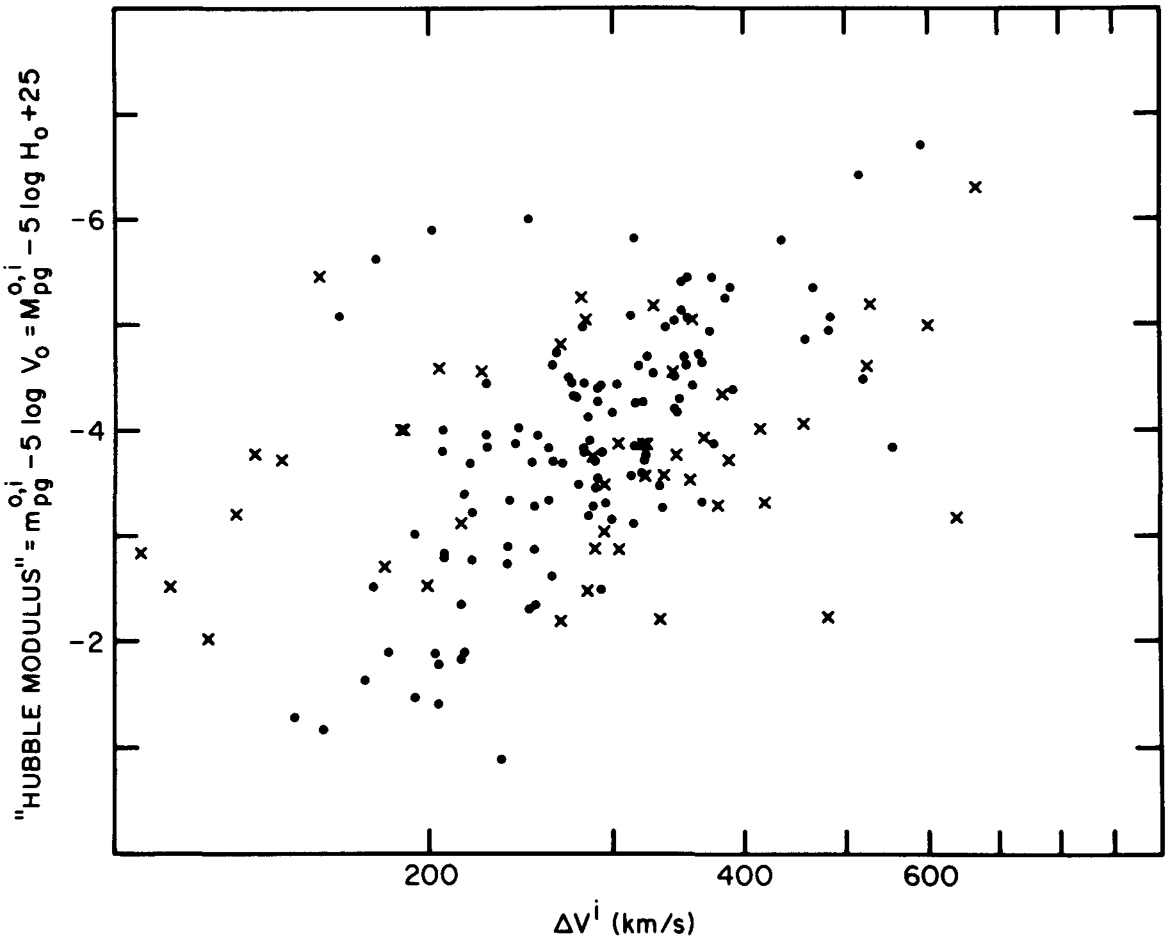

Fig. 8.— Hubble modulus-HI profile diagram for field galaxies of types Sc to Sd, with $|b| > 25$ and $300 < V_0 < 3000$ km/s. Dots, $45^{\circ} < i < 90^{\circ}$; crosses, $0^{\circ} < i < 45^{\circ}$.

52 to 74 galaxies. The "Hubble modulus"-HI profile width correlation for this sample is shown in Figure 9. Superimposed are the straight line drawn through <u>all</u> the calibrators seen in Figure 6 and open circles indicating the placement of those three calibrators which satisfy the type and inclination criteria of this sample. The vertical placement of the line is the visual best fit of the straight line, and the rms scatter (excluding two points) is $\pm\ 0^{m}.6$. The fit corresponds to a Hubble constant of 78 km/s/Mpc.

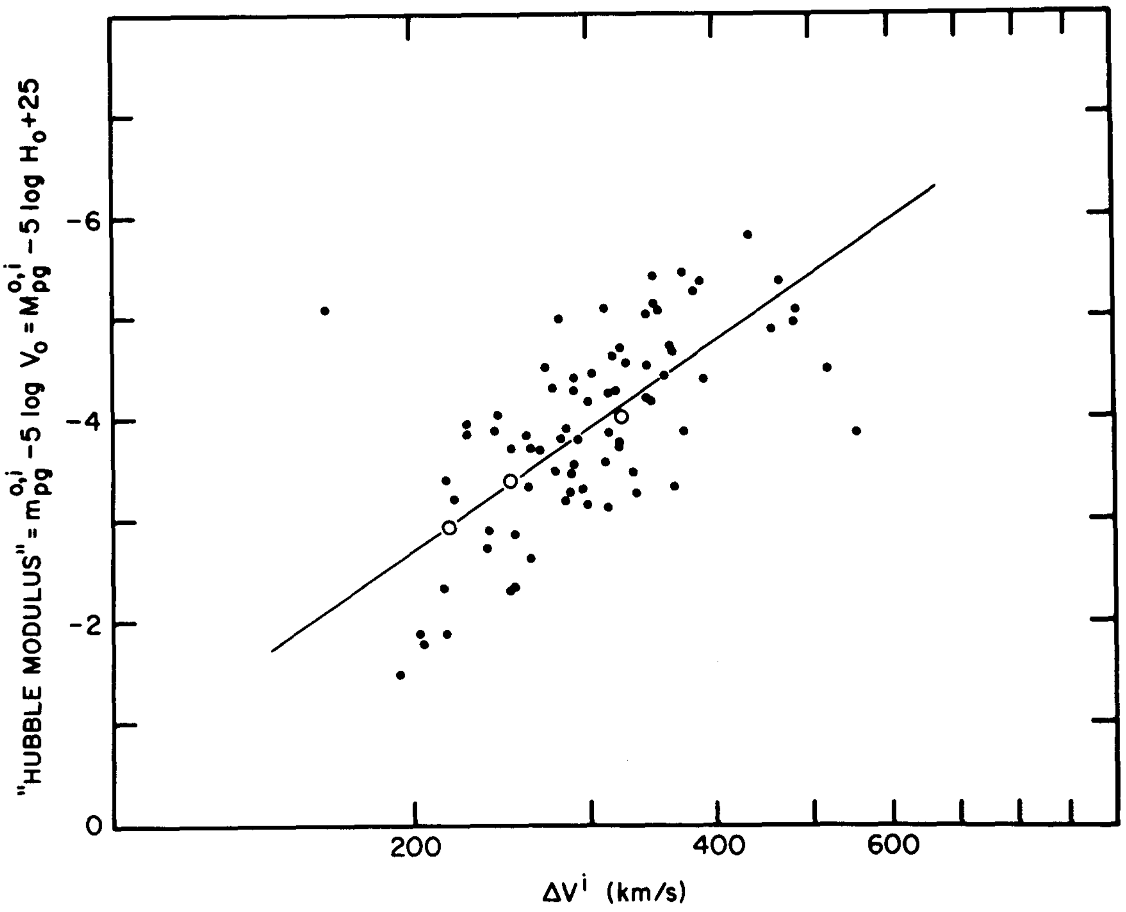

Fig. 9.—Hubble modulus-HI profile diagram for field galaxies of types Sc to Sd, with $|b| > 25^0$ and $300 < V_0 < 3000$ km/s, and inclinations $45^0 < i < 75^0$. The straight line is the same as seen through the calibrators in Figure 6, and the visual fit corresponds to a Hubble constant of 78 km/s/Mpc. The open circles locate the three calibrators which satisfy the type and inclination criteria of the field sample.

Possible Anisotropy in the Field Galaxies

It appears likely that not all the scatter in Figure 9 is random. There may be a small systematic difference between the Hubble modulus of systems within the supercluster and of those beyond. In Figure 10 systemic velocities are plotted against distances derived with our method. Means and dispersions of the mean are given for groups of ten points clumped by distance. Systems north of the galactic plane (filled circles) and south of the plane (open circles) are identified. There is a selection effect beyond a distance of about 25 Mpc due to the velocity cut-off to the sample at 3000 km/s.

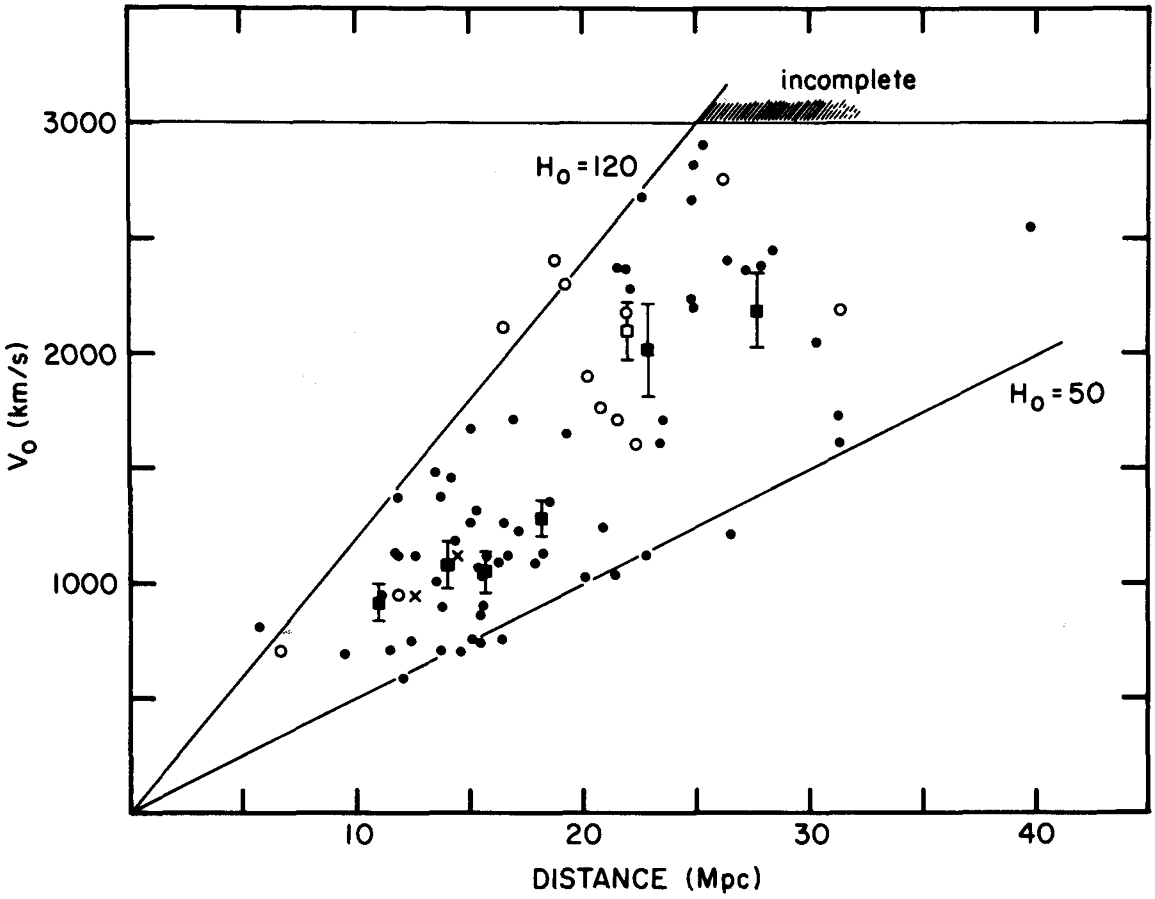

Fig. 10.—Systemic velocities vs. distances by our method. Our sample has a velocity cut-off at 3000 km/s. Filled circles, b > 0°; open circles, b < 0°; crosses, Virgo and Ursa Major clusters; squares, means of ten individual points grouped by distance (error bars are deviations of the mean).

It may be noticed that the filled circles with distances less than 20 Mpc tend to lie relatively toward the lower boundary in Figure 10 while open circles at all distances and filled circles beyond 20 Mpc tend to lie toward the upper boundary. This effect is seen more clearly in Figure 11 where values for the Hubble constant, derived from each of those mean points in Figure 10, are plotted against distance. Our results for the Virgo and Ursa Major clusters are also included.

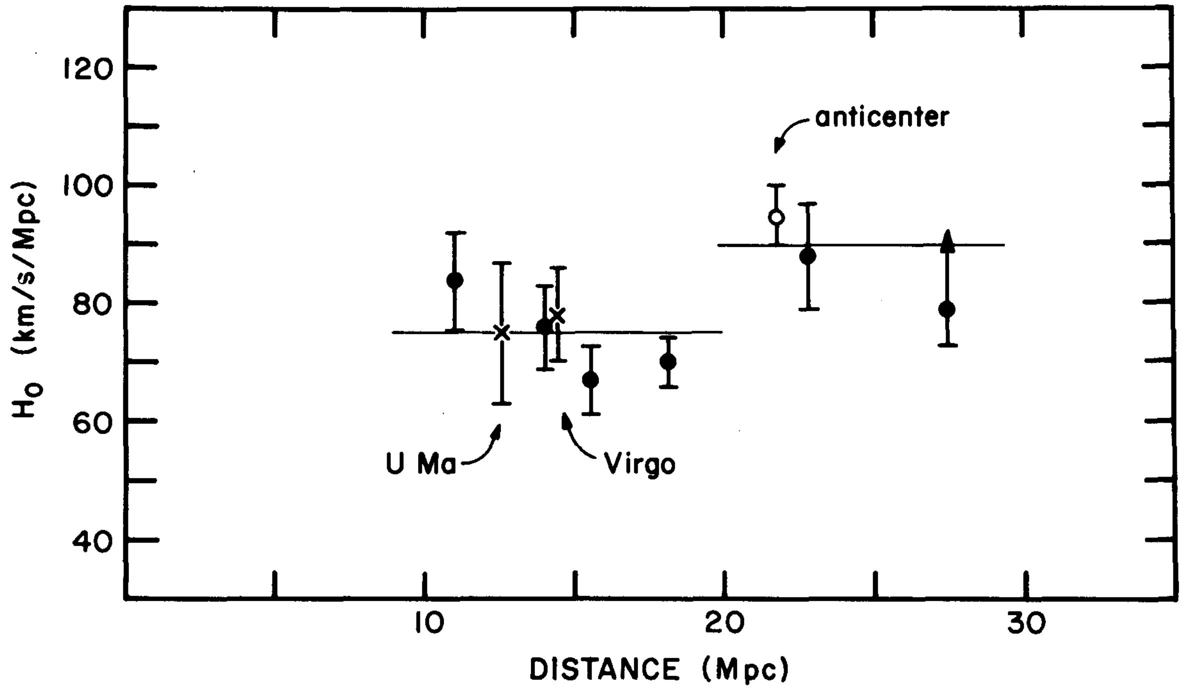

Fig. 11.—The means in Figure 10 are interpreted as values of the Hubble constant and plotted as a function of distance. Filled circles, b > 0; open circles, b < 0; crosses, Virgo and Ursa Major clusters. Straight lines are drawn through the points identified with the supercluster ($H_0 \sim 75$ km/s/Mpc) and the points outside the supercluster ($H_0 \sim 90$ km/s/Mpc).

A single best value for the Hubble constant based on all the points in Figure 11 would be $H_0 \sim 80$ km/s/Mpc (± 10%). However, there is an indication that the supercluster is having a dynamical effect: for galaxies toward the center of the supercluster with distances between 10 and 20 Mpc our method suggests $H_0 \sim 75$ km/s/Mpc while for galaxies toward the anticenter or with distances between 20 and 30 Mpc, $H_0 \sim 90$ km/s/Mpc. The scatter of the mean points in Figure 11 about these values are about 7-8% so the suggested variation of $\Delta H/H_0 \sim 17\%$ has a significance of about 2 sigma. This result is not inconsistent with the work of Sandage and Tammann (1975a, ST V) who conservatively state only that $\Delta H/H_0 < 30\%$. The same effect was suggested by de Vaucouleurs (1975) and, in fact, our results are in good quantitative agreement with his.

We have looked for the phenomenon suggested by Rubin, et al. (1976); a strong dependency of the Hubble modulus on direction in the sky which they interpret as being due to the motion of our local group of galaxies toward $\ell \sim 163^{\circ}$, $b \sim -11^{\circ}$ with a velocity of 450 km/s. We have looked in various velocity ranges including one with $1600 < V_0 < 3000$ in which Rubin, et al. claim the effect exists, but fail to see it. However, we do not have the good sky coverage required to analyze this problem properly. In particular, our lack of magnitudes at negative declinations leaves us without candidates near their antapex.

Conclusions

Either we have an important systematic error in our method or Sandage and Tammann do in theirs. The possibility of a systematic error in our work has been illustrated by the separation between edge-on and face-on spirals first brought to light in ST VII. However, the tightness of all our plots when only edge-on systems are considered suggests the problem is in inclinations and should not seriously affect results where the projection corrections to the HI profile widths are small. This very real problem aside, our method for determining distances to galaxies has an advantage over the one used by Sandage and Tammann in that it is much more straightforward.

Our preliminary results with field galaxies suggest (2 sigma significance) that there is a small perturbation to the Hubble expansion between the supercluster and the area just beyond it. From galaxies within the supercluster we get $H_0 \sim 75$ km/s/Mpc. From galaxies just outside, we get $H_0 \sim 90$ km/s/Mpc.

This discussion is based on the Hyades distance accepted in ST I-VI. If the Hyades cluster is taken to be 10% more distant (ST VII; Hanson, 1975) then the Hubble constant should be reduced by this amount.

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DISCUSSION

G. BURBIDGE: Do you assume that the velocity you measure is all due to rotation? Have you found evidence for non-circular motion in the plane or motion in or out of the plane?

R.B. TULLY: There is no explicit assumption about the origin of the line width since our relationship is strictly empirical, although our physical interpretation is that we are seeing circular velocities only. Of course for very face-on galaxies random z-motions could provide a significant component which would cause a spuriously large line width upon projection to edge-on. Occasionally confused or obviously disrupted galaxies have unusual profiles which might be attributed to non-circular motions. We ignore these systems in applying our method.

H. ARP: Do those edge-on galaxies that you are having trouble with in Virgo include the negative velocity galaxies?

R.B. TULLY: I think the arguments that these are members of the Virgo cluster is good.

H. ARP: I accept that they are members of the Virgo cluster but I still believe that they are considerably closer to us than the center of the cluster.

R.B. TULLY: If the line-of-sight depth of the Virgo cluster compares with the extent on the plane of the sky then the depth is about ± 10 per cent of the distance (i.e. $\pm\ 0\overset{m}{.}2$).

G.O. ABELL: Over how large a region of the sky do you consider galaxies as belonging to the "Virgo cluster"?

R.B. TULLY: A region of 6° radius.

S. SHOSTAK: Could I have a clarification of the derivation of inclinations used for your distance criterion? In your original manuscript all but two of the inclinations in the Virgo sample were determined by your own methods, rather than by using axial ratios. Using the latter instead, one finds rather little difference between Sandage and Tammann's distance to Virgo and yours. Could you explain why your inclinations were systematically greater than those obtained using axial ratios?

R.B. TULLY: It is true that in Paper I our inclinations deviated systematically from what would be derived from axial ratios. Rather than defend our position there, let me say that in the analysis here we have accepted inclinations from Sandage and Tammann (i.e. from axial ratios) in all but three cases - one from our original sample, one added from ST VII with i $> 45^{\circ}$, and one with i $< 45^{\circ}$. In these three cases there are well defined spiral structure or rings and deprojecting this structure suggests that the inclination is significantly (roughly 10°) more edge-on than suggested by axial ratios. We suspect that inclinations from axial ratios may be susceptible to systematic errors but we do not yet understand what is going on.

G.A. TAMMANN: The 21 cm line method requires two inclination corrections, one to obtain the true width and one for internal absorption to obtain the true magnitude of the galaxy. The former correction causes random errors of the true line width due to observational errors in i. The latter, amounting in nearly edge-one galaxies to $> 1^m$, may be systematic (in addition to possibly large random scatter) because our knowledge of internal absorption is not satisfactory. In fact edge-on galaxies give consistently smaller distances than less inclined objects. This suggests strongly that edge-on galaxies are overcorrected and should be given very low weight.

As to your sample of late-type field galaxies it should be stressed that we shall never come to an understanding of the distance scale if we do not begin to clearly define our samples. For an unbiased distance - limited sample one can define a mean absolute magnitude $\langle M \rangle$ and the galaxies of such a sample follow a magnitude - redshift relation with the slope 5. For an unbiased magnitude - limited sample the $\langle M \rangle$ is brighter by the Malmquist correction and its galaxies do not follow a line of slope 5 in the magnitude-redshift relation. The properties of an ill-defined sample are uncontrollable.

R.B. TULLY: Regarding the tilt corrections: it does not seem to us that systematic errors in magnitudes are likely to cause the observed discrepancy between edge-on and face-on galaxies, given that we get a tight relationship with the edge-on sample, where we apply corrections of $0^m.4$ to $1^m.1$, and get a large, systematic scatter with the face-on galaxies, where we apply corrections of 0^m to $0^m.4$. However, I have already indicated that

I do suspect systematic errors, and in particular those cases with inclinations from rectification of the spiral structure lie tightly with the edge-on sample where they did not, using inclinations from the axial ratios. In any event, all the calibrators save M 101 are edge-on more than 50° and it would seem appropriate to compare galaxies of similar inclination in lieu of understanding the systematic differences between edge and face-on.

Regarding the sample selection: Here is the implication in your question that, because there is a suggestion of an increase in the "Hubble constant" with distance in our data, it might be due to the Malmquist effect. Again I stress that our cut-off was based on the structure of the galaxy - as if we made a judgement of the luminosity class and used that to estimate a distance. So we have a fuzzy distance cut-off. However all normal Sc and Sd galaxies within this range are well above the magnitude limit (visability on the Palomar Survey) and diameter limit (1 arc min) of our sample. Given that we are considering such a small part of the universe as two supercluster diameters, because there may be small departures from a Hubble flow need not necessarily imply that the data are wrong.

L. GOUGUENHEIM: When considering the data given by Tully and Fisher and by ST VII, it appears that:

1 - Their relations (ΔV_o, M) are in agreement for the nearby calibrating galaxies, though ST VII use different magnitude corrections.

2 - The disagreement for the Virgo cluster comes essentially from the fact that face-on galaxies, for which the determination of the inclination is decisive for the ΔV_o determination, have been included by ST VII and not by Tully and Fisher.

3 - For the galaxies in common, the values of the inclination agree, on the mean.

4 - For these galaxies, ST VII adopted systematically larger values of the observed 21 -cm line width, which contribute to increase the Virgo cluster distance.

R.B. TULLY: Regarding point 3, there was actually a 3° systematic difference between us. Regarding point 4, the two most significant disagreements are illustrated in Figure 4.

J.P. VIGIER: At what level of significance is your center - anticenter difference? A similar difference has appeared (at the 2.8σ level) in Sandage and Tammann's own sample. We have found a similar increase in the Hubble modulus for type I and Type II supernovae.

R.B. TULLY: Let us just say that our data is suggestive of a center - anticenter difference.

V. RUBIN: 1) For our sample of over 100 Sc I (almost all with $i \leq 60^{\circ}$), we determine a very small magnitude correction for internal extinction, compared with the classical Holmberg value. What you require, of course, is a correction suitable for your own sample and you could evaluate this directly by plotting Hubble modulus vs sec i. This should eliminate all uncertainty about the proper magnitude corrections for inclination.

2) Diameters of Sc I galaxies measured from the Palomar Sky Survey suffer systematic effects as a function of galactic latitude and zenith distance. Suitable corrections should be applied to your diameter measures.

3) Your Virgo/Anti-Virgo plot showed values of $< H >$ which increase with distance. This is just what one finds for a sample which is not bias free, i.e. more distant galaxies come preferentially from the high luminosity tail of the luminosity distribution and hence give artificially high values of H. Before your result is accepted as an anisotropy, I think the sample must be examined for the presence (or absence) of a Malmquist bias.

R.B. TULLY: To your points 1) and 2), I agree that we have the capacity with our data to look deeper into the various correction parameters and we will do so. For the moment we wanted to hold to the corrections made by Sandage and Tammann so as not to further obscure where differences may lie. Further on the Malmquist bias point: the Malmquist effect is only important if there is significant dispersion in magnitudes in the distance indicator being used. Our calibrator plot offers the hope (admittedly only that because of the small sample) that the vertical dispersion due to magnitudes is small in our method.

B.M. LEWIS: I am impressed by the dependence of all calibrations upon the properties of small groups of galaxies. Since there is only one group for which most techniques of distance estimation have been applied, and no groups for which independent distances are available for more than one member, I feel it is vital to determine independent distances to as many members of the two or three closest groups as possible. This is particularly important if any dispersion in the value of the different distance indicators is to be evaluated and is essential if the calibrations are to be made independent of our present uncertainty about the membership of small groups and about the possibly large dispersion of members along the line of sight.