CORRESPONDENCE.

REVERSIONARY LIFE INTERESTS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Mr. Sprague, in his interesting and important paper on the Valuation of Reversionary Life Interests, read before the Institute of Actuaries on the 27th ultimo (see page 107), gave two expressions for the value of a reversionary annuity calculated so as to yield to a buyer 6 per-cent until the time of falling into possession and 5 per-cent afterwards. In the discussion which followed the reading of the paper, I mentioned that the difference between the two formulas numbered (2) and (4) appeared to me to be, that in No. (2) it is assumed the buyer will make 6 per-cent interest on his outlay until one year after the death of the life-tenant, and in formula (4) that he will make 6 per-cent only until the end of the year of death of the life tenant—that is, on an average, for a period less by six months than in the

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former case. The point may not appear of great importance, bearing in mind the wide differences of opinion which prevail as to the values of these interests. I hold it, however, to be a fundamental principle of actuarial science that, given the basis on which an actuarial estimate is to be made, the formulas and methods of valuation employed shall be as accurate as the circumstances will reasonably admit. I trust, therefore, you will forgive my troubling you with an examination of the difference between the two formulas.

First consider formula No. (2). We know that, if we have a curtate unit annuity in possession, $\frac{1}{P+d_5}$ is the value, at any time when a premium is just due, of the policy and the remainder of the annuity, including the year's payment just accrued. This is, therefore, the value of a reversionary curtate annuity at the end of the year of death of the life tenant. Allowing for 6 per-cent interest until the same epoch, the present value of such a reversionary annuity is

$$\frac{A_{xy(6)}}{P+d_5}-\frac{P(1+a_{xy})_6}{P+d_5} \quad \text{(see } J.I.A.\text{, xiv, 427),}$$

or

$$\frac{1-(P+d_6)(1+a_{xy})_6}{P+d_5}$$

Now in the complete reversionary annuity of practice, each payment falls due on an average six months later than in the curtate annuity, and therefore if 6 per-cent is to be realized until the first payment of the complete annuity falls due, its value is

$$\sqrt{v_6}\,\frac{1-(P+d_6)(1+a_{xy})_6}{P+d_5}$$

as given in Mr. Sprague's formula No. (2); but if 6 per-cent is to be realized only until the end of the year in which the life-tenant dies, the value of the complete annuity is

$$\sqrt{v_5}\,\frac{1-(P+d_6)(1+a_{xy})_6}{P+d_5} \quad . \quad . \quad . \quad . \quad . \quad (a)$$

Next consider formula No. (4). The most direct way to obtain it appears to me to be the following:

Let V be the value of a complete life annuity I in possession, and S the sum to be assured to cover the buyer of such an annuity, so that the buyer's outlay is PS+V. The income must suffice to pay interest and premiums; therefore

$$i_5(PS+V)+PS=I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

Again, the capital must be replaced and the last year's interest upon it paid at the end of the year of death of the annuitant. Therefore

$$(PS+V)(1+i_5)=\tfrac{1}{2}I+S \quad . \quad . \quad . \quad . \quad . \quad (c)$$

These two equations give us (*b*) and (*c*) in terms of the other quantities. Solving them, we obtain

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$$V = I\frac{v_5 - \frac{1}{2}P}{P + d_5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

$$S = I\frac{1 - \frac{1}{2}d_5}{P + d_5} = V + \frac{1}{2}I \quad . \quad . \quad . \quad . \quad . \quad (e)$$

$$PS + V = I\frac{v_5(1 + \frac{1}{2}P)}{P + d_5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

Also $V + I$, that is, $I\frac{1 + \frac{1}{2}P}{P + d_5}$ is the value, at any time when a premium is due, of the policy and the remainder of the annuity including the payment just accrued. Formula (*d*) is that of Mr. Baden. It and those which follow it are given in Mr. Sprague's paper, and on reading his investigation of formula (4), deduced from them, it will be seen that this formula assumes 6 per-cent interest is to be realized until the end of the year of death of the life-tenant. It is practically identical with formula (*a*) given above.

The method of finding the value of a life annuity in possession, under which equations (*b*) and (*c*) are obtained, appears to me to be worth noting, since it is easy of application whether the annuity is curtate or complete, and on any hypothesis as to date of payment of the sum assured—*e.g.*, if the sum assured may be assumed to be payable three months after the death of the annuitant, the equations become

$$i_5(PS + V) + PS = I$$

$$(PS + V)(1 + \tfrac{3}{4}i_5) = \tfrac{1}{2}I + S$$

from which we obtain

$$V = I\left\{\frac{1 + \frac{1}{2}P}{d_5 + P(v_5 + \frac{3}{4}d_5)} - 1\right\}$$

$$S = I\frac{v_5 + \frac{1}{4}d_5}{d_5 + P(v_5 + \frac{3}{4}d_5)}$$

$$PS + V = I\frac{v_5(1 + \frac{1}{2}P)}{d_5 + P(v_5 + \frac{3}{4}d_5)}$$

It may not be out of place to notice that under the formulas just given, or under those deduced from (*b*) and (*c*), if a life annuity in possession and the policy stand in a purchaser's books at the value $PS + V$, there will, in certain cases, be a deficit when the policy money becomes payable. For example, take the formulas (*b*), (*c*), (*d*), (*e*), and (*f*), and suppose the death of the annuitant occurs immediately after payment of a premium. Then, on the hypothesis made as to date of payment of claims, the amount to be provided when the claim falls due is $(PS + V)(1 + i_5) = I\frac{1 + \frac{1}{2}P}{P + d_5}$, and the amount forthcoming to meet this is $S = I\frac{1 - \frac{1}{2}d_5}{P + d_5}$, so that there is a deficit of $\frac{1}{2}I$, as is otherwise apparent. There will, of course, in other cases be a surplus, and with Mr. Baden's formula there will be an exact balance only when the annuitant dies six months after payment of a premium.

I am, Sir,
Your obedient servant,
A. W. SUNDERLAND.

2, *King William Street*,
London, E.C.
1 *March* 1888.

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