

Trades and defining sets: theoretical and computational results

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Given a particular combinatorial structure, there may be many distinct objects having this structure. When investigating these, two natural questions to ask are:

- ▷ Given two objects, where and how do they differ?
- ▷ How much of an object is necessary to uniquely identify it?

These two questions are obviously related, with the first leading to the concept of a *trade*, and the second to that of a *defining set*. In this thesis we study trades and defining sets, in the context of t - (v, k, λ) designs. In our enquiries, we make use of both theoretical and computational techniques, with theoretical results leading to more efficient programmes and computational results suggesting lines of theoretical enquiry.

We investigate the *spectrum* of trades, and prove an extant conjecture regarding this [1]. Our results also suggest a more general version of this conjecture [2]. A t - (v, k, λ) design where $\lambda = 1$ is called a *Steiner design*, and the related trades are called Steiner trades. In the case $t = 2$, we establish the spectrum of Steiner trades for each value of k , except for a finite number of values in each case [3, 4].

The connection between trades and defining sets is used to obtain some new theoretical results on defining sets of designs, and is exploited throughout the thesis [5, 6]. We also consider the collections of all trades and all defining sets in a design.

A *simple design* is one which is a set, as opposed to a multiset. We present an algorithm to enumerate all the trades in simple designs. For non-simple designs we introduce the concept of a *discriminating set*, and present an algorithm to enumerate these. Output from these algorithms was used to investigate the trades and defining sets of a number of designs. An error in a published result was discovered, and some new results were obtained [7].

Given part of a design, its *completions* are all those designs that contain it. An existing algorithm to complete partial designs is examined, and a heuristic yielding a much improved algorithm for Steiner designs is discussed. This completion routine was used to investigate a number of designs, and new information on the size and distribution of their defining sets was obtained [8].

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