

# The Construction of Stäckel Potentials for Galactic Dynamical Modelling

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**Abstract.** We present a set of axisymmetric Stäckel potentials which can be used for Galactic dynamical modelling. Each of them has a halo-disk structure with a flat rotation curve.

## 1. Introduction

Dynamical modelling of stellar populations in the bulge of our Galaxy necessitates a potential that approximates as much as possible our idea of the true potential. As is well known, of all requirements that we must impose on a potential, a flat rotation curve and the presence of a third integral are foremost. Therefore, we construct Stäckel potentials with flat rotation curves. Besides, the rotation curve must remain flat over a large distance (see Rubin et al. 1985).

To adapt such a potential to the true potential of our own Galaxy, one can use the force component  $K_{\varpi}$  and  $K_z$ , which can be estimated from velocity measurements in the solar neighbourhood, as is well known.

## 2. Method

An axisymmetric Stäckel potential separates the Hamilton–Jacobi equation in spheroidal coordinates  $(\lambda, \varphi, \nu)$  (see Dejonghe and de Zeeuw 1988a). Those coordinates are defined by two constants  $a$  and  $c$  which are related to the oblate or prolate shape of the system. For simplicity, we assign to both components a Kuzmin–Kutuzov potential (Kuzmin 1962), which is written in spheroidal coordinates as follows:

$$V(\lambda, \nu) = -\frac{GM}{\sqrt{\lambda} + \sqrt{\nu}}$$

with  $G$  the gravitational constant and  $M$  the total mass. To handle the different shape of disk and halo and to preserve the Stäckel formalism, two spheroidal coordinate systems are considered which are connected by  $\lambda_{halo} = \lambda_{disk} - q$ ,  $\nu_{halo} = \nu_{disk} - q$ ,  $q \geq 0$ . If  $k$  measures the relative contribution of the disk mass versus the total mass, the resulting potential can be written as:

$$V(\lambda, \nu) = -GM \left( \frac{k}{\sqrt{\lambda} + \sqrt{\nu}} + \frac{1-k}{\sqrt{\lambda-q} + \sqrt{\nu-q}} \right)$$

To obtain potentials which are compatible with what we know about the Galactic potential, we calculate for the solar neighbourhood the spatial and projected surface density and the circular velocity for each potential from the selected set and compare the values with the Bahcall–Schmidt–Soneira model in a Stäckel version (see Dejonghe and de Zeeuw 1988b).

### 3. Results

A survey of the parameter-space consisting of the disk and halo flatness, disk and halo mass produces a set of potentials with a flat rotation curve. The large variety in shape of the rotation curves led us to select the flat curves by simple inspection (see Fig.1). The following results, we should emphasize, are results for Stäckel potentials and the axis ratio mentioned is the ratio of the surfaces of constant density near the center:

- The requirement of flat rotation curves implies flat disks, with axis ratio larger than 1.5
- The halo must be nearly spherical: axis ratio smaller than 1.009
- The disk cannot contribute more than 0.15% to the total mass.

The Galactic potential is best fit with a disk which is rather flattened (axis ratio  $\approx 4.4$ ), while the halo is almost spherical. The total mass can be estimated to be  $4 \times 10^{11} M_{\odot}$  and about 90% of that mass is contained in the halo. The spatial density in the solar vicinity equals  $0.07 M_{\odot} \text{pc}^{-3}$  while the projected surface density is  $43 M_{\odot} \text{pc}^{-2}$ .

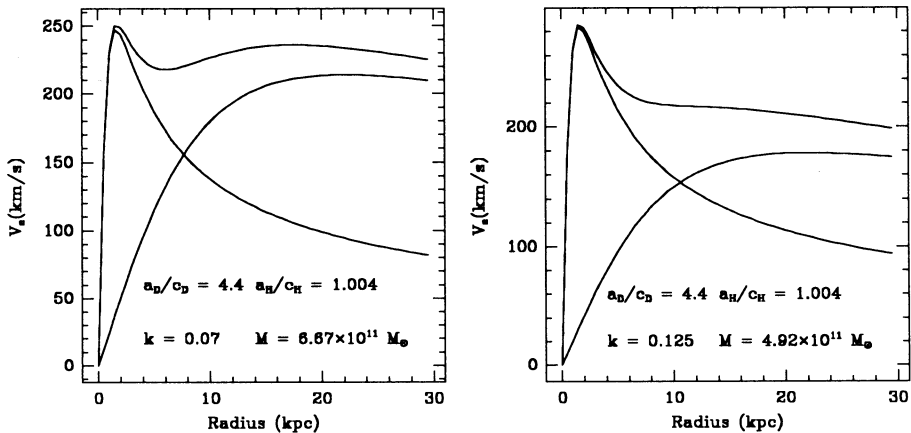


Fig. 1. Rotation curves of some of the retained potentials. Symbols are explained in the text.

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