Introduction to differential geometry and Riemannian geometry, by E. Kreyszig. University of Toronto Press, 1968. xii + 370 pages.

There is no shortage now of high-level monographs on modern differential geometry. The challenging task of presenting the fruits of the revolution which occurred in this field during the past twenty years on a medium level - suitable for advanced undergraduates - has also been successfully tackled recently by various authors (Auslander; Bishop/Goldberg; Flanders; Guggenheimer; Hicks; O'Neill; Singer/Thorpe. A volume by S.-T. Hu is in preparation).

A common feature of the "new" treatment is the invariant definition of tensors in the framework of multilinear algebra. Even if this is not done via universal mapping properties, it can be achieved painlessly and, for most purposes, satisfactorily by identifying a tensor with a multilinear map. This presentation leads naturally to a global approach to the relevant problems.

In this and other respects the present book has a curiously outdated flavour. Surely the fact that sometimes a calculation is still simpler when carried out in a particular adapted basis is no excuse for introducing an intrinsic geometric object by its components in an arbitrary system of coordinates. Having been raised myself on "definitions" of the type "A tensor is if something behaves such and such", I feel that, to inflict on students of the year A.D. 1968, the author's nightmarish definitions on pages 102-110, might be considered justified cause for students' riots! Also it seems extraordinary that the most powerful instrument for studying intrinsically geometric problems on a manifold, viz. the exterior calculus of differential forms, is not even mentioned. Naturally, no integration theory on manifolds is then available either. Without an invariant definition of tensors the introduction of covariant differentiation and affine connexions becomes a formal manipulation. Compare the neat modern treatment in Bishop/Goldberg, page 221 (following Koszul's axioms and avoiding the fiber space approach).

My remaining comments concern details only:

1) The author insists (pages 101-110) that coordinate transformations form a group, although he does not use this condition. Except for trivial cases, they cannot even be combined due to different domains and ranges. I do not think that the introduction of the correct concept of a pseudogroup (Ehresman) would have been beyond the intended level.

2) There is no attempt to show the relation between "metric spaces" as defined on page 6 and the "Riemannian metrics" of page 71. Although on page 71 quotation marks are used once to distinguish between the two concepts, they are dropped in the next sentence. Although one may argue that the proof is too difficult for this level, I feel that the importance of this point deserves at least that it be mentioned - particularly so, since otherwise the remarks on pages 204-205 do not make sense.

3) The definition of a covering on pages 58 and 275 contains redundant conditions.

4) In the definition of an atlas on pages 59 and 276, it is superfluous to add the condition that the Jacobian should not vanish. (Interchange 'i' and 'j'.)

5) For the term "embedding" used on page 60, one should substitute the generally accepted "immersion".

6) Of course, the author has the right to define a manifold to be connected (page 275). But a student will discover to his dismay in other courses that many classical Lie groups are not connected and are still called manifolds. Cf. also the frame bundle of a (connected) manifold.

7) Submanifolds are required (page 277) to carry the relative topology. A great many investigations owe their existence solely to the fact that this is too restrictive a condition in very natural circumstances (cf. Helgason, Chapter II).

8) The bibliography mentions a disproportionate amount of obscure authors from the 18th and 19th centuries, but I found practically no reference to any modern textbook on the medium level. The omission of some of the outstanding modern treatises strikes me also as curious (Bishop/Crittenden; Favard; Helgason; Sternberg; Wolf).

I feel that the shortcomings of this text are so numerous as to outweigh its advantages, but I must mention in fairness some of the latter: it contains introductions to the subjects of Bergman's metric, modular surfaces of analytic functions, the mappings used in cartography, a wealth of (mostly traditional) exercises, and many good pictures.

H.G. Helfenstein, University of Ottawa

Singularities of smooth maps, by James Eells, Jr. Gordon and Breach, 150 Fifth Avenue, New York 10011, 1967. ix + 104 pages. Hardcover - U.S. $5.50, prepaid - U.S. $4.40; paperback – U.S. $3.00, prepaid - U.S. $2.40.

This book is an excellent introduction to the study of singularities of smooth maps; this branch of differential topology is rapidly developing and unfortunately there is no easy introduction to a first year graduate student (except for some original papers of Morse, Whitney, Pontrjagin and Thom, to mention a few). The author has done an excellent job. Here is a chapter-wise summary:

Chapter I is the standard introduction to calculus in E_n and definition of manifolds and their tangent spaces.

Chapter II is devoted to the study of singularities of smooth maps: Whitney's imbedding theorem is proved. He then studies generic maps, i.e. maps which have only non-generate singular points.

Chapter III studies the relation between the topology of a smooth manifold and of singularities of a smooth real valued function on it. The author discusses the theorem of Morse (Morse inequalities) and the homology properties of critical points (for example, the Index theorem). Finally, he gives some applications to Differential Geometry and Algebraic Geometry (Lefschetz's theorem).

K. Srinivasacharyulu, Université de Montréal