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SUMSETS CONTAINING A TERM OF A SEQUENCE

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Abstract

Let $S = \{s_1, s_2, \ldots\}$ be an unbounded sequence of positive integers with s_{n+1}/s_n approaching α as $n \to \infty$ and let $\beta > \max(\alpha, 2)$. We show that for all sufficiently large positive integers l, if $A \subset [0, l]$ with $l \in A$, $\gcd A = 1$ and $|A| \geq (2 - k/\lambda\beta)l/(\lambda + 1)$, where $\lambda = \lceil k/\beta \rceil$, then $kA \cap S \neq \emptyset$ for $2 < \beta \leq 3$ and $k \geq 2\beta/(\beta - 2)$ or for $\beta > 3$ and $k \geq 3$.

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1. Introduction

For two sets A, B of integers and $a \in \mathbb{Z}$, define

$$A + B = \{a + b : a \in A, b \in B\},$$

and

$$a - B = \{a - b : b \in B\}.$$

For a positive integer $h \geq 2$, let

$$hA = \{a_1 + \cdots + a_h : a_1, \ldots, a_h \in A\}.$$

A set A of nonnegative integers is called *normal* if $0 \in A$ and the greatest common divisor of all elements of A is 1.

In 1990, Erdős and Freiman [2] proved a conjecture of Erdős and Freud: if a set A of integers is a subset of $[1, n]$ and $|A| > n/3$, then a power of 2 can be written as the sum of elements of A. In 1989, Nathanson and Sárközy [6] improved this result by showing that 3504 elements of A is enough. Finally, Lev [4] obtained the best possible result that a power of 2 is the sum of at most four elements of A. In 2004, Abe [1] extended Lev's result to a power of m. In 2006, Pan [7] generalised the results of Lev and Abe.

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THEOREM 1.1 [7, Theorem 1]. *Let $k, m, n \geq 2$ be integers. Let A be a normal subset of $[0, n]$ satisfying*

$$|A| > \frac{1}{l+1}\Big(\Big(2 - \frac{k}{lm}\Big)n + 2l\Big),$$

where $l = \lceil k/m \rceil$. If $m \geq 3$, or $m = 2$ and k is even, then kA contains a power of m.

Pan conjectured that Theorem 1.1 still holds for $m = 2$ and k is odd. In 2012, Wu and Chen [8] made some progress towards this conjecture.

In 2010, Kapoor [3] extended Pan's result for $2A$ to general sequences. He proved the following two results.

THEOREM 1.2 [3, Theorem 1]. *Let $\{a_1, a_2, a_3, \ldots\}$ be an unbounded sequence of positive integers. Assume that a_{n+1}/a_n approaches some limit α as $n \to \infty$, and let $\beta > 2$ be some real number greater than α. Then for sufficiently large $x \geq 0$, if A is a set of nonnegative integers less than or equal to x containing 0 and satisfying*

$$|A| \geq \Big(1 - \frac{1}{\beta}\Big)x,$$

then $2A$ contains an element of $\{a_n\}$.

THEOREM 1.3 [3, Theorem 2]. *Let $\{a_1, a_2, a_3, \ldots\}$ be an unbounded sequence of positive integers such that $a_{n+1}/a_n \leq \beta$ for some constant $\beta \geq 2$. Then for any $x \geq 0$, if A is a set of nonnegative integers less than or equal to x containing 0 and satisfying*

$$|A| > \Big(1 - \frac{1}{\beta}\Big)x + \frac{1}{\beta} \cdot \Big\lfloor \frac{a_1 - 1}{2} \Big\rfloor + 1,$$

then $2A$ contains an element of $\{a_n\}$.

We extend Pan's result for kA ($k \geq 3$) to general sequences that grow like the powers of a real number greater than or equal to 2.

THEOREM 1.4. *Let $\beta > 2$ be a real number and let $S = \{s_1, s_2, \ldots\}$ be an unbounded sequence of positive integers such that $\lim_{n\to\infty} s_{n+1}/s_n = \alpha < \beta$. Let $k \geq 3$ be a positive integer. For large enough l, let A be a normal subset of $[0, l]$ with $l \in A$ such that*

$$|A| \geq \frac{1}{\lambda + 1}\Big(2 - \frac{k}{\lambda\beta}\Big)l,$$

where $\lambda = \lceil k/\beta \rceil$. If $2 < \beta \leq 3$, then $kA \cap S \neq \emptyset$ for all $k \geq 2\beta/(\beta - 2)$. If $\beta > 3$, then $kA \cap S \neq \emptyset$ for all $k \geq 3$.

THEOREM 1.5. *Let $\beta > 2$ be a real number and let $S = \{s_1, s_2, \ldots\}$ be an unbounded sequence of positive integers such that $s_{n+1}/s_n \leq \beta$. Let $k \geq 3$ and l be positive integers such that*

$$l\Big(\frac{k}{\beta} - \lambda + 1\Big) \geq \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor + 1. \tag{1.1}$$

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Let A be a normal subset of $[0, l]$ with $l \in A$ satisfying

$$|A| > \frac{2}{\lambda\beta(\lambda+1)}\Big(\Big\lfloor \frac{s_1-1}{2} \Big\rfloor + \frac{\beta}{2}\Big) + \frac{1}{\lambda+1}\Big(\Big(2-\frac{k}{\lambda\beta}\Big)l + 2\lambda\Big), \tag{1.2}$$

where $\lambda = \lceil k/\beta \rceil$. If $2 < \beta < 3$, then $kA \cap S \neq \emptyset$ for all $k \geq 2\beta/(\beta-2)$. If $\beta \geq 3$, then $kA \cap S \neq \emptyset$ for all $k \geq 3$.

2. Lemmas

LEMMA 2.1 [5, Corollary 1]. *If A is a normal subset of $[0, l]$ with $l \in A$ and $\rho = \lceil (l-1)/(|A|-2) \rceil - 1$, then*

$$|hA| \geq \begin{cases} B_h(|A|) & \text{if } h \leq \rho, \\ B_\rho(|A|) + (h-\rho)l & \text{if } h \geq \rho, \end{cases}$$

where $B_h(x) = \frac{1}{2}h(h+1)(x-2) + h + 1$.

LEMMA 2.2. *Let $\beta \geq 2$ be a real number. Let $S = \{s_1, s_2, \ldots\}$ be an unbounded sequence of positive integers such that $s_{n+1}/s_n \leq \beta$. Let a, b be two positive integers. Suppose A and B are sets of integers satisfying $A \subseteq [0, a]$, $B \subseteq [0, b]$ and $0 \in A \cap B$. If*

$$|A| + |B| > 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1-1}{2} \Big\rfloor + \max\Big\{a, b, \Big(1-\frac{1}{\beta}\Big)(a+b)\Big\}, \tag{2.1}$$

then $(A+B) \cap S \neq \emptyset$.

PROOF. We may assume that $a \leq b$. If $0 \leq b < s_1$, put $x_0 = \lfloor (s_1-1)/2 \rfloor$. If $b < x_0$, then

$$\begin{aligned} |A| + |B| &> 2 + \frac{2}{\beta}\Big\lfloor \frac{s_1-1}{2} \Big\rfloor + \Big(1-\frac{1}{\beta}\Big)(a+b) \\ &> 2 + \frac{1}{\beta}(a+b) + \Big(1-\frac{1}{\beta}\Big)(a+b) = 2 + a + b, \end{aligned}$$

which is a contradiction since $|A| + |B| \leq a + b + 2$. Thus, $x_0 \leq b < s_1$. If $a + b < s_1$, then

$$\begin{aligned} |A| + |B| &> 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1-1}{2} \Big\rfloor + \Big(1-\frac{1}{\beta}\Big)(a+b) \\ &\geq 3 + \frac{1}{\beta}(s_1-2) + \Big(1-\frac{1}{\beta}\Big)(a+b) \\ &\geq 3 + \frac{1}{\beta}(a+b-1) + \Big(1-\frac{1}{\beta}\Big)(a+b) \\ &\geq 3 - \frac{1}{\beta} + a + b > a + b + 2, \end{aligned}$$

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which is again impossible. Thus, $a + b \geq s_1$. Then,

$$\begin{aligned}|A| + |B| &> 3 + \frac{1}{\beta}(s_1 - 2) + \Big(1 - \frac{1}{\beta}\Big)(a + b)\\ &\geq 3 + \frac{1}{\beta}s_1 - \frac{2}{\beta} + \Big(1 - \frac{1}{\beta}\Big)s_1\\ &= 3 - \frac{2}{\beta} + s_1 \geq s_1 + 2.\end{aligned}$$

Since $B, s_1 - A \subseteq [0, s_1]$, we must have $s_1 \in A + B$.

Next, we consider $b \geq s_1$. Choose r such that $s_r \leq b < s_{r+1}$. We proceed by induction on $a + b$.

If $a + b = s_1 + 1$, then $A \subseteq [0, 1]$ and $B \subseteq [0, s_1]$. Since $s_1 - A, B \subseteq [0, s_1]$ and

$$\begin{aligned}|A| + |B| &> 3 + \frac{1}{\beta}(s_1 - 2) + \Big(1 - \frac{1}{\beta}\Big)(a + b)\\ &= 3 - \frac{3}{\beta} + s_1 + 1 > s_1 + 2,\end{aligned}$$

we have $s_1 \in A + B$.

Now, assume that $a + b > s_1 + 1$ and that the lemma holds for the sets $A' \subseteq [0, a_1]$ and $B' \subseteq [0, b_1]$ with $a_1 + b_1 < a + b$. Suppose that $(A + B) \cap S = \emptyset$.

Case 1: $a < s_r$. Write $A_1 = s_r - A$. Thus, $A_1 \subseteq [s_r - a, s_r] \subseteq [0, b]$. If $|A_1| + |B| > b + 1$, then

$$|A_1 \cap B| = |A_1| + |B| - |A_1 \cup B| \geq b + 2 - (b + 1) = 1.$$

Thus, $s_r \in A + B$, which is impossible. Hence, $|A| + |B| = |A_1| + |B| \leq b + 1$, which contradicts the hypothesis (2.1).

Case 2: $a \geq s_r$ and $a + b > s_{r+1}$. Write

$$A_1 = [0, b] \cap (s_{r+1} - A),\ B_1 = [s_{r+1} - a, b] \cap B,$$

$$A_2 = [0, s_{r+1} - b - 1] \cap A,\ B_2 = [0, s_{r+1} - a - 1] \cap B.$$

Then,

$$|B| = |B_1| + |B_2| \tag{2.2}$$

$$\begin{aligned}|A| &= |A_2| + |[s_{r+1} - b, a] \cap A|\\ &= |A_2| + |s_{r+1} - ([s_{r+1} - b, a] \cap A)|\\ &= |A_2| + |[s_{r+1} - a, b] \cap (s_{r+1} - A)|\\ &= |A_1| + |A_2|. \end{aligned} \tag{2.3}$$

Since $A_1, B_1 \subseteq [s_{r+1} - a, b]$ and $s_{r+1} \notin A_1 + B_1$,

$$|A_1| + |B_1| \leq b - s_{r+1} + a + 1. \tag{2.4}$$

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If $b = s_{r+1} - 1$, then $|A_2| = 1$ and $|B_2| \le s_{r+1} - a$. By (2.2)–(2.4),

$$|A| + |B| \le s_{r+1} - a + 1 + b - s_{r+1} + a + 1 = b + 2,$$

which contradicts the hypothesis (2.1). Thus, $b \le s_{r+1} - 2$.

Since $(A + B) \cap S = \emptyset$, we have $(A_2 + B_2) \cap S = \emptyset$. Noting that

$$s_{r+1} - b - 1 + s_{r+1} - a - 1 < 2(a + b) - a - b - 2 < a + b,$$

it follows from the hypothesis that if

$$\max\Big\{s_{r+1} - b - 1, s_{r+1} - a - 1, \Big(1 - \frac{1}{\beta}\Big)(2s_{r+1} - a - b - 2)\Big\} = s_{r+1} - a - 1,$$

then

$$|A_2| + |B_2| \le s_{r+1} - a - 1 + 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor. \tag{2.5}$$

By (2.2)–(2.5),

$$|A| + |B| \le b - s_{r+1} + a + 1 + s_{r+1} - a - 1 + 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor = b + 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor,$$

which contradicts (2.1). If

$$\begin{aligned}\max\Big\{s_{r+1} - b - 1, s_{r+1} - a - 1, \Big(1 - \frac{1}{\beta}\Big)(2s_{r+1} - a - b - 2)\Big\}& \\ = \Big(1 - \frac{1}{\beta}\Big)(2s_{r+1} - a - b - 2),&\end{aligned}$$

then

$$|A_2| + |B_2| \le \Big(1 - \frac{1}{\beta}\Big)(2s_{r+1} - a - b - 2) + 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor. \tag{2.6}$$

By (2.2)–(2.4) and (2.6),

$$\begin{aligned}|A| + |B| &\le b - s_{r+1} + a + 1 + \Big(1 - \frac{1}{\beta}\Big)(2s_{r+1} - a - b - 2) + 3 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor \\ &= s_{r+1} - \frac{2}{\beta}s_{r+1} + \frac{1}{\beta}(a + b) + \frac{2}{\beta} + 2 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor \\ &\le a + b - \frac{2}{\beta}(a + b) + \frac{1}{\beta}(a + b) + \frac{2}{\beta} + 2 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor \\ &= \Big(1 - \frac{1}{\beta}\Big)(a + b) + \frac{2}{\beta} + 2 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor \\ &\le \Big(1 - \frac{1}{\beta}\Big)(a + b) + 3 + \frac{2}{\beta}\Big\lfloor \frac{a_1 - 1}{2} \Big\rfloor,\end{aligned}$$

which again contradicts (2.1).

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Case 3: $a \geq s_r$ and $a + b \leq s_{r+1}$. Write

$$A_1 = [0, s_r] \cap A, \ B_1 = [0, s_r] \cap B,$$

$$A_2 = (s_r, a] \cap A, \ B_2 = (s_r, b] \cap B.$$

Since $(A + B) \cap S = \emptyset$, it follows that $(A_1 + B_1) \cap S = \emptyset$ and so $|s_r - A_1| + |B_1| \leq s_r + 1$. Thus,

$$|A| + |B| \leq a + b - 2s_r + s_r + 1 = a + b - s_r + 1.$$

However, by (2.1),

$$\begin{aligned} |A| + |B| &> 2 + \Big(1 - \frac{1}{\beta}\Big)(a + b) \\ &= a + b - \frac{1}{\beta}(a + b) + 2 \\ &\geq a + b - \frac{1}{\beta} s_{r+1} + 2 \\ &\geq a + b - s_r + 2, \end{aligned}$$

which is a contradiction. Hence, we have $(A + B) \cap S \neq \emptyset$.

This completes the proof of Lemma 2.2. □

REMARK 2.3. If s_1 is odd, this lower bound can be improved to

$$|A| + |B| > 2 + \frac{2}{\beta}\Big\lfloor \frac{s_1 - 1}{2} \Big\rfloor + \max\Big\{a, b, \Big(1 - \frac{1}{\beta}\Big)(a + b)\Big\}.$$

3. Proof of Theorem 1.5

Let $k_1 = \lfloor k/2 \rfloor$ and $k_2 = \lceil k/2 \rceil$. Then,

$$k_1 A \subseteq \Big[0, \Big\lfloor \frac{k}{2} \Big\rfloor l\Big], \quad k_2 A \subseteq \Big[0, \Big\lceil \frac{k}{2} \Big\rceil l\Big].$$

Since $\beta > 2$, if $k \geq \beta/(\beta - 2)$, then

$$\Big\lceil \frac{k}{2} \Big\rceil \leq (\beta - 1)\Big\lfloor \frac{k}{2} \Big\rfloor.$$

In particular, if $\beta \geq 3$, then

$$k \geq 3 \geq 1 + \frac{2}{\beta - 2} = \frac{\beta}{\beta - 2}.$$

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Hence, if $2 < \beta < 3$ and $k \geq \beta/(\beta - 2)$, or $\beta \geq 3$, then $\lceil k/2 \rceil \leq (\beta - 1)\lfloor k/2 \rfloor$. So,

$$\left\lfloor \frac{k}{2} \right\rfloor \leq \left\lceil \frac{k}{2} \right\rceil \leq \left(1 - \frac{1}{\beta}\right)\left(\left\lceil \frac{k}{2} \right\rceil + \left\lfloor \frac{k}{2} \right\rfloor\right). \tag{3.1}$$

Since $0 \in A$,

$$k_1A + k_2A = \left(\left\lfloor \frac{k}{2} \right\rfloor + \left\lceil \frac{k}{2} \right\rceil\right)A = kA.$$

To show $kA \cap S \neq \emptyset$, by Lemma 2.2 and (3.1), it is sufficient to show that

$$|k_1A| + |k_2A| > 3 + \frac{2}{\beta}\left\lfloor \frac{s_1 - 1}{2} \right\rfloor + \left(1 - \frac{1}{\beta}\right)kl.$$

By (1.2),

$$(\lambda + 1)(|A| - 2) > \frac{2}{\lambda\beta}\left(\left\lfloor \frac{s_1 - 1}{2} \right\rfloor + \frac{\beta}{2}\right) + \left(2 - \frac{k}{\lambda\beta}\right)l - 2. \tag{3.2}$$

Noting that $\lambda \geq k/\beta$,

$$(\lambda + 1)(|A| - 2) > \left(2 - \frac{k}{\lambda\beta}\right)l - 2 \geq l - 2. \tag{3.3}$$

Write

$$\rho = \lceil (l - 1)/(|A| - 2) \rceil - 1.$$

Then by (3.3),

$$\rho < \frac{l - 1}{|A| - 2} \leq \lambda + 1. \tag{3.4}$$

If $\beta > 2$ and $k \geq 2\beta/(\beta - 2)$, then

$$k\left(\frac{1}{2} - \frac{1}{\beta}\right) \geq 1 > \left\lceil \frac{k}{\beta} \right\rceil - \frac{k}{\beta}$$

and so $k/2 > \lceil k/\beta \rceil$. If $\beta \geq 3$ and $3 \leq k < 2\beta$, then $\lceil k/\beta \rceil \leq k/2$. If $\beta \geq 3$ and $k \geq 2\beta$, then

$$\lambda = \left\lceil \frac{k}{\beta} \right\rceil < \frac{k}{\beta} + 1 = \frac{k}{2} - \frac{(\beta - 2)k}{2\beta} + 1 \leq \frac{k}{2}.$$

Thus, $\lambda \leq k/2$ for $\beta \geq 3$ or $2 < \beta < 3$ and $k \geq 2\beta/(\beta - 2)$. Hence, by (3.4),

$$\rho \leq \lambda \leq \left\lfloor \frac{k}{2} \right\rfloor = k_1.$$

By Lemma 2.1,

$$|k_iA| \geq B_\rho(|A|) + (k_i - \rho)l \quad \text{for } i = 1, 2. \tag{3.5}$$

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If $\lambda = \rho$, then by (3.2) and (3.5),

$$\begin{aligned}|k_1A| + |k_2A| &\geq \lambda(\lambda+1)(|A|-2) + 2\lambda + 2 + (k-2\lambda)l \\ &> \lambda\Big(\frac{2}{\lambda\beta}\Big(\Big\lfloor\frac{s_1-1}{2}\Big\rfloor + \frac{\beta}{2}\Big) + \Big(2 - \frac{k}{\lambda\beta}\Big)l - 2\Big) + 2\lambda + 2 + (k-2\lambda)l \\ &= 3 + \frac{2}{\beta}\Big\lfloor\frac{s_1-1}{2}\Big\rfloor + \Big(1 - \frac{1}{\beta}\Big)kl.\end{aligned}$$

Now suppose that $\rho \leq \lambda - 1$. Then, $0 \leq \rho \leq \lceil k/\beta \rceil - 1$. Hence, by (1.1),

$$\begin{aligned}|k_1A| + |k_2A| &\geq \rho(\rho+1)(|A|-2) + 2\rho + 2 + (k-2\rho)l \\ &\geq \rho(l-1) + 2\rho + 2 + (k-2\rho)l \\ &= kl - \rho(l-1) + 2 \\ &> kl - \Big(\frac{k}{\beta} + \Big\lceil\frac{k}{\beta}\Big\rceil - \frac{k}{\beta} - 1\Big)l + 2 \\ &= kl - \frac{k}{\beta}l + \Big(\frac{k}{\beta} - \Big\lceil\frac{k}{\beta}\Big\rceil + 1\Big)l + 2 \\ &\geq \Big(1 - \frac{1}{\beta}\Big)kl + 3 + \frac{2}{\beta}\Big\lfloor\frac{s_1-1}{2}\Big\rfloor.\end{aligned}$$

This completes the proof of Theorem 1.5.

4. Proof of Theorem 1.4

Let β^- be some constant satisfying $\alpha < \beta^- < \beta$, and assume that

$$\frac{s_{n+1}}{s_n} \leq \beta^- \quad \text{for all } n \geq 1. \tag{4.1}$$

Then for any l so large that

$$\frac{1}{\lambda+1}\Big(2 - \frac{k}{\lambda\beta}\Big)l \geq \frac{1}{\lambda+1}\Big(\Big(2 - \frac{k}{\lambda\beta^-}\Big)l + 2\lambda\Big) + \frac{2}{\lambda(\lambda+1)\beta^-}\Big(\Big\lfloor\frac{s_1-1}{2}\Big\rfloor + \frac{\beta^-}{2}\Big),$$

we see that Theorem 1.5, using the constant β^-, gives the conclusion of Theorem 1.4. If (4.1) does not hold for all $n \geq 1$, then as $s_{n+1}/s_n \leq \beta^-$ for sufficiently large n, a simple relabelling of the terms of the sequence, omitting finitely many terms at the beginning, would suffice.

This completes the proof of Theorem 1.4.

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