In a similar manner may all the joint life formulæ be derived from the single, without the trouble of investigating each particular case independently of the single life formulæ.

Obs.—*Formulæ* (2), (3), (4), and (7), are, I believe, *new*, as I have never seen them in any work that has come under my notice; (2) and (4) will be found very serviceable in computing a table of reversions by means of a reciprocal probability table, such as Table XXIV. given in the first volume of Jones, page 550, and the ordinary life annuity tables.

W. C. O.

ON CERTAIN COMMUTATION FORMULÆ.

To the Editor of the Assurance Magazine.

SIR,—The communications of Messrs. Laundy and Sprague,* in the last Number of the *Assurance Magazine*, have induced me to send you a few "Commutation Table" formulæ which I have had occasion to make use of in practice, and which, as not lying quite on the surface of the subject, may prove a useful contribution to those interested in them.

The notation used is that employed by Professor De Morgan, where "l_x" represents the number living at age x, "$p_{x.n}$" the probability of a life aged x living n years, "v"$=\dfrac{1}{1+r}$, the present value of £1 due a year hence, &c.

1. Endowment of £1 payable to (x) if alive n years hence; premium to be returned in the event of death; p per £1 Office commission added.

Single premium, $\dfrac{(1+p)\,\mathrm{D}_{x+n}}{\mathrm{D}_x-(1+p)\,(\mathrm{M}_x-\mathrm{M}_{x+n})}$.

Annual premium, $\dfrac{(1+p)\,\mathrm{D}_{x+n}}{(\mathrm{N}_{x-1}-\mathrm{N}_{x+n-1})-(1+p)\,(\mathrm{R}_x-\mathrm{R}_{x+n}-n\,\mathrm{M}_{x+n})}$.

2. *Life assurance.*—Annual premium payable until death, on which the whole of the premiums paid (without interest) are to be returned. Commission of p per £ on net value.

$$\pi=\frac{(1+p)\,\mathrm{M}_x}{\mathrm{N}_{x-1}-(1+p)\,\mathrm{R}_x}.$$

3. Assurance for n years on (x), with endowment payable should he survive that term, but the benefit payable only provided (y) be alive.

Single premium, $\dfrac{\mathrm{D}_{x+n,\,y+n}}{\mathrm{D}_{xy}}\,.\,(1-\mathrm{A}_{\overline{x+n,\,y+n}}^{\,1})+\mathrm{A}_{\overline{x.y}}^{\,1}$.

Annual premium, $\left\{\mathrm{D}_{x+n,\,y+n}(1-\mathrm{A}_{\overline{x+n,\,y+n}}^{\,1})+\mathrm{D}_{x.y}\mathrm{A}_{\overline{x.y}}^{\,1}\right\}\div$
$(\mathrm{N}_{x-1,\,y-1}-\mathrm{N}_{x+n-1,\,y+n-1})$.

For the Carlisle 3 per cent. rates, the values $\mathrm{A}_{\overline{x.y}}^{\,1}$, &c., are got by inspection in Gray, Smith, and Orchard's tables.

4. Endowment of £1 payable to (y) should he survive n years, and provided (x) die before.

* Mr. Sprague has overlooked the circumstance that his (joint life) formula previously appeared in Professor De Morgan's able paper in the *Companion to the Almanac* for 1842.

To suit the published tables, the numerator must here change with the relation the ages bear to each other; thus:—

$x <$ or $= y$. $\quad D_{x,\,y+n} - D_{x+n,\,y+n}$.

$x < (y+n) > y$. $\quad v^{x-y}\, D_{x,\,y+n} - D_{x+n,\,y+n}$.

$x >$ or $= y+n$. $\quad v^{n}\, D_{x,\,y+n} - D_{x+n,\,y+n}$.

For *single premium*, the denominator must in each case be $D_{x.y}$, and for *annual premium*, $N_{x-1,\,y-1} - N_{x+n-1,\,y+n-1}$.

5. Assurance and endowment payable at the first death of (x) or (y), if within n years, or at the end of that term if they are both alive.

$$\text{Single premium,}\quad 1 - \frac{(1-v)\,N_{x-1,\,y-1} - N_{x+n-1,\,y+n-1})}{D_{x\,y}}.$$

$$\text{Annual premium,}\quad \frac{D_{x.y}}{N_{x-1,\,y-1} - N_{x+n-1,\,y+n-1}} - (1-v).$$

6. *Educational endowments.*—Annual premium: k premiums in all for annuity of £1 per annum to (y), to be entered on at death of (x), if within n years, and thence to continue until annuitant attains age $y+n$.

$$x > y. \qquad \frac{l_x v^{x-y}(N_y - N_{y+n}) - (N_{x\,y} - N_{x+n,\,y+n})}{N_{x-1,\,y-1} - N_{x+k-1,\,y+k-1}}.$$

7. A current assurance for £s on the joint lives of (x) and (y), on which an annual premium of P is payable, is to be converted into an assurance payable at the death of (x) only. The reduced premium is

$$\frac{P(1+a_{x\,y}) - s(A_{x\,y} - A_x)}{1+a_x},$$

and, in the "Commutation Table" formula, becomes

$$\frac{PN_{x-1,\,y-1} - s(vN_{x-1,\,y-1} - N_{x\,y} - M_x l_y)}{l_y N_{x-1}};$$

or,

$$\frac{N_{x-1,\,y-1}[P + s(1-v)] - s(D_{x.y} - l_y M_x)}{l_y N_{x-1}}.$$

To suit the published tables, if $x < y$, the factor "l_y" in the formula must, in each case, be changed to $l_y v^{y-x}$.

8. A current assurance for £s on the life of (x), on which an annual premium of P is payable, is required to be converted into an assurance of the same amount payable at the first death of (x) or (y). Here the new benefit being of increased value, the Office commission of p per £1 on it has to be provided for, and the annual premium, payable during joint lives only, becomes

$$\frac{P(1+a_x) + s(1+p)(A_{x\,y} - A_x)}{1+a_{x\,y}};$$

or, by the "Commutation Table" formula,

$$\frac{l_y(PN_{x-1} - M_x) - s(1+p)\,N_{x\,y}}{N_{x-1,\,y-1}} + s(1+p)\,v.$$

To suit the published tables, where $x < y$, the factor "l_y" must be altered to $l_y v^{y-x}$.

I am, Sir,

Your most obedient servant,

Aberdeen, 30*th December*, 1858. H. A. S.