SYSTEMATIC CORRECTIONS TO THE FUNDAMENTAL CATALOGUE DUE TO THE PRECESSION ERROR AND THE EQUINOX CORRECTION

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ABSTRACT. Fundamental catalogues of stars have systematic errors due to the precession error and the equinox correction. The formula for these errors to be applied to the FK4 is presented.

1. Introduction

Fundamental catalogues of stars have been compiled mainly from meridian observations. The adopted precession constant is obtained from analyses of stars' proper motions, and the adopted position at any epoch of the zero-point in right ascension (equinox) is obtained from analyses of positional observations of the Sun, Moon and planets in addition to a study of stars' proper motions. When a new fundamental catalogue is constructed, a decision is made whether the precession constant and/or the position of the equinox should be changed or not. In the case of the Fifth Fundamental Catalogue (FK5) (Fricke *et al.* 1988), the correction to Newcomb's precession constant obtained by Fricke (1977) and the correction to the position of the FK4 equinox derived by Fricke (1982) are adopted.

In the course of investigating the transformation from FK4 system to FK5 system, Sôma and Aoki (1990) have obtained the formula for the systematic correction to the FK4 due to the errors in the precession constant and in the location of the equinox. In deriving the formula they assumed the equinox correction (error in the location of the equinox) to be expressed by a linear function of time, but the linearity cannot be assumed *a priori.*

In this paper we derive the formula for the systematic correction to the FK4 without assuming the linearity of the equinox correction and show that Sôma and Aoki's assumption is valid up to the 10^{-10}/century^2.

2. Derivation of the Formula

We will deal with the systematic corrections to the FK4 due to the errors in the precession and in the location of the equinox. For this purpose we will ignore the systematic corrections to the FK4 at the epoch of B1950.0. In this paper it is assumed that the E-terms of aberration (the elliptic part of aberration due to the eccentricity of the Earth's orbit) are already removed from the positions and proper motions.

https://doi.org/10.1017/S0252921100064204 Published online by Cambridge University Press

The position vector $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ in this paper are related to the right ascension α, declination δ, proper motions in right ascension μ_α (in radians/tropical century) and in declination μ_δ (in radians/tropical century), radial velocity V (in km/sec), and parallax π (in radians) by the following formulae:

$$\boldsymbol{r} = \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \quad \text{and} \quad \boldsymbol{v} = \mu_\alpha \begin{pmatrix} -\cos\delta\sin\alpha \\ \cos\delta\cos\alpha \\ 0 \end{pmatrix} + \mu_\delta \begin{pmatrix} -\sin\delta\cos\alpha \\ -\sin\delta\sin\alpha \\ \cos\delta \end{pmatrix} + 21.094502\pi V\boldsymbol{r}.$$

The 3×3 unit matrix is denoted by I.

The position vector $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ of a star in the FK5 system are related to the position vector $\boldsymbol{r}_1$ and velocity $\boldsymbol{v}_1$ of the star in the FK4 system by the following equations:

$$\begin{aligned} P_{\mathrm{IAU76}}(\mathrm{B1950.0},t)(\boldsymbol{r}+\boldsymbol{v}t)|_{t=0} &= R_3(-E(t))P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(\boldsymbol{r}_1+\boldsymbol{v}_1t)|_{t=0}, \\ \frac{d}{dt}[P_{\mathrm{IAU76}}(\mathrm{B1950.0},t)(\boldsymbol{r}+\boldsymbol{v}t)]|_{t=0} &= \frac{d}{dt}[R_3(-E(t))P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(\boldsymbol{r}_1+\boldsymbol{v}_1t)]|_{t=0}, \end{aligned} \tag{1}$$

where the vectors $\boldsymbol{r}$, $\boldsymbol{v}$, $\boldsymbol{r}_1$, and $\boldsymbol{v}_1$ are evaluated at the epoch of B1950.0, $P(T_1, T_2) = R_3(-z_A)R_2(\theta_A)R_3(-\zeta_A)$ is the 3×3 precession matrix for the equatorial coordinates from the epoch T_1 to the epoch T_2 based on either Newcomb's precession (subscript NEWC) or the IAU (1976) precession (subscript IAU76), R_i is the standard 3×3 rotation matrix about the i_{th} axis, $E(t)$ is the equinox correction, and t is the time reckoned from B1950.0 in tropical centuries. The variables ζ_A, z_A and θ_A denote the equatorial precession parameters (see Lieske *et al.* 1977). The above equations were given by Aoki *et al.* (1983) and their correctness has been confirmed by Sôma and Aoki (1990).

Since

$$P(\mathrm{B1950.0},t)|_{t=0} = I \quad \text{and} \quad \frac{d}{dt}P(\mathrm{B1950.0},t)|_{t=0} = \begin{pmatrix} 0 & -m & -n \\ m & 0 & 0 \\ n & 0 & 0 \end{pmatrix}, \tag{2}$$

we obtain from solving Eq. (1) the following:

$$\boldsymbol{r} = M\boldsymbol{r}_1 \quad \text{and} \quad \boldsymbol{v} = N\boldsymbol{r}_1 + M\boldsymbol{v}_1, \tag{3}$$

where

$$\begin{aligned} M &= \begin{pmatrix} \cos E_0 & -\sin E_0 & 0 \\ \sin E_0 & \cos E_0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ N &= \begin{pmatrix} (m^N - m^O - \dot{E})\sin E_0 & (m^N - m^O - \dot{E})\cos E_0 & n^N - n^O\cos E_0 \\ -(m^N - m^O - \dot{E})\cos E_0 & (m^N - m^O - \dot{E})\sin E_0 & -n^O\sin E_0 \\ -n^N\cos E_0 + n^O & n^N\sin E_0 & 0 \end{pmatrix}. \end{aligned} \tag{4}$$

In the above equations E_0 and $\dot{E}$ are the values of the equinox correction and its first derivative at B1950.0 derived by Fricke (1982), and m and n are the rates of general precession in right ascension and declination, respectively, based on the Newcomb precession (superscript O) and the IAU (1976) precession (superscript N). The quantities m and n are obtained from

$$m = \frac{\partial}{\partial t}(\zeta_A(T,t) + z_A(T,t))|_{t=0} \quad \text{and} \quad n = \frac{\partial}{\partial t}(\theta_A(T,t))|_{t=0}, \tag{5}$$

where T is evaluated at the epoch of B1950.0.

The position vector $s_{\mathrm{FK4}}(t)$ in the FK4 system with respect to the mean equinox of date is given by

$$s_{\mathrm{FK4}}(t) = P_{\mathrm{NEWC}}(\mathrm{B1950.0}, t)(\boldsymbol{r}_1 + \boldsymbol{v}_1 t). \tag{6}$$

The position vector $s_{\mathrm{FK5}}(t)$ in the FK5 system with respect to the mean equinox of date is given by

$$\begin{aligned} s_{\mathrm{FK5}}(t) &= P_{\mathrm{IAU76}}(\mathrm{B1950.0}, t)(\boldsymbol{r} + \boldsymbol{v}t) \\ &= P_{\mathrm{IAU76}}(\mathrm{B1950.0}, t)[(M + Nt)\boldsymbol{r}_1 + Mt\boldsymbol{v}_1]. \end{aligned} \tag{7}$$

The difference between (6) and (7) is mainly due to the equinox correction at the epoch t.

The systematic correction other than the equinox correction expressed as $E + \dot{E}t$ is obtained by

$$\begin{aligned} & s_{\mathrm{FK5}}(t) - R_3(E_0 + \dot{E}t)s_{\mathrm{FK4}}(t) \\ &= \left[\begin{pmatrix} +0.0003 & +0.3170 & -0.1235 \\ -0.3170 & +0.0001 & +5.9231 \\ +0.1234 & -5.9233 & +0.0002 \end{pmatrix} \times 10^{-8}t^2 \right. \\ &\quad + \begin{pmatrix} +0.0059 & +0.0991 & -0.0657 \\ -0.0415 & +0.0071 & +0.0009 \\ -0.0668 & -0.0005 & -0.0013 \end{pmatrix} \times 10^{-8}t^3 \\ &\quad \left. + \begin{pmatrix} +0.0016 & -0.0001 & +0.0000 \\ +0.0001 & +0.0016 & +0.0000 \\ +0.0000 & +0.0007 & +0.0001 \end{pmatrix} \times 10^{-8}t^4 \right] \boldsymbol{r}_1 \\ &+ \left[\begin{pmatrix} +0.0000 & +1.1539 & -2.1113 \\ -1.1539 & +0.0000 & +0.0247 \\ +2.1113 & -0.0247 & +0.0000 \end{pmatrix} \times 10^{-6}t^2 \right. \\ &\quad + \begin{pmatrix} +0.0053 & +0.0034 & -0.0018 \\ -0.0032 & +0.0258 & +0.0120 \\ +0.0012 & -0.0480 & -0.0205 \end{pmatrix} \times 10^{-6}t^3 \\ &\quad \left. + \begin{pmatrix} +0.0001 & +0.0006 & +0.0000 \\ -0.0004 & +0.0001 & +0.0000 \\ -0.0006 & +0.0000 & +0.0000 \end{pmatrix} \times 10^{-6}t^4 \right] \boldsymbol{v}_1, \end{aligned} \tag{8}$$

where the difference is expanded to the polynomial of t. The $\boldsymbol{r}_1$-term is the regional systematic correction, and $\boldsymbol{v}_1$-term is the correction depending on the star's velocity which contributes to the individual correction.

3. Comparison with the Previous Result

Eq. (8) shows the systematic corrections to the FK4 due to the errors in precession and in the location of the equinox. The corresponding difference in accord with the discussion by

Sôma and Aoki (1990) is obtained from their Eqs. (12a) and (13):

$$
\begin{aligned}
&P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)[(X_2 - X_1X_0^{-1}X_1)X_0^{-1}t^2 + (X_3 - X_2X_0^{-1}X_1)X_0^{-1}t^3]\boldsymbol{r}_1 \\
&\quad + P_{\mathrm{NEWC}}(\mathrm{B1950.0},t)(X_1X_0^{-1}t^2 + X_2X_0^{-1}t^3)\boldsymbol{v}_1 \\
&= \left[\begin{pmatrix} +0.0002 & +0.3148 & -0.1354 \\ -0.3148 & +0.0001 & +5.9231 \\ +0.1354 & -5.9232 & +0.0002 \end{pmatrix} \times 10^{-8}t^2\right. \\
&\quad + \begin{pmatrix} +0.0058 & +0.1006 & -0.0579 \\ -0.0430 & +0.0070 & +0.0007 \\ -0.0746 & -0.0007 & -0.0014 \end{pmatrix} \times 10^{-8}t^3 \\
&\quad \left. + \begin{pmatrix} +0.0017 & -0.0001 & -0.0001 \\ +0.0001 & +0.0016 & +0.0002 \\ +0.0000 & +0.0007 & +0.0001 \end{pmatrix} \times 10^{-8}t^4 \right] \boldsymbol{r}_1 \\
&+ \left[\begin{pmatrix} +0.0000 & +1.1539 & -2.1112 \\ -1.1539 & +0.0000 & +0.0247 \\ +2.1112 & -0.0247 & +0.0000 \end{pmatrix} \times 10^{-6}t^2\right. \\
&\quad + \begin{pmatrix} +0.0053 & +0.0034 & -0.0019 \\ -0.0031 & +0.0258 & +0.0120 \\ +0.0014 & -0.0480 & -0.0205 \end{pmatrix} \times 10^{-6}t^3 \\
&\quad \left. + \begin{pmatrix} +0.0001 & +0.0002 & -0.0007 \\ +0.0001 & +0.0001 & -0.0001 \\ +0.0000 & +0.0000 & +0.0000 \end{pmatrix} \times 10^{-6}t^4 \right] \boldsymbol{v}_1. \qquad (9)
\end{aligned}
$$

Note that Eq. (12a) of Sôma and Aoki is with respect to the reference frame of B1950.0 and therefore the difference is multiplied by P_{NEWC} to express the difference with respect to the frame of date.

The difference between (8) and (9) is less than 10^{-10}/century^2 which is equal to 2×10^{-5}arcsec/century^2. Therefore the linearity assumption of the equinox correction is valid within this accuracy.

The calculations were carried out on the FACOM M780/10S of the National Astronomical Observatory of Japan.

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