MATTER IN SUPERSTRONG MAGNETIC FIELDS

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Abstract. We consider the structure of atoms and atomic chains in the presence of ultra-strong magnetic fields as might be found in pulsars or neutron stars. Some consequences of these models for neutron star surfaces are mentioned.

1. Electrons in Superstrong Magnetic Fields

The enormous range of known 'strong' magnetic fields which exist in the universe is sampled in Table I. In magnetic fields greatly exceeding 10^9 G the nature of matter is qualitatively different from that of our normal experience: magnetic forces on electrons become stronger than Coulomb forces in atoms. The surface of a neutron star may consist of such a rather unique form of matter and this is the prime motivation for considering the structure of atoms, molecules, and above all compact matter, in superstrong fields. The nature of such matter seems reasonably susceptable to detailed analysis, if worth the effort. Any possible effects on observations or electrodynamics of pulsars is much less clear.

TABLE I

Very strong magnetic fields

Location	Field (Gauss)
Iron magnet	10^4
Superconducting magnet	10^5
Solar surface	1–10
Outer solar interior	10^3
Magnetic white dwarf surface	10^6–10^7
Neutron star (pulsar) surface	10^{12}–10^{13}
Neutron star interior	$? \gg 10^{13}$
Nuclear surface and interior	10^{15}–10^{16}

Classically a sufficiently huge magnetic field (**B**) confines a free electron to motion along the field like a bead on a straight wire. Quantum mechanical zero point motion perpendicular to **B** limits the magnetic confinement. 'Weak' electric fields can accelerate electrons only parallel to **B** or give a slow unaccelerated drift velocity perpendicular to **B**.

The essential features of the quantum mechanical description of the motion of electrons in superstrong **B** and weak atomic electric fields is clear in the Bohr model.

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A non-relativistic electron of mass m_e in a uniform **B** has an equation of motion

$$\frac{m_e v^2}{\varrho} = \frac{ev}{c} B \tag{1}$$

with ϱ the cylindrical radius perpendicular to **B** and **v** the perpendicular velocity. (Along **B** the electron, of course, moves as a free particle.) The quantization condition is

$$\mathbf{p} \times \boldsymbol{\varrho} = n\hbar \mathbf{B}/B \tag{2}$$

with

$$\mathbf{p} = m_e \mathbf{v} + \frac{e\mathbf{A}}{c}. \tag{3}$$

A convenient gauge for the vector potential is

$$\mathbf{A} = \frac{B\varrho}{2} \hat{\boldsymbol{\phi}}. \tag{4}$$

The quantized energies are

$$E_n = n\hbar\omega_c; \quad n = 1, 2, \ldots, \infty \tag{5}$$

with the 'cyclotron frequency'

$$\omega_c \equiv \frac{eB}{m_e c}. \tag{6}$$

The circular orbits have quantized radii

$$r_n = (2n)^{1/2} \hat{\varrho}$$

with

$$\hat{\varrho} \equiv \left(\frac{\hbar c}{eB}\right)^{1/2} = \frac{2.6 \times 10^{-10}}{B_{12}^{1/2}} \text{ cm}, \tag{7}$$

where B_{12} is the field in units of 10^{12} G. We shall generally consider fields so large that neither thermal energies nor perturbative electric fields are sufficient to give significant occupation to any except the lowest energy state with $E = \hbar\omega_c$. (For $B = 2 \times 10^{12}$ G, $\hbar\omega_c \sim 25$ keV and $r_1 = 3.6 \times 10^{-10}$ cm.)

By choosing other gauges for **A** the centers of the circular orbits can be moved anywhere. This degeneracy is most simply resolved by considering first the effect of a weak uniform electric field **E** perpendicular to **B**. The classical electron will then have a cycloidal motion with an average drift velocity

$$\mathbf{v}_d = \frac{\mathbf{E} \times \mathbf{B}}{B^2} c. \tag{8}$$

In a weak radial electric field, say from an unshielded nucleus at the origin $\varrho = 0$, the electron's uniform cycloidal motion becomes slightly bent along a large circle so that

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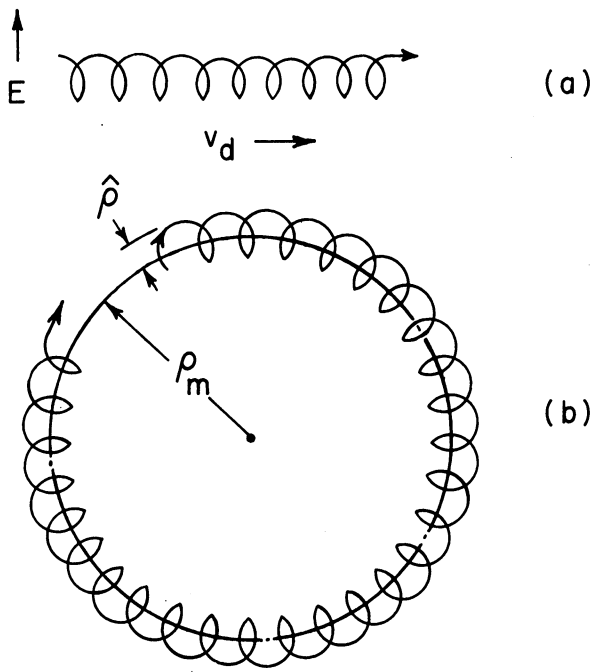
the classical electron's cycloidal motion takes it slowly around the nucleus as shown in Figure 1. These orbits are just a superposition of the circular orbits with origins $\varrho \neq 0$ but with the orbit centers guided to revolve about the origin of **E**. The strength of the (weak) electric field controls only the rate of revolution about the origin and has only a very slight effect upon the electron confinement or energy. For the lowest energy orbits whose guiding center radius $\gg r_1$ the quantization condition becomes

$$2\pi m\hbar = \oint \mathbf{p}\cdot d\boldsymbol{\varrho} \sim \oint \frac{e\mathbf{A}}{c}\cdot d\boldsymbol{\varrho}\,. \tag{9}$$

The quantization of (9) thus determines the guiding center radii to be

$$\varrho_m = (2m)^{1/2}\,\hat{\varrho}\,; \quad m = 1, 2, \ldots, \infty \tag{10}$$



Figs. 1a–b. (a) Motion of an electron in crossed electric and magnetic fields. (b) Motion in a radial electric field with perpendicular **B** out of the plane.

for a set of states all of which in the limit $\mathbf{E}\to 0$ have energy

$$E = \hbar\omega_c \quad (\text{all } m). \tag{11}$$

The angular momentum quantum number m mainly determines how far away the electron is from the origin rather than its $m_e v\varrho$. The width of the cycloidal orbit $\sim \hat{\varrho}$ for all m.

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The criterion for a 'weak' electric field is that the difference in potential across the orbit satisfy

$$eE\hat{\varrho} \ll \hbar\omega_c . \tag{12}$$

Applied to an atom this is equivalent to

$$\hat{\varrho} \ll a_0 , \tag{13}$$

i.e., the magnetic confinement radius is much less than that from the atomic electric fields, the Bohr radius a_0. Equation (13) gives

$$B \gg \left(\frac{e^2}{\hbar c}\right)^2 \frac{m_e^2 c^3}{\hbar e} = 4 \times 10^9 \text{ G} . \tag{14}$$

This is the criterion that the magnetic field be sufficiently large that all electrons in ground state atoms or condensed matter be in the lowest magnetic energy states, given by (11).

The magnetic confinement will force the electrons to be relativistic when $\hat{\varrho} \lesssim \hbar/m_e c$ or $\hbar\omega_c \gtrsim m_e c^2$. These criteria imply that a necessary condition for electrons to be non-relativistic is

$$B \ll \frac{m_e^2 c^3}{\hbar e} = 4.4 \times 10^{13} \text{ G} . \tag{15}$$

The inequalities (14) and (15) are probably well satisfied for neutron star surfaces. (Relativity does not, however, introduce great complications into atomic calculations when fields exceed 4.4×10^{13} G.)

The wave mechanical description of the classical orbit of Figure 1b uses the 'Landau orbital' (Landau and Lifschitz, 1965) solutions of the Schroedinger equation with uniform magnetic field (in the z-direction):

$$\psi_{m,k} = \exp\left(\frac{-\varrho^2}{4\hat{\varrho}^2}\right) \varrho^{|m|} \exp(\mathrm{i}m\phi) \exp(ikz) . \tag{16}$$

This wave function represents a free particle in the z-direction, magnetically confined in the perpendicular direction with a maximum probability at

$$\bar{\varrho} = (2m+1)^{1/2} \hat{\varrho} ; \quad m = 0, 1, \ldots, \infty . \tag{17}$$

The angular momentum $m\hbar$ is mainly $e\boldsymbol{\varrho} \times \mathbf{A}/c$. The energy of the states represented by the wave function of (16) is

$$E = \frac{p_z^2}{2m_e} + \hbar\omega_c + \frac{e\boldsymbol{\sigma} \cdot \mathbf{B}}{m_e c} \tag{18}$$

with $\boldsymbol{\sigma}$ the Pauli spin matrices. All states not represented by (16) are higher in energy by an integral multiple of $\hbar\omega_c$, tens of keV on a pulsar surface. Similarly, parallel and anti-parallel electron spin differ by $\hbar\omega_c$. A complete set of magnetic ground state wave

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functions is given by (16) with anti-parallel electrons. From (18) the corresponding electron energies are

$$E = \frac{P_z^2}{2m_e} \quad (\text{all } m). \tag{19}$$

2. Atoms

The hydrogen atom in a superstrong magnetic field contains an electron whose binding in the z-direction is caused by Coulomb attraction but whose confinement in the perpendicular direction is magnetic. The ground state is composed of an $m=0$ Landau orbital except that $\exp(ikz)$ is replaced by a real function of z chosen to minimize the total energy. The probability distribution is roughly a long cylinder of length l and radius $\hat{\varrho}$ with $l \gg \hat{\varrho}$ (Figure 2). Then, very roughly,

$$E \sim \frac{\hbar^2}{m_e^2 l} - \frac{e^2}{l} \ln\left(\frac{l}{\hat{\varrho}}\right) \tag{20}$$

and minimizing with respect to l yields

$$l \sim \frac{a_0}{\ln\left(\frac{a_0}{\hat{\varrho}}\right)} \tag{21}$$

and

$$E \sim -\frac{\hbar^2}{m_e a_0^2}\left[\ln\left(\frac{a_0}{\hat{\varrho}}\right)\right]^2 \tag{22}$$

(cf. Haines and Roberts, 1969). In the limit of huge fields $a_0 \gg l \gg \hat{\varrho}$ and the binding energy E grows like $l \ln^2 B$.

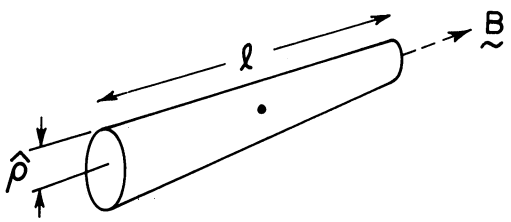

Fig. 2. Schematic hydrogen atom in a huge magnetic field.

(An analogue of the hydrogen atom in superstrong **B** is obtained within some solids with achievable **B** because the dynamic mass of an electron m^* can be enormously smaller than m where the curvature in an energy band is great. Then (14) gives a critical **B** smaller by $(m^*/m)^2$. An additional reduction can come from dielectric reduction of the effective e. Bound electron-hole pairs [excitons] can be qualitatively altered in 10^4–10^5 G fields.) A more quantitative variational estimate of the hydrogen binding energy is given in Figure 3 for the regime $B \gg 10^{10}$ G (Cohen *et al.*, 1970). A

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free hydrogen atom on a pulsar surface would be expected to have a binding energy of a few hundred eV.

There are two classes of excited states of such simple atoms, neither of which involve excitation out of the ground magnetic state. Instead of the smallest radius cylinder $m=0$, higher values of m corresponding to cylinders of radius $\sim(2m+1)^{1/2}\,\hat{\varrho}$ of (17), can be used for the electron. From (22) the binding energy depends only logarithmically on this radius, so that for superstrong **B** these states are bound almost as tightly as the ground state. The second class of excited state consists in replacing the nodeless

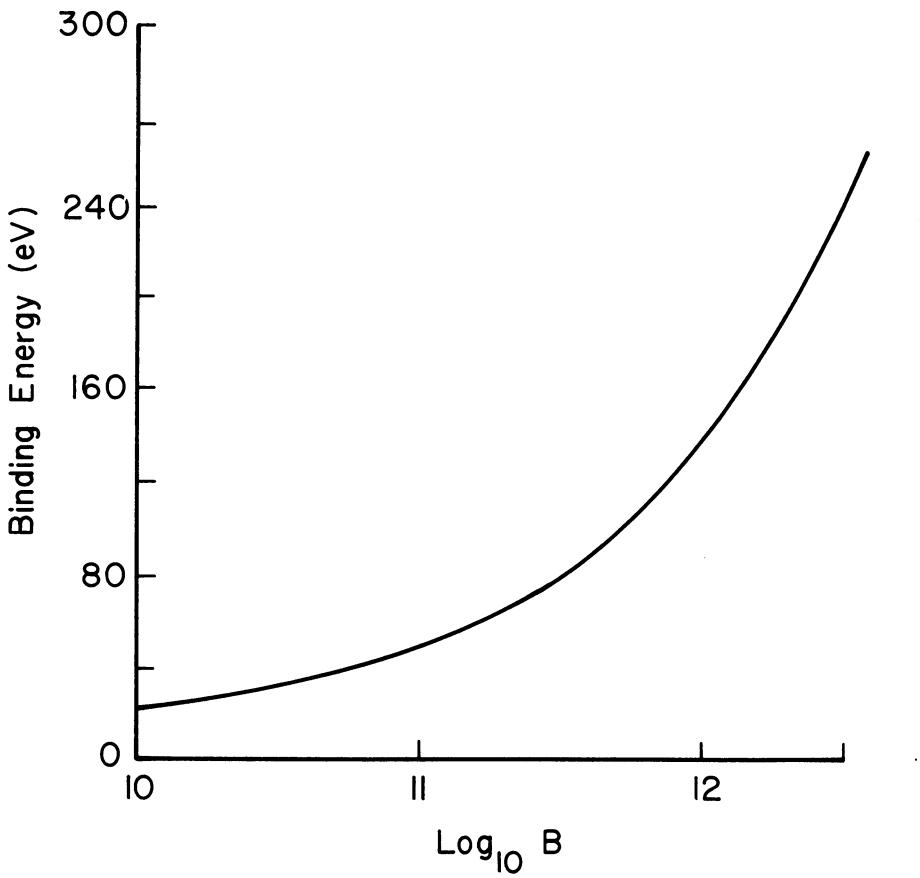

Fig. 3. Binding energy of a hydrogen atom in superstrong magnetic fields.

z-dependent part of the wave function by an orthogonal function with $\nu=1, 2, \ldots$ etc. nodes. All of these states are relatively very weakly bound (cf. Figure 4) and have a z-extent of order a_0 or greater rather than the l of (21). A simple way to estimate the energies of the second class of excited state has been pointed out by L. Spruch. Normal hydrogenic s-states satisfy the reduced wave equation $-\phi''-(2\,m_e/\hbar^2 r)\,\phi=E\phi$ with $\psi=\phi/r$. Therefore $\phi(0)=0$. The quasi-one dimensional atom in superstrong **B** satisfies

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almost the same equation with $r=z$ except that $\psi \leftrightarrow \phi$ and not the ground but the first excited odd state has the node at $z=0$. Therefore this one node excited state has about the same binding energy as the ground state of the normal atom. Multinode states are less bound as the number of nodes increases.

The simplest multielectron atoms are formed by filling consecutively the $m=0, 1, 2, \ldots, Z$ Landau orbitals with a single (spin anti-parallel to **B**) electron in each. From Figure 4 it is evident that putting two electrons in the same orbital by introducing excited states in the z-dependent component costs much more energy than putting the electrons slightly further away in ϱ. This continues to remain true only until the distance ϱ becomes comparable to $a_0 Z$; the ground states of heavy multinode wave atoms utilize functions to put many electrons into a Landau orbital.

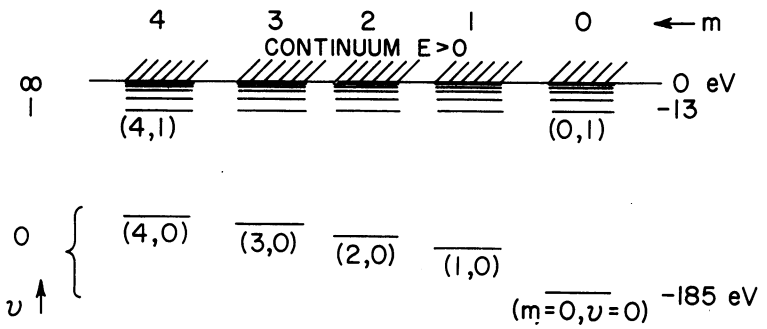

Fig. 4. Energy levels of a hydrogen atom in a 2×10^{12} G field (not to scale).

For the light atoms in the Hartree approximation each electron has its own pure Landau orbital. A variational calculation with such wave functions in a 2×10^{12} G magnetic field gives atoms whose shape and size is modeled in Figure 5 (Cohen *et al.*, 1970). The length of the cylindrical atoms is not sensitive to Z while the radii increase roughly as $(2Z+1)^{1/2}$. These atoms have all electron spins anti-parallel to **B**, huge quadrupole moments, and volumes of order 10^{-3}–10^{-4} that of normal atoms. The ionization energy (E_1) of the last electron of light atoms in $B=2 \times 10^{12}$ G has been estimated as

$$E_1 \sim 160 + 70 \ln Z \text{ eV}. \tag{23}$$

The $\ln Z$ term is the exchange energy contribution, calculated in perturbation theory. Calculations including exchange *ab initio* are in progress (Kaplan and Glasser, 1972). The ionization energy of (23) is typically two orders of magnitude greater than that for single ionization of normal atoms. It is also a smooth function of Z with none of the usual steep valleys and peaks. The absence of significant 'chemical' differences between atoms with neighboring Z's is a consequence of the one-dimensional nature of the filling of orbitals: there are, for example, no closed shells, which depend upon special degeneracies among three-dimensional angular momentum components.

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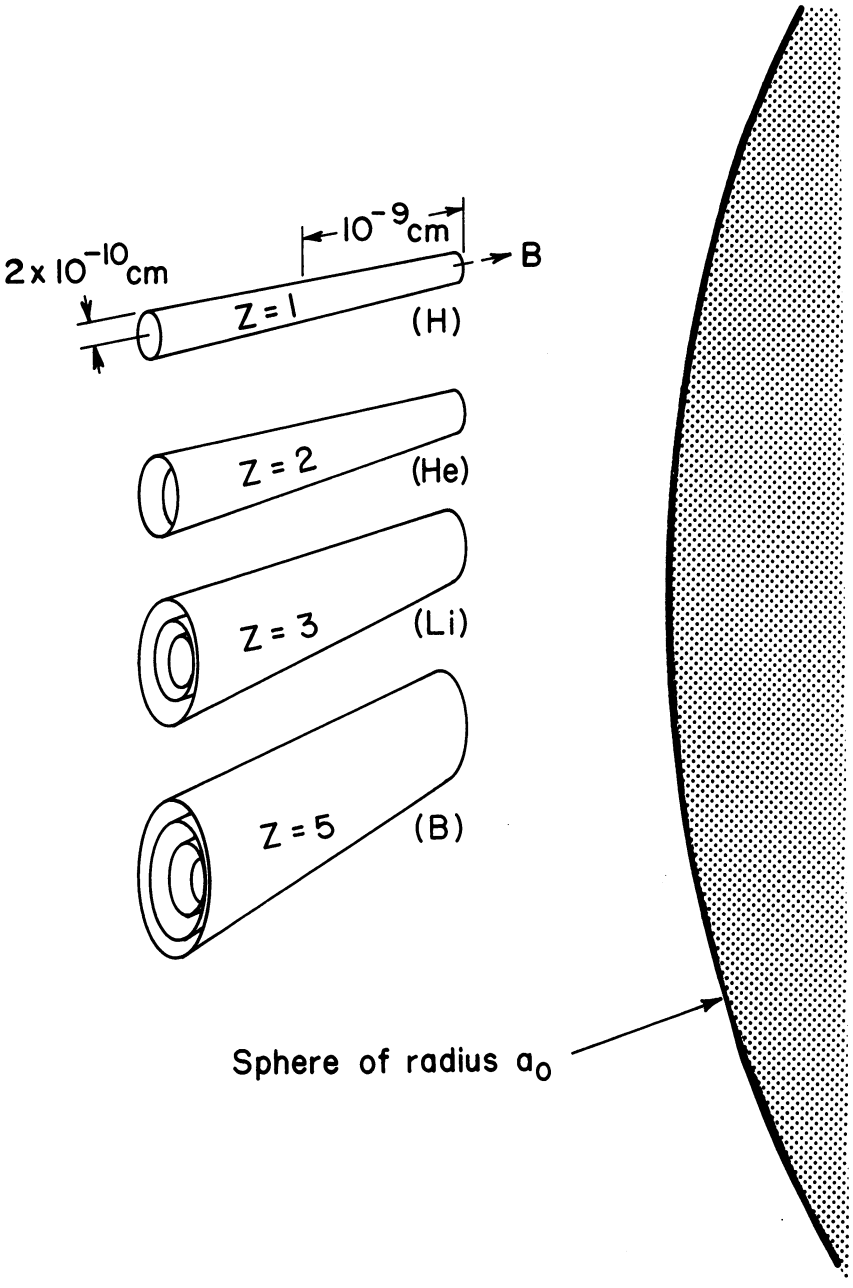

Fig. 5. Shapes and sizes of light atoms in a 2×10^{12} G field.

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For heavier atoms the recipe of achieving the lowest energy state of an atom by putting only one electron in each Landau orbital fails. If it did not the last electron would be on a cylinder whose minimal approach to the nucleus is the radius

$$\varrho_z = (2Z+1)^{1/2}\,\hat{\varrho}\,. \tag{24}$$

If, instead, the last electron were put into the closest orbital in its first excited state (i.e., a node at $z=0$ in the z-component part of the wave function), then its typical distance to the nucleus is a_0/Z. The dimensionless ratio

$$\eta \equiv \frac{a_0}{Z\varrho_z} = \left(\frac{B}{4.6\times 10^9\, Z^3}\right)^{1/2} \tag{25}$$

characterizes the different kinds of atoms which can exist (at $B\sim 5\times 10^{12}$ G, $\eta\sim 0.3$ for $Z=26$ and $\eta=33$ for $Z=1$).

(i) The regime $\eta \gg 1$, corresponding to $B \gg 4.6\times 10^9\, Z^3$ G, is that described above with one electron per orbital. The total binding energy of such an atom has been estimated (Kadomtsev and Kudryavtsev, 1971a) as

$$E \sim -\frac{9}{8}\frac{Z^3 e^2}{a_0}\ln^2\eta\,. \tag{26}$$

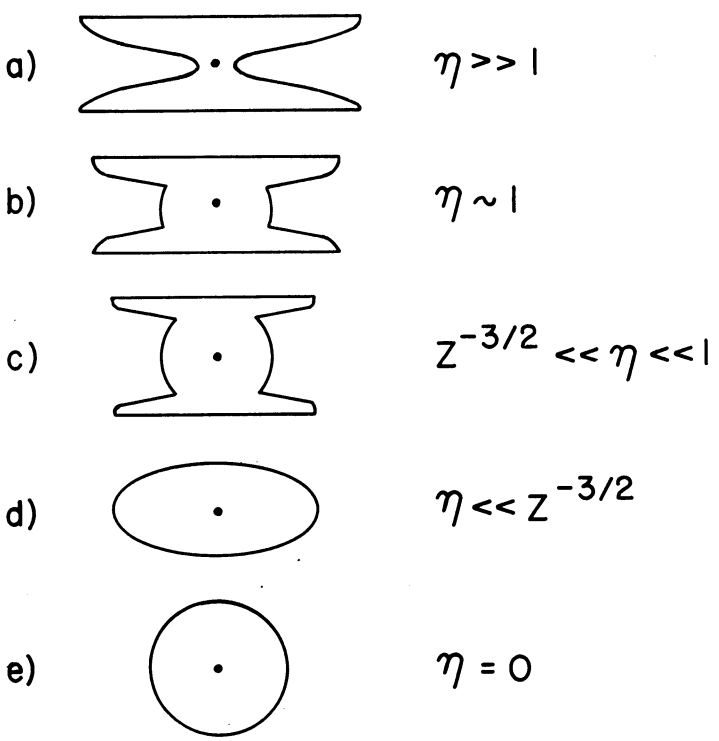

Fig. 6. Shapes of atoms in superstrong magnetic fields as a function of η of Equation (25) (not to scale).

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The shape of the atom as revealed by cutting the atom in half with a slice through its symmetry axis parallel to **B** is given in Figure 6a. Dr H.-H. Chen finds this regime appropriate for iron ($Z=26$) in a field of a few times 10^{12} G (Columbia Univ. doctoral thesis 1973).

(ii) The regime $\eta \sim 1$ is characterized by emptying some of the outer orbitals of regime (i) and placing those electrons into excited states in inner orbitals. This intermediate regime has not yet been considered quantitatively. Its presumed shape is that of Figure 6b.

(iii) The regime $1 \gg \eta \gg Z^{-3/2}$ still has sufficiently strong B that no excited magnetic states (n of equation [5]>1) are significantly populated. But all of the inner Landau orbitals contain many electrons, so many that a Fermi-Thomas treatment of the electrons in each level appears reasonable. A Fermi-Thomas treatment of atoms (or ions) in superstrong **B** gives a spherical shape of radius

$$R \sim \frac{a_0}{Z\eta^{4/5}} \tag{27}$$

and total energy

$$E \sim -150(B_{12})^{2/5} Z^{9/5} \text{ eV} \tag{28}$$

(Kadomtsev, 1970; Mueller *et al.*, 1971). The Landau orbitals furthest from the nucleus have too few electrons to be treated in this approximation. These electrons 'see' a spherical ion core of net charge $\hat{Z} < Z$ which is essentially impenetrable because of the Pauli principle. These extracore electrons fill successive Landau orbitals singly, beginning with that orbital whose cylinder radius $\varrho \sim a_0/\hat{Z}\eta^{4/5}$ (Ruderman, 1971). There are not yet quantitative descriptions for their wave functions. They form a cylindrical sheath surrounding a spherical core to give a shape sketched in Figure 6c. The thickness of the sheath (δ) is of order $\eta^{4/5}R$.

(iv) When $\eta \ll Z^{-3/2}$, $B \ll 4 \times 10^9$ G, and perturbation theory is adequate. The perturbative part of the Hamiltonian is

$$H' = \frac{e\mathbf{B}\cdot\mathbf{L}}{mc} + \frac{e^2B^2\varrho^2}{8mc^2}. \tag{29}$$

When spin orbit coupling is negligible its effect on atomic shape is a slight additional confinement perpendicular to **B**.

3. Condensed Matter

Atoms in superstrong fields can bind to each other extraordinarily tightly. Even if all other quantum effects are ignored, the huge quadrupole moments of isolated atoms will result in a very strong electrostatic interatomic attraction for certain orientations. In an orthorhombic configuration near b.c.c. nearest neighbors attract and next nearest repel. But the quadrupole-quadrupole force falls off so rapidly with spacing (r^{-6}) that the attraction dominates. However a quantum mechanical binding analogous to conventional covalent bonding appears to contribute the main binding energy.

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In ordinary matter the binding among atoms is essentially determined by the state of the least bound (valence) electrons. When these are shared between neighbors the binding energy is generally very much less than the total binding energy of the atoms. The wavelengths of almost all the atomic electrons, except the valence ones, are very much less than the interactomic spacing so that the state of most of the electrons is insensitive to the environment of the atom which contains them. This situation is very different for atoms in superstrong magnetic fields, especially when $\eta \gg 1$. All of the atomic electrons can be effective in binding, and in the limit $B \to \infty$ the binding between atoms can greatly exceed the total binding energy of an isolated atom (Kadomtsev and Kudryavtsev, 1971b).

The main difference between 'magnetic atoms' and conventional ones is the effect of the Pauli principle when atoms approach each other. Two hydrogen atoms with

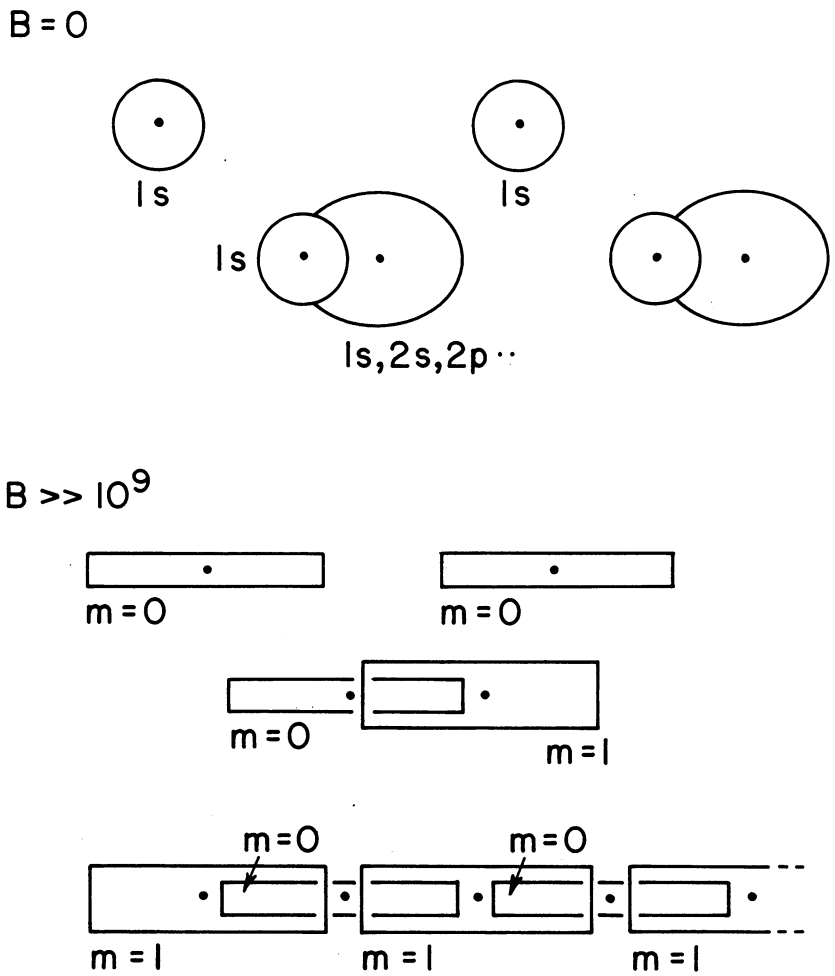

Fig. 7. Binding of magnetic hydrogen atoms into molecules and polymer chains. (All electron spins are antiparallel to **B**.)

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parallel electron spins do not bind because in order to have the electrons close to each other one of the electrons has to be promoted out of its normal 1 s state to the 2 s, 2 p, etc. in order to satisfy the exclusion principle. This takes a relatively enormous energy, i.e., one comparable to the binding of the 1 s electron. Magnetic atoms which approach each other along their symmetry axes can satisfy the exclusion principle with a relatively small energy expenditure (Figure 7). An excitation which consists just in changing a Landau orbital from state m to $m+1$ changes the average cylinder radius $\bar{\varrho}$ by only $\delta\varrho \sim (2m)^{-1}\bar{\varrho}$. This changed radius, according to (22), enters only logarithmically into the electron binding energy. In the regime $\eta \gg 1$ the energy needed to excite in this way is negligible next to the electron binding energy. Thus it is inexpensive to satisfy the exclusion principle and both electrons can be shared by two nuclei. A similar argument holds for atoms with $Z>1$: it does not cost an excessive amount of energy to alter Landau orbital quantum numbers m in order to make room for more electrons.

When two atoms join in this way, additional atoms can attach at the ends (cf. Figure 7) to extend the diatom to a chain of arbitrary length (Ruderman, 1971). The magnetic polymer formed in this way can be described as a chain of nuclei of some spacing (l) surrounded by a sheath of electrons which occupy Landau orbitals out to some spatial distance (r). Magnetic confinement holds them near to the symmetry axis along which the nuclei are distributed. The electrons are free to move in the z-direction subject only to the restrictions of Fermi statistics. They behave as if in a one-dimensional metal. For an infinitely long magnetic jelly roll polymer of this sort, the energy per atom (i.e., per cell of Z electrons) is

$$E_a = -\frac{(Ze)^2}{l}\left[\ln\frac{2l}{r} - \left(\gamma - \frac{5}{8}\right)\right] + \frac{2Z^3\pi^2\hbar^2}{3m_e l^2}\left(\frac{\hat{\varrho}}{r}\right)^4 \tag{30}$$

with γ the Euler constant.* A minimization with respect to l and r yields (in the limit $\eta \to \infty$)

$$r \sim \frac{1.3\, a_0}{Z\eta^{4/5}} \tag{31}$$

$$l \sim \frac{2.4\, a_0}{Z\eta^{4/5}} \tag{32}$$

and a total binding per atom

$$E_a \sim -\frac{0.5\, Z^3 e^2}{a_0}\eta^{4/5} \tag{33}$$

This binding energy of a magnetic polymer atom is greater than the estimate of the binding energy of the isolated magnetic atom given in (26) when $\eta \gg 1$. The polymer is,

* I am grateful to Mr Hsing-Hen Chen for pointing out that the fraction $\frac{3}{4}$ in the published result should be as in (30).

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therefore, extremely tightly bound. Its density (σ) is

$$\sigma \sim \frac{Z^4 m_n}{6a_0^3}\eta^{12/5} \sim 10^6\left(\frac{Z}{26}\right)^4 \eta^{12/5}\ \mathrm{g\ cm^{-3}}, \tag{34}$$

enormously greater than the 1–10 g cm^{-3} range for conventional matter.

The nuclei in the surface of a neutron star are probably not hydrogen. Most scenarios for the birth of a neutron star suggest iron peak elements with, perhaps, a very thin covering layer of helium. For helium $\eta \sim 12$ so that the approximations leading to (33) may be valid. (The $|E_a|$ computed there is at least a minimum total binding energy for any η.) Then $\sigma \sim 10^4$ g cm^{-3} and $E_a \sim -800$ eV in a 5×10^{12} G field. But if the nuclear constituent is iron with $Z=26$, $\eta=0.3$ and regime (ii) or (iii) obtains. Only the outer

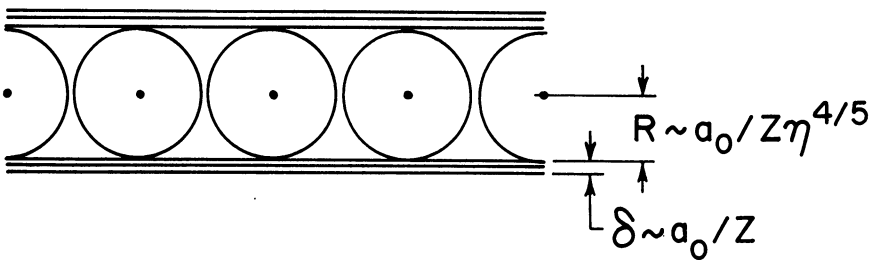

Fig. 8. Magnetic polymer chains in the regime $\eta \lesssim 1$.

sheath electrons contribute to the binding described above with the core electrons remaining almost unaffected (Figure 8). A very rough estimate for an iron magnetic polymer in a 5×10^{12} G field gives $E_a \sim -30$ keV and $\sigma \sim 4\times 10^4$ g cm^{-3}. (See note added in proof on page 131.)

The strength or Young's modulus of a single polymer chain $\sim E_a/l \sim 10$–10^2 dyne. A bundle of polymers has a modulus $\sim 10^{19}$ dyne cm^{-2}, about 10^7 that of steel.

A polymer chain will attract a neighboring one electrostatically even if van der Waals polarization attraction is ignored. The periodically distributed nuclear charges

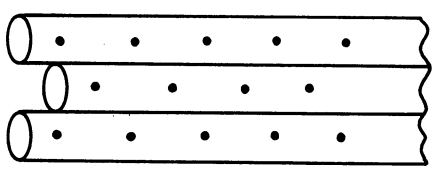

Fig. 9. Arrangement of adjacent magnetic polymer chains in condensed matter.

are imperfectly screened by the surrounding electron sheath. Therefore a magnetic polymer chain will be attracted to one right alongside itself but displaced half a lattice length along the field (Figure 9). (The resulting distribution of nuclei is close to that of a b.c.c. lattice, expected for nuclei in an approximately uniformly distributed electron background.) Matter formed of such chains will have the density σ in the range 10^4–10^5 g cm^{-3}.

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4. Neutron Star Surfaces

Even under its almost zero vapor pressure the magnetic matter described in Section 3 remains in its condensed high density form up to very high temperatures; only when kT approaches the energy released when an atom is bound into a magnetic polymer chain does condensed magnetic matter become a gas. For iron this will happen near $T \sim 10^8$ K; even for helium such a T approaches 10^7 K. Both of these temperatures are much higher than estimated surface temperatures of all expect possibly very young pulsars, those whose age is less than or of order 10^3 yr. Therefore if a neutron star surface region were to be undisturbed except for thermal agitation it would have an abrupt edge. Unlike a gaseous surface the pressure can drop to zero even though the matter is dense and hot. The neutron star surface would more closely resemble that of the earth than that of a normal star. The solid (or liquid) magnetic polymer surface of a neutron star has a thickness of only a few meters. Below this the pressure from the weight of this layer compresses the underlying matter to above 10^5 g cm^{-3}, where the electron Fermi energy exceeds $\hbar\omega_c \sim 25$ keV. Then magnetic effects no longer qualitatively alter the matter equation of state. (The nuclei may arrange themselves in a crystalline lattice but this is a solid caused by the extreme pressure.)

The electrical properties of the magnetic polymerized surface are those of a one-dimensional metal: a good conductor parallel to **B**, a poor one perpendicular to it. Like a metal, and unlike an ionized gas, it has a work function which opposes the emission of electrons by appropriately directed electric fields. The electron work function (W) is typically expected to be of order a few hundred eV to a keV. In addition, an electric field whose direction is such that it would pull positive ions from a hot stellar surface plasma may be entirely ineffective in pulling nuclei out of the condensed metallic surface.

In a pulsar older than, say, 10^5 yr the surface temperature is expected to be below 10^5 K so that $kT \ll W$. Then field emission from the surface would be the main source of electrons injected into the pulsar magnetosphere if bombardment of the stellar surface by energetic particles and radiation is ignorable. Only surface electric fields parallel to **B** are effective. Unfortunately, it is just this component that is extremely model dependent in magnetosphere models. A conducting sphere (of radius R with surface dipole field B_s) spinning in a vacuum with angular frequency Ω has a surface electric field component $\mathbf{E}_s^0$ given by

$$\mathbf{E}_s^0 \cdot \mathbf{B}_s \sim \frac{\Omega R}{c} B_s^2 . \tag{35}$$

This would give a vacuum electric field component parallel to $\mathbf{B}_s$ of about 10^{12} V cm^{-1} for the Crab pulsar and about 10^{10} V cm^{-1} for an old pulsar whose period is 3 s. The actual electric field component parallel to $\mathbf{B}_s$ may be very much less. Thus, we write

$$\mathbf{E}_s \cdot \mathbf{B}_s \equiv \varepsilon \mathbf{E}_s^0 \cdot \mathbf{B}_s , \tag{36}$$

where $\varepsilon = 1$ for no magnetosphere, $\varepsilon = 0$ for an exactly corotating magnetosphere and

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ε is near zero for models with almost corotating magnetospheres. The field emission electron current from the electric field of (35) and (36) is

$$j \sim 10^3 F^2 \exp\left(-\frac{30 WP}{\varepsilon B_{12}}\right), \tag{37}$$

where j is the current in A cm^{-2}, F is $\mathbf{E}_s \cdot \hat{\mathbf{B}}_s$ in V cm^{-1}, W is the magnetic metallic surface's work function in keV, and P is the pulsar period in seconds. (For most models of the Crab pulsar $j \sim 10^5$–10^7 A cm^{-2}.) From (37) long period pulsars ($P \gtrsim 3$ s) might have their electric current emission turned off and be unobservable. Faster pulsars could have sufficient field emission of electrons but the outgoing flow of negative charge would be balanced by electron inflow along other channels rather than positive ion outflow as conventionally assumed. Unfortunately uncertainties about the external flow of energetic particles and radiation onto the stellar surface preclude firm conclusions at present.

Note added in proof. Dr Chen in his thesis (*loc. cit.*) finds the additional binding energy for a chain of iron atoms in 2×10^{12} G to be about 10 keV/atom. The lattice spacing is 0.42 Å; 8 electrons per atom remain localized around each iron nucleus while the other 18 move freely in the surrounding cylindrical sheath of radius 0.25 Å.

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