

and the intermediate field extensions of a Galois extension. This leads immediately to the question of the extent to which  $L(G)$  determines the group  $G$ . Röttlander showed that there are nonisomorphic groups  $G_1, G_2$  and a lattice homomorphism  $\phi: L(G_1) \rightarrow L(G_2)$  such that  $|U:V| = |U^\phi:V^\phi|$  whenever  $V \leq U \leq G$ .

The two basic questions in the theory are:

- (I) Given a lattice  $L$  satisfying property  $\mathcal{X}$  what can one say about groups  $G$  with  $L(G) \cong L$ ?
- (II) Given a class of groups  $\mathcal{Y}$  what can one say about the lattice  $L(G)$  for  $G \in \mathcal{Y}$ ?

An early example of a result of type (I) is Ore's theorem that if  $L(G)$  is distributive then  $G$  is locally cyclic. Further results concern  $M$ -groups or groups with modular subgroup lattice. Iwasawa characterized both locally finite and nonperiodic  $M$ -groups. For periodic groups the author has given a characterization in terms of extended Tarski groups.

In  $M$ -groups every subgroup is modular but modular subgroups arise throughout in more general situations. One usually wishes to get information about the normal structure of a group but if  $N$  is a normal subgroup of  $G$  and  $\phi$  is a projectivity from  $G$  to  $\bar{G}$  (i.e., a lattice isomorphism from  $L(G)$  to  $L(\bar{G})$ ) then the image  $V^\phi$  need not be a normal subgroup of  $\bar{G}$ . However the image is modular and this leads to consideration of the structure and embedding of  $M/M_G$  and  $M^G/M_G$  for a modular subgroup  $M$  of a group  $G$ .

There are many examples given of the second type of result in which well-known classes of groups are characterized by giving conditions on the lattice of subgroups.

For finite groups one can characterize simple, perfect, soluble and supersoluble groups. The author remarks that Ore's theorem and this characterization of finite supersoluble groups are the only results known in which the lattice-theoretic and group-theoretic conditions both seem to be natural conditions. Most of these characterizations have been extended to infinite groups in recent years, results being obtained for simple, perfect, polycyclic, finitely generated soluble groups, etc. However there is still no lattice-theoretic characterization for the class of soluble groups.

The above can only give a taste of the results contained in this book and can give little impression of the range of techniques which have been developed.

The only previous volume devoted to this topic is that by Suzuki in the *Ergebnisse* series from 1956. That ran to 92 pages with 3 pages of references. This book is six times the length with 18 pages of references, reflecting the development of the subject over the last forty years.

Most group theorists will find results and techniques of interest here but the aim of the book is as a reference for specialists in the area. It will be particularly important for graduate students working in the area and suggests a number of possible directions for further investigation.

M. J. TOMKINSON

ROSENBLUM, M. and ROVNYAK, J. *Topics in Hardy classes and univalent functions* (Birkhäuser Advanced Texts, Birkhäuser, Basel, Berlin, Boston 1994) 264 pp, 3 7643 5111 X, £36.

It was generally believed that with the proof of the Bieberbach conjecture by Louis de Branges the principal motivation for the study of extremal problems for univalent functions had gone. Instead, the subject has been stimulated by the revelation of the close connection between such problems and the much studied problems of operator theory. The appearance of the generalized Jacobi polynomials like some *deus ex machina* in de Brange's original proof is now seen as perfectly natural. Indeed the polynomials never appear explicitly in the authors' presentation, the crucial step being a positivity result for the generalized hypergeometric function  $F(a, b, c, d, e | x)$ . This result is due to Askey and Gasper, but the history of such positivity results is long, going back beyond Fejér to the masters of the 19th century.

The book is essentially in two parts and all of the above is clearly set forth in the last three chapters. The crucial early idea was due to Löwner, who realised that much important

information on individual univalent functions can be obtained by embedding them by subordination in the so-called Löwner families and then studying the semi-group structure. I can think of no authors better able than the present ones to give an exposition of the modern theory of univalent functions and they have succeeded admirably. They conclude by remarking, "Since publication of the proof of the Bieberbach conjecture there has been a search for deeper significance of the methods. . . . The search continues, and it seems safe to say that the final chapter remains to be written." One only hopes that, when it *is* written, it is as well written as the present volume.

The first six chapters were originally intended for the authors' book *Hardy classes and operator theory* but even read on their own they give a cogent presentation of the role of Function Theory in Operator Theory. Many of the landmarks of the subject are considered, including the Nevanlinna and Hardy classes, the F. and M. Riesz theorem, Beurling's theorem on the shift operator and the Szegő–Kolmogorov–Krein theorem on weighted trigonometric approximation. There is a chapter on the Phragmén–Lindelöf principle applied to functions of exponential type in a half-plane.

The book is written for graduate students. It is a compelling introduction to this fascinating subject and is warmly recommended.

J. M. ANDERSON

BAYLIS, W. E. *Theoretical methods in the physical sciences: an introduction to problem solving using Maple V* (Birkhäuser, Basel, Berlin, Boston 1994) 304 pp, (a floppy disk is included), softcover, 3 7643 3715 X, £36.

The stated aim of this book is to cover the areas of mathematics which students of the physical sciences will need 'throughout their careers', which are accessible at (Scottish) second year level, but which may not be covered in more traditional Mathematical Physics courses. For some topics the examples are taken from physical situations and exercises lead the reader toward deeper exploration of these. The purpose of introducing Maple into the course is (I paraphrase) to remove some of the drudgery from manipulations so that the effects of different options can be more readily explored. I sympathise with this aim and read the book hoping to pick up a few tips for my own teaching.

The contents of the book are as follows. The first two chapters form an introduction to Maple and in addition cover such topics as units, dimensional analysis and radioactive decay as an example. Next follow chapters on approximations to a real function, vectors, basic statistics, curve fitting, integration and complex numbers. The final chapter is devoted to Clifford algebras with the Pauli algebra and its application in special relativity and electromagnetism as an example.

In addition to the book there is a 3.5 inch disk with Maple worksheets for each chapter. A comparison of Chapter 3 (*Approximating Real Functions*) in the book and the corresponding worksheet shows that there is quite a lot of overlap, so that points of mathematics are explained in both and identical Maple commands are in both. One could use only the worksheets and find out much of what is in the book and at the same time see the mathematics come to life. Indeed, since there are no Maple outputs or plots in the text, the worksheets are needed to appreciate what is going on. However an example in the book on simple harmonic motion is omitted from the worksheet and the section numbers in the two formats do not correspond.

The introduction of Maple is a help in avoiding discouraging amounts of pencil and paper work in areas such as approximating functions and curve fitting. The plotting routines are of great value in the investigation of differential equations. In some chapters however there is not much use of Maple, for example the ones on vectors and, surprisingly, statistics.

The shape of the book is determined by areas of mathematics and as a result the description of Maple commands is a bit unsystematic. It would have been a help to put the commands in the index at the end of the book. Most are introduced through examples and the syntax is