FAST WINDS IN PLANETARY NEBULAE

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ABSTRACT

A planetary nebula consists mainly of gas ejected slowly by a red giant. Its dynamics is dominated by the hot central star which is left behind later. In particular a fast wind from this star forms a bubble of hot gas which fills the inner part of the nebula and pushes the envelope into a shell. This shell remains only partly ionized for a considerable time. Its non-ionized part is subject to a Rayleigh-Taylor instability, and is expected to break up into fragments which remain behind in the HII part of the nebula.

I. INTRODUCTION

Good evidence exists that the central stars of planetary nebulae produce fast winds, having speeds of the order of 1000 km s^{-1}; these winds sweep into the gas which had previously been ejected, with much lower speed (10 km s^{-1}), by the red giant which was the progenitor of the nebula and its central star. The phenomena are discussed in some detail by Sun Kwok (1983) in a recent paper. Infra-red observations of circum-stellar masers confirm that red giants expel a flow of gas with the right sort of speed (Sun Kwok, 1983). For an early paper on the effect of a fast wind on an HII region see Dyson (1978).

The dynamics of the interaction between the fast wind and the slow envolope is the subject of a thesis by Lazareff (1981). His general model is that the fast wind shocks close to the central star and forms a bubble of hot gas. Material from the slow envelope is swept up into a shell of gas around the bubble. Lazareff considers at some length whether the hot gas can cool appreciably by contact with the HII region which forms on the inner side of the shell. He finds that this effect is present, but not dominant. The energy of the hot gas is therefore largely expended in pushing outward the envelope of slow-moving gas. Lazareff considers various sets of parameters, and finds that by and large the sequence of events is always much the same.

D. R. Flower (ed.), Planetary Nebulae, 305–316.

As it happens the most sensible model is also the simplest to handle. It has two main phases; from time $-t_o$ to time zero the central star ejects a mass M_o of gas, with low terminal speed U. During this period the star is a red giant.

After time t = 0 the central star has evolved to a more compact structure, and has a high surface temperature (T ∿ 50 000 K). It also has a fast wind, which carries off a fraction of a per cent of the energy output. This mechanical luminosity is relatively small, but its importance lies in the fact that the wind soon shocks and turns into a hot gas (T ∿ 3 000 000 K) which is slow to cool.

In first approximation one finds that the shocked shell of gas moves out into the envelope at constant speed. A closer examination shows that the expansion of the HII part of the shell causes the neutral part of the shell to accelerate noticeably. An obvious consequence is that the neutral shell becomes subject to the Rayleigh-Taylor instability. The prediction is therefore that the nebula will lose its simple structure of nested shells, and that more and more pockets of non-ionized gas will be left behind in the HII region as the nebula evolves.

II. THE SIMPLE WIND-DRIVEN MODEL

The primary injection of gas from the central star comprises a mass M_o with terminal speed U, released during time $-t_o < t < 0$. It gives rise to a density distribution

$$\rho = \omega / r^2 \qquad \text{with} \qquad \omega = M_o / 4\pi U t_o \, . \tag{1}$$

From time t = 0 onwards a fast wind blows. The central star is now hot and has a luminosity L, the wind speed is $V (\gg U)$ and the rate of input of energy into the wind is ηL. Typical values are as follows:

$$L = 5 \times 10^{36} \text{ erg s}^{-1}, \quad \eta L = 2 \times 10^{34} \text{ erg s}^{-1}, \; M_o = 4 \times 10^{32} \text{ gm},$$
$$U = 10^6 \text{ cm s}^{-1}, \; V = 2 \times 10^8 \text{ cm s}^{-1}, \; t_o = 3 \times 10^{11} \text{ s}.$$

It is generally found, in the case of main-sequence O stars, that the wind energy output is of order V/c times the luminosity (Cassinelli, 1979). This has been taken to apply here also, and explains the choice of the value for η.

The fast wind drives a shock into the primary gas, and sweeps it into a shell. In this simple treatment the shell is taken to be thin; let r be its radius at time t. Another shock facing inwards sits in the fast wind close to the star. The overwhelming bulk of the material in the fast wind is shocked and hot, and expands at a very subsonic speed. The equations governing the motion of the system are:

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Energy in the shocked fast wind gas

$$\frac{d}{dt}(2\pi P r^3) = \eta L - 4\pi P r^2 \dot{r} . \qquad (2)$$

Here P is the pressure, and a balance is struck between the energy input from the stellar wind and the work done by the pressure in expanding the shell. There is no allowance made for radiative or conductive losses from the hot gas. This approximation is justified later.

Mass of the swept up shell:

$$\frac{dM}{dt} = 4\pi \omega (\dot{r} - U) = \frac{M_o}{U t_o}(\dot{r} - U) \qquad (3)$$

Pressure balance at the outer shock:

$$P = \frac{M \ddot{r}}{4\pi r^2} + \frac{\omega}{r^2}(\dot{r} - U)^2 \qquad (4)$$

The two terms on the right hand side come from the acceleration of the gas in the shell and from the transfer of momentum to the newly swept up gas.

There is a simple solution to these equations with

$$r = \lambda U t \quad \text{and} \quad \dot{r} = \lambda U, \qquad (5)$$

$$P = \varpi / t^2 \quad \text{where} \quad \varpi = \frac{(\lambda - 1)^2 M_o}{4\pi \lambda^2 U t_o}, \qquad (6)$$

and

$$M = (\lambda - 1) M_o t / t_o . \qquad (7)$$

The parameter λ satisfies the equation

$$\lambda(\lambda - 1)^2 = \frac{2 \eta L t_o}{3 M_o U^2} \equiv N, \text{ say,} \qquad (8)$$

whose solution is tabulated below:

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N	0	2	5	10	20	40
λ	1	2	2.44	2.87	3.42	4.12

With the typical values quoted before, N = 10. In this calculation the parameter λ is set equal to 3, in reasonable approximation. The shocked shell therefore moves outwards at 30 km s^{-1}.

Now to check the various assumptions.

i) The backward facing shock sits in the fast wind close to the star at radius r_c. To produce the correct post shock pressure, which is given by (6), requires that

$$\frac{3\dot{\mathcal{M}}V}{16\pi r_c^2} = \left(\frac{\lambda-1}{\lambda}\right)^2 \frac{M_o}{4\pi U t_o t^2}, \tag{9}$$

where $\dot{\mathcal{M}} \equiv 2\eta L/V^2$ is the rate of injection of mass into the fast wind. There is a small error (of about 6 per cent) in this equation because there is no allowance made for the fact that the newly shocked gas, at r_c, has a finite speed. It follows from (9) that

$$r_c = \left(\frac{3}{2}\right)^{1/2} \frac{\lambda}{\lambda-1} \left(\frac{\eta L U t_o}{M_o V}\right)^{1/2} t, \tag{10}$$

and that the ratio of the radius of the inner shock to that of the shell is

$$\frac{r_c}{r} = \frac{1}{(\lambda-1)}\left(\frac{3\eta L t_o}{2 M_o U V}\right)^{1/2} = \frac{3N^{1/2}}{2(\lambda-1)}\left(\frac{U}{V}\right)^{1/2} = \frac{3}{2}\left(\frac{\lambda U}{V}\right)^{1/2} \tag{11}$$

and is always small; with the present assumed physical values it equals 0.18. The hot shocked wind gas therefore occupies all but 0.6 per cent of the volume enclosed by the swept-up shell.

ii) The density of the hot shocked gas is

$$\rho_h = \frac{3\dot{\mathcal{M}}t}{4\pi\lambda^3U^3t^3} = \frac{3\eta L}{2\pi\lambda^3U^3V^2t^2} \tag{12}$$

and therefore the temperature is

$$T_h = \frac{\bar{m}P}{k\rho_h} = \frac{\bar{m}\lambda(\lambda-1)^2M_oU^2V^2}{6k\eta L t_o} \equiv \frac{\bar{m}V^2}{9kN}. \tag{13}$$

With the present numbers, and with $\bar{m} = 10^{-24}$ gm for a fully ionized gas

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$T_h = 3.2 \times 10^6$K.

The cooling of a gas at a temperature in the million degree range can be simply described in terms of the adiabatic parameter K ($\equiv P/\rho^{5/3}$) by the equation

$$\frac{d}{dt} K^{3/2} = -q \tag{14}$$

(Kahn, 1976), with $q = 4 \times 10^{32}$ ($cm^6\ gm^{-1}\ s^{-4}$) for a gas with the usual cosmic composition. From relations (6) and (12)

$$K^{3/2} \equiv \frac{P^{3/2}}{\rho^{5/2}} = \frac{\pi}{2^{1/2}\ 3^{5/2}} \frac{\lambda^{9/2}(\lambda-1)^3 M_o^{3/2} V^5 U^6 t^2}{(\eta L)^{5/2}\ t_o^{3/2}}, \tag{15}$$

The gas cannot cool effectively when $K^{3/2}$ much exceeds qt, or when

$$t \gg \frac{2^{1/2}\ 3^{5/2}(\eta L)^{5/2}\ t_o^{3/2}\ q}{\lambda^{9/2}(\lambda-1)^3 M_o^{3/2} V^5 U^6} = \frac{81}{2} \frac{\eta L q}{\lambda^3 V^5 U^3}, \tag{16}$$

that is t much exceeds 3.75×10^7s, in the present case, or about one year. In our model it has been assumed that there is a sudden change from a slow wind to a fast wind at time $t = 0$. But it seems unrealistic to think that the switch-over occurs in a time less than a year. Radiative cooling by the hot gas is therefore never important, in practice.

iii) There is heat loss at the "evaporation front" between the hot shocked wind and the HII part of the compressed shell. The effect has been studied by Lazareff (1981) who treated the problem in terms of particle-particle interactions, and did not consider collective plasma effects, other than the need to maintain space-charge neutrality. His conclusion was that there are noticeable energy losses, but that they have no decisive influence on the evolution of the planetary nebula.

It seems probable, though, that mirror and/or firehose instabilities will occur in the hot shocked wind. A large conduction flux in this gas sets up an anisotropic velocity distribution. The local magnetic field is likely to be weak, and so the instabilities occur very easily. Individual charged particles are then scattered by the inhomogeneities that are set up, and the conductive flux is reduced. This effect strengthens Lazareff's conclusion that heat losses at the evaporation front are not significant.

III. THE HII PART OF THE SHELL

So far the compressed shell has been regarded as being thin, but nevertheless it has important structural features. Its inner part will be ionized by the Ly-c flux from the central star. For a stellar temperature in the likely range, the rate of production of Ly-c photons is $j \equiv 10^{10}$ photons per erg emitted. The gas density in the HII shell is

$$\rho_i = \frac{\varpi}{c_i^2 t^2} = \frac{(\lambda-1)^2}{\lambda^2} \frac{M_o}{4\pi U c_i^2 t_o t^2} , \tag{17}$$

where c_i is the isothermal sound speed, say 10 km s^{-1}. If the shell is ionization limited, then the mass of ionized gas M_i is given by

$$jL = \frac{b M_i \rho_i}{m_a^2} ; \tag{18}$$

here m_a is the average atom or ion mass. Thus

$$M_i = \frac{4\pi \lambda^2}{(\lambda-1)^2} \frac{jL m_a^2 U c_i^2 t_o t^2}{b M_o} . \tag{19}$$

The fraction of the mass in the shell which is ionized at time t is

$$\xi = \frac{M_i}{M} = \frac{4\pi \lambda^2}{(\lambda-1)^3} \frac{jL m_a^2 U c_i^2 t_o^2 t}{b M_o^2} , \tag{20}$$

The shell has swept up all the primary injection gas at time $t = t_o/(\lambda - 1)$, when equation (20) gives formally

$$\xi = \xi_* = \frac{4\pi \lambda^2}{(\lambda-1)^4} \frac{jL m_a^2 U c_i^2 t_o^3}{b M_o^2} . \tag{21}$$

With the values being used here, $\xi_* = 1.19$. Of course ξ cannot exceed unity: the interpretation has to be that the shell is completely ionized when

$$t = t_i = \frac{t_o}{(\lambda-1)\xi_*} = \frac{0.84 t_o}{\lambda - 1} . \tag{22}$$

Up to that time the primary injection gas beyond the shell would be neutral, but soon after it will be overtaken by an R-type ionization

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front. For other parameters of the system it is of course possible that the shell sweeps up all the gas before it becomes fully ionized itself.

The neutral gas in the shocked shell will be cool: Lazareff thinks that a temperature of 100K is rather on the high side. In any case this gas will be highly compressed and confined to a thin layer. But the finite thickness of the ionized part of the shell can have important dynamical effects; at time t it equals

$$\Delta = \frac{M_i}{4\pi\lambda^2 U^2 t^2 \rho_i} = \frac{4\pi\lambda^2}{(\lambda-1)^4} \; \frac{j L m_a^2 c_i^2 t_o^2 t^2}{b M_o^2} \, . \tag{23}$$

The ratio of the shell thickness to the shell radius is

$$\frac{\Delta}{\lambda U t} = \frac{4\pi\lambda}{(\lambda-1)^4} \; \frac{j L m_a^2 c_i^4 t_o^2 t}{b M_o^2 U} = \frac{\xi c_i^2}{\lambda(\lambda-1) U^2} \, . \tag{24}$$

Here c_i^2/U^2 is about unity, $\lambda(\lambda - 1) = 6$, and so Δ is always small compared with λUt during the phase in which the shell is partially ionized ($\xi < 1$). Nevertheless the gradual thickening of the shell has interesting dynamical consequences.

IV. DYNAMICAL EFFECTS OF SHELL THICKENING

When the HII part of the shell expands it restricts the volume available for the hot shocked stellar wind, and therefore raises the pressure. As a consequence the non-ionized part of the shell accelerates outwards. This effect can be treated as a perturbation of the flow pattern described in Section II. The linearized treatment is strictly valid for early stages of the motion, when t is small. At that time only a small fraction of the mass of the shell is ionized. This has the advantage that the pressure difference across the HII part of the shell can be ignored. Further, ionized gas enters the HII shell via the ionization front, with speed v_i relative to the neutral shell. The speed v_i is important in connection with the stability at this front, but it can be ignored in calculating the momentum flux carried by the newly ionized gas.

Then let the neutral shell be at radial distance $r = \lambda Ut + R_a$, and the hot shocked wind/HII interface at $r = \lambda Ut + R_b$, and let the pressure be $P = \varpi/t^2 + \Pi$. From relation (23)

$$\Delta = R_a - R_b = \frac{4\pi\lambda^2}{(\lambda-1)^4} \; \frac{j L m_a^2 c_i^4 t_o^2 t^2}{b M_o^2} \, , \tag{25}$$

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The energy equation for the hot shocked wind gas is now

$$\frac{d}{dt}\left[2\pi(\lambda Ut+R_b)^3\left(\frac{\varpi}{t^2}+\Pi\right)\right]=\eta L-4\pi(\lambda Ut+R_b)^2(\lambda U+\dot{R}_b)\left(\frac{\varpi}{t^2}+\Pi\right) \quad (26)$$

Pulling the first order part out of equation (26) gives, after some simplification, that

$$\lambda Ut^3\dot{\Pi}+5\lambda Ut^2\Pi+5\varpi\dot{R}_b+4\varpi R_b/t=0. \quad (27)$$

It is clear from relation (25) that R_a and R_b should vary like t^2 ; relation (27) then shows that Π varies like t^{-1} , and therefore

$$R_b=X_bt^2,\quad \Pi=-\frac{7\varpi X_b}{2\lambda Ut}=-\frac{7(\lambda-1)^2M_oX_b}{8\pi\lambda^3U^2t_ot}. \quad (28)$$

The momentum equation at the neutral shell is

$$\frac{\varpi}{t^2}+\Pi=\frac{M_o(\lambda-1)t\ddot{R}_a}{4\pi\lambda^2U^2t^2t_o}+\frac{M_o}{4\pi Ut_o}\frac{[(\lambda-1)U+\dot{R}_a]^2}{(\lambda Ut+R_a)^2} \quad (29)$$

provided that the bulk of shocked gas is still non-ionized. The first order part of equation (29) gives that

$$\Pi=\frac{M_oX_a}{2\pi U^2t_ot}\frac{(2\lambda+1)(\lambda-1)}{\lambda^3} \quad (30)$$

The comparison between relations (28) and (30) shows that

$$X_b=-\frac{4(2\lambda+1)}{7(\lambda-1)}X_a \quad (31)$$

and so, from equation (25)

$$X_a=\frac{28\pi\lambda^2\dot{j}Lm_a^2c_i^4t_o^2}{3(\lambda-1)^3(5\lambda-1)6M_o^2}, \quad (32)$$

The expansion of the HII shell produces an acceleration $2X_a$ of the neutral shell; it is independent of the time t , and can be expressed in terms of the time t_i , when the shell is fully ionized, by

$$g_a = 2X_a = \frac{14 c_i^2}{3(5\lambda - 1)\, U t_i} \tag{33}$$

With the values adopted in this paper, $t_i = 1.3 \times 10^{11}$ s and the acceleration equals 2.6×10^{-6} cm s^{-2}.

V. STABILITY OF THE NEUTRAL SHELL

The gas in the neutral part of the shell is much denser than the HII gas which is accelerating it outwards. Taking c_o (= 1 km s^{-1}) to be the sound speed in the neutral gas, one gets that the thickness of the shell is

$$\delta = \frac{M c_o^2}{4\pi \lambda^2 U^2 \varpi} = \frac{c_o^2 t}{(\lambda - 1) U} \tag{34}$$

The maximum growth rate for the Rayleigh-Taylor instability is

$$\sigma_g \sim \sqrt{g_a / \delta} = \left[\frac{14(\lambda - 1)}{3(5\lambda - 1)}\right]^{1/2} \frac{c_i}{c_o} (t_i t)^{-1/2} \tag{35}$$

The growth is opposed, to a certain extent, by a damping effect associated with the ionization front. Imagine a corrugated front in which a particular portion projects a distance z beyond the mean surface. If n_i is the ion density in the HII region then this portion is illuminated by an additional number $b n_i^2 z$ of Ly-c photons, per unit area and unit time. This extra flux tends to remove the corrugation because the HI gas, with atom density n_o, must supply an extra number of ions; if this effect were present alone, i.e., no Rayleigh-Taylor instability, then we should have

$$n_o\, dz/dt = -b n_i^2 z . \tag{36}$$

The particle densities n_o and n_i are related by the condition of pressure equilibrium which demands that

$$n_i / n_o = c_o^2 / c_i^2$$

and so the damping rate due to this effect is

$$\sigma_d = b n_i c_o^2 / c_i^2 . \tag{37}$$

Since $n_i \equiv \rho_i/m_a$ it now follows from relation (17) that

$$\sigma_d = \frac{(\lambda-1)^2 \, b M_o c_o^2}{4\pi \lambda^2 \, m_a U c_i^4 t_o t^2} \tag{38}$$

or in terms of t_i

$$\sigma_d = \frac{jL m_a c_o^2 t_i t_o}{(\lambda-1) M_o c_i^2 t^2} . \tag{39}$$

Clearly the Rayleigh-Taylor instability and the damping process work in opposite directions. The net effect is that the instability will grow when σ_g exceeds σ_d , that is when

$$\xi \equiv \frac{t}{t_i} > \left[\frac{3(5\lambda-1)}{14(\lambda-1)^3}\right]^{1/3} \left(\frac{jLt_o}{\mathcal{N}_o}\right)^{2/3} \frac{c_o^2}{c_i^2} . \tag{40}$$

Here $\mathcal{N}_o \equiv M_o/m_a$ is the number of atoms + ions in the nebula, typically 2×10^{56}. The first factor on the right hand side in (40) is of order unity, unless $\lambda - 1$ is very small which would imply that the stellar wind is very weak. With our chosen values the condition is $\xi > 0.20$.

There is therefore a substantial period in the evolution of a planetary nebula during which the shocked shell is only partially ionized, and the neutral part of the shell is subject to the Rayleigh-Taylor instability. The fastest growing disturbances have a length scale comparable with the thickness of the shell; in the present case this is of the order of

$$\delta = 10^{15} \text{ cm}.$$

At a typical stage the mass per unit area in the neutral shell is some 3×10^{-4} gm cm^{-2}: the individual neutral fragments therefore have a rather low mass, typically 3×10^{26} gm.

VI. CONCLUSION

A planetary nebula originates in the slow ejection of gas from a red giant. It becomes ionized when the central star has shrunk to a small hot object. If, as seems likely, this star also produces a fast wind, then the structure of the nebula is dominated by the bubble of hot gas that forms around the star. The HII part of the nebula will at first be confined to the inner part of the compressed shell that surrounds the hot bubble. At later stages there are two possibilities: if the shell becomes fully ionized before it has swept up all the gas in the envelope, in this case the remaining gas in the nebula becomes fully ionized soon afterwards. Alternatively the whole envelope is swept up before the shell is fully ionized. This simple picture ignores a Rayleigh-Taylor instability which can break up the non-ionized part of the swept-up shell. The instability sets in at a relatively early stage, perhaps after one-fifth of the time needed to sweep up the whole envelope. It results in the formation of blobs of non-ionized gas which tend to be left behind in the HII region.

There is also the possibility for more violent instabilities to occur at a later stage. Consider a case where the shocked shell reaches the boundary of the envelope while it is still only partly ionized. It now suffers a considerable acceleration because there is no longer any drag from the newly swept up gas. There has not been time to analyse this effect in the present talk, but clearly it must be significant in nebulae which contain a large mass of gas.

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BEGELMAN: What are the prospects for observing soft X-rays from the shocked wind?

KAHN: The hot shocked wind cools quite slowly, mainly by excitation of spectral lines of various impurities in the far ultraviolet. Bremsstrahlung contributes only a small ($\approx$ 1%) fraction to this low cooling rate. So the prospects are not too good.

SEATON: Could one expect to observe a coronal-type spectrum from the hot bubble?

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KAHN: For a PN of age 10^3 y, the hot bubble cools by radiation on a time scale of 10^6 y, mainly by electron excitation of various metal ions. For a wind luminosity of, say, 10 $L_\odot$ this gives a luminosity of 10^{-2} $L_\odot$ in the cooling lines. For different times, t, the cooling rate varies like $1/t^2$.

SEATON: What might one expect to be the observable differences in properties of nebulae with and without winds?

KAHN: When there is a fast wind, the nebular gas is partly swept into a shell. For young PN, only the inner part of the shell is ionized. Later the shell should fragment, leaving behind neutral globules, each surrounded by an ionized layer. An individual globule would evaporate in about 10^3 y.

NUSSBAUMER: We made a preliminary investigation of the possibility of observing hot ($T > 10^6$ K) gas in the young PN V1016 Cyg. The observed X-ray flux can be explained by the hot central star ($T_* = 160\,000$ K) alone. A search for forbidden Fe X and Fe XIV in the visual spectrum has been negative. Thus, at present, there are no observational indications of a very hot gas. However, the uncertainties involved in interpreting the "Einstein" X-ray data and the quality of the visual data are such that the existence of such hot gas cannot be ruled out.

KWOK: In the calculations that I did, conduction of the Lorentz gas is assumed to be the major cooling mechanism and the temperature of the hot zone is found to vary as $t^{-2/7}$. Do you obtain similar results?

KAHN: Following Lazareff, I did not include the dynamical effect of this cooling process.

HARRINGTON: Once globules have been produced inside the hot shocked region, they will be important in cooling the gas. This problem merits more careful attention.

KAHN: Yes, the combined areas of the interfaces between the ionized jackets of the globules and the hot gas could be quite large. One should repeat Lazareff's estimate of the heat loss to be expected at such contact surfaces.

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