

Precise and accurate interpolated stellar oscillation frequencies on the main sequence

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Abstract. High-quality data from space-based observatories present an opportunity to fit stellar models to observations of individually-identified oscillation frequencies, not just the large and small frequency separations. But such fits require the evaluation of a large number of accurate stellar models, which remains expensive. Here, we show that global-mode oscillation frequencies interpolated in a grid of stellar models are precise and accurate, at least in the neighbourhood of a solar model.

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Asteroseismology from space presents an opportunity to fit stellar models using sets of individual mode frequencies for a large and growing number of solar-like oscillators. This wealth of data can tightly constrain stellar models but most fitting methods require either a large number of model evaluations for a single star or a fixed grid of low resolution to model many stars. Here, we show that interpolating in a grid of main-sequence models can provide precise and accurate stellar oscillation frequencies. The interpolation is much faster than the calculation of similarly accurate stellar models and overcomes deficiency in the grid resolution. Though stellar model frequencies have previously been interpolated linearly along evolutionary tracks (e.g. Kallinger *et al.* 2010), and thus in age, we are unaware of any previous attempts to interpolate in other stellar model parameters.

We computed a grid of models using the stellar evolution code CESTAM (Marques *et al.* 2013) on a regular grid with 61 ages t from 3 to 6 Gyr, eleven masses M from $0.975 M_{\odot}$ to $1.025 M_{\odot}$, eleven initial metallicities Z from 0.016 to 0.021, eleven initial hydrogen contents X from 0.69 to 0.74 and seven mixing-length parameters α from 1.6 to 1.9. These were chosen such that the central model is Sun-like and the range is large enough to characterize parameter uncertainties. The oscillation frequencies were calculated with ADIPLS (Christensen-Dalsgaard 2008). For any set of parameters within the grid's boundaries, we interpolate the oscillation frequencies of the corresponding model with cubic splines.

As a first test of the accuracy of the interpolation, we computed dense sequences (50 times the grid resolution) of models by taking the central parameter values and varying one of M , Z , X or α at a time. (Age is discussed in the next paragraph.) The fractional differences between the oscillation frequencies of these models (for modes $17 \leq n \leq 25$, $0 \leq l \leq 2$) for the metallicity Z are shown in Fig. 1. We note two points. First, there is scatter on the order of 10^{-6} . We attribute this to the accuracy of the stellar evolution code, which has a numerical tolerance of 10^{-6} in the mass co-ordinate. Second, there only appears to be one curve because the results for 27 different oscillations modes are plotted

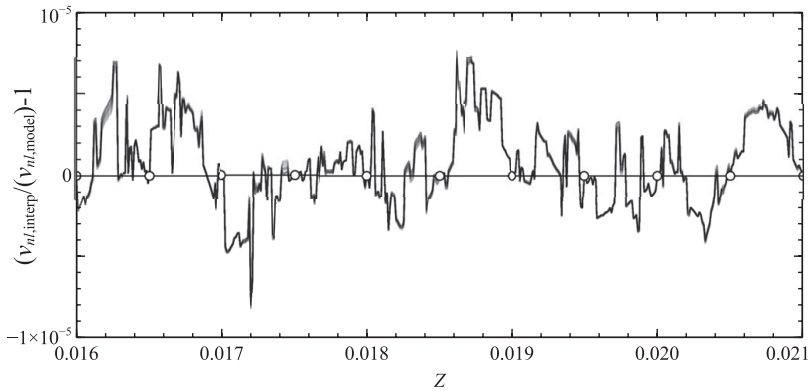


Figure 1. Fractional differences between modelled and interpolated frequencies along the metallicity Z , with the other parameters held fixed at $t = 4.5$ Gyr, $M = 1 M_{\odot}$, $X = 0.715$ and $\alpha = 1.75$. There are 27 lines plotted, each corresponding to a different oscillation mode ($17 \leq n \leq 25$, $0 \leq l \leq 2$) but the lines lie mostly on top of each other and cannot be distinguished. The solid white circles show the interpolation points. The interpolation error is dominated by scatter that we attribute to numerical noise.

over each other. The error induced by interpolation thus behaves like a nearly-perfectly correlated fractional error in all the frequencies.

To model the interpolation error, we divided the grid into two parts. The first contained models spanning the grid with twice the step in each parameter, e.g., six metallicities instead of eleven. We then used these models to interpolate at the parameter values of the other models and computed the differences in the frequencies between the stellar models and the interpolated values. For each frequency, the fractional errors are approximately normally distributed with the same scatter of about 4×10^{-6} for all modes. We assume the errors are perfectly correlated and construct a covariance for the model errors. This is added to the observed covariance matrix when fitting stellar models to observed frequencies. Ideally, the interpolation error would be everywhere much smaller than the observed error. For our interpolation routine, the fractional errors correspond to absolute errors up to about 10 nHz, which is several tens times smaller than frequencies derived from typical *Kepler* or *CoRoT* observations.

For the accuracy in the age t , we used every fourth model in the sequence of the central model to interpolate along that model’s evolutionary track. The output is qualitatively similar to Fig. 1 but accurate everywhere to a fractional error smaller than 10^{-6} .

Thus, stellar oscillation frequencies can be precisely and accurately interpolated, provided that the stellar models are themselves accurately calculated. This may increase the computational cost of the grid but not of the interpolation itself. Finally, the interpolation should be tested for any given grid and, if non-negligible, appropriately characterized, noting that the model errors might be strongly correlated.

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