

ON THE AVERAGE AMOUNT OF A SUM INVESTED AT COMPOUND INTEREST FOR THE LIFE OF THE INVESTOR.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—“If in a group of individuals sufficiently large to insure an average mortality, each invests £1 at interest, what will the several sums amount to, one with another, at the end of the year of the death of each investor?”

Assuming Annual contributions, Mr. Morgan, under a very vague description computed a table for dealing with this question, of which Professor De Morgan gives a more lucid account in the *Companion to the Almanac* for 1842, reprinted in the *Journal*, vol. xiii., p. 141.

A leading Mutual Office having lately adopted the above principle in disposing of a portion of its ascertained profits, has drawn attention to the subject when viewed in its relation of an investment by single Premium, and I have taken the trouble of computing the examples annexed to show the probable result.

The function required for each age  $x$  is  $(l_x - l_{x+1})(1+i) + (l_{x+1} - l_{x+2})(1+i)^2 + (l_{x+2} - l_{x+3})(1+i)^3 + \dots$  &c., divided by  $l_x$ . Multiplying both numerator and denominator by  $(1+i)^x$ , and then summing the terms of the numerator as in the Annuity Col. N, we obtain a table of the ordinary Commutation form, from which the value for any age is found by division. The examples given below are based on the last (male) Government Table, taking interest at three per cent.

| Age. | I<br>Assurance for<br>£100 Premium. | II<br>Accumulation<br>of £100. | III<br>Ratio of<br>accumulation to<br>Assurance Premium |
|------|-------------------------------------|--------------------------------|---|
| 20   | 279·767                             | 373·460                        | 133·490   |
| 25   | 259·274                             | 332·249                        | 128·146   |
| 30   | 239·774                             | 296·010                        | 123·454   |
| 35   | 221·180                             | 264·079                        | 119·396   |
| 40   | 203·650                             | 236·022                        | 115·896   |
| 45   | 187·377                             | 211·494                        | 112·871   |
| 50   | 172·451                             | 190·166                        | 110·272   |
| 55   | 159·125                             | 171·895                        | 108·025   |
| 60   | 147·255                             | 156·295                        | 106·140   |

The table reads as follows:—

I. In a group of persons aged 20, each paying £100 into a common fund, on the condition that all should share in the proceeds alike (an ordinary Life Assurance), the representatives of each would receive at the end of the year of death £279·767.

II. Were the payments invested independently (at the same rate of interest) they would amount, one with another, at the end of the year of death, to £373·460.

III. The Single Premiums which would assure £100 payable to each of this age, would, if separately invested by the individual contributors, realize for them, one with another (being the average of the one year's amount receivable by some, and the much larger accumulation obtainable by a few) the sum of £133·490.

The last column affords by inspection facility for contrasting the benefits at different ages.

As the benefit contains that element of uncertainty which offers an attraction to many, it is matter of surprise that no other Office has before now hit upon this novel mode of dealing with its surplus.

I am, Sir,

Your most obedient servant,

5, *Lothbury*,  
London, 7th December, 1867.

H. AMBROSE SMITH.

P.S.—It is worthy of note that the numerator of the first expression above, may be put into the following form :

$$l_x(1+i) + i\{l_{x+1}(1+i) + l_{x+2}(1+i)^2 + l_{x+3}(1+i)^3 + \dots \&c.\}$$

This form is more elegant than the other, and being quite as easy of computation it might be used if we had to deal with only a single age. But it is unsuited for the formation of a Commutation Table as the terms do not represent *annual* values.

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#### ON THE ADJUSTMENT OF PREMIUMS FOR LIFE ASSURANCE IN REFERENCE TO EXTRA RISKS.

*To the Editor.*

SIR,—There are two kinds of Extra Risk with which Assurance Companies in the ordinary transaction of their business have to deal, viz., that which arises from deteriorated health on the one hand, and that which consists in exposure to danger from some external cause, such as Climate, Military or Naval service, &c., on the other. The first risk is sometimes met by assuming an addition of a certain number of years to the actual age, and sometimes by a fixed addition to the annual premium, that is, fixed as regards the age and the nature of the assurance, but proportional to the sum assured. The second risk almost invariably by the latter method, except in those cases where the office is possessed of special Tables of Mortality founded upon observations made in the countries in which the parties reside, as India, for instance.

The adjustment of the premium by the former method,—that is, by a fixed addition to the age,—is open in some respects to considerable objection. For instance, if the life be young, and the assurance for a “Short Term,” the addition of even several years would have comparatively but little effect upon the rate of premium. Nay, if Mr. Bailey’s theory be correct, and it should ever be carried into practice, we should sometimes by this method obtain a *diminished* premium as a provision for a supposed *additional* risk! The second method, viz., that of a constant addition to the yearly premium irrespective of the age and the nature of the assurance, is not open to the particular objection just referred to,—but others, perhaps quite as strong, may be urged against it. In proof of this, it is only necessary to mention the case of “Endowment Assurances,”—to which indeed neither of the methods in question is applicable. There are two distinct benefits comprised in assurances of this description, viz., a Term Assurance and an Endowment; and the extra risk under the former is partly compensated by the diminished risk under the latter. The addition to the age which would be made if the case were that of an ordinary whole