

## CORRESPONDENCE

The Editor,

*Journal of Glaciology*

SIR,

*Multiple flow states for ice masses:  
reply to Dr Fowler's comments*

The determination of temperature and flow in an ice mass undergoing sub-solidus creep is a coupled thermomechanical problem. Both quantities must be found simultaneously and self-consistently; temperature affects flow because the effective viscosity of ice is highly temperature-dependent and flow affects temperature because shear in the flow leads to frictional heating. One of the major purposes of our paper (Yuen and Schubert, 1979) was to explore the properties of self-consistent solutions to a tractable mathematical model of the deformation of an ice mass. It was found that the family of mathematical solutions is multiple-valued in that two (or three) values of surface velocity or basal temperature can correspond to one value of ice thickness. The existence of mathematical multiple steady states does not imply, of course, the existence of physical multiple steady states for real ice masses (Fowler, 1980). Environmental circumstances, including the integrated mass flux arising from the net of accumulation and ablation, will determine a particular value of ice thickness. However, while such environmental factors remove the indeterminacy of the steady state, they do not help us to understand how the ice mass will evolve when environmental conditions undergo sudden and dramatic changes. The determination of steady state and the stability of the steady state are distinct issues. To decide whether a steady state is stable, one must subject the steady state to finite-amplitude changes in the environmental factors such as accumulation- and ablation-rates and follow the temporal evolution of the ice mass in a self-consistent thermomechanical calculation. We believe that thermal runaway can be shown to occur in idealized mathematical models of ice-sheet deformation when the ice is subject to finite-amplitude changes in certain environmental factors. However, thermal runaway may not in fact occur in a real ice mass because the appropriate changes in environmental conditions may not be realizable. Caution is certainly required in attributing such phenomena as glacial surges to the thermal runaway instability.

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## REFERENCES

- Fowler, A. C. 1980. The existence of multiple steady states in the flow of large ice masses. *Journal of Glaciology*, Vol. 25, No. 91, p. 183-84.  
Yuen, D. A., and Schubert, G. 1979. The role of shear heating in the dynamics of large ice masses. *Journal of Glaciology*, Vol. 24, No. 90, p. 195-212.

SIR,

*Multiple flow states for ice masses*

By illustrating the importance of correctly posing instability problems, Fowler (1980) has made a useful contribution. The possible existence of thermal instabilities in glaciers remains an unsettled question and we agree that further study with more realistic models is warranted. As Fowler has pointed out, there are obvious shortcomings in our work (Clarke and others, 1977) and that of Yuen and Schubert (1979); however we do not accept some of his suggestions.

In our work and that of Yuen and Schubert, ice thickness is kept constant and it is shown that multiple solutions for ice flux can exist. Fowler correctly questions the assumption of holding thickness constant and shows that the instability may possibly disappear if different assumptions are made. According to his interpretation of Yuen and Schubert's figure 3, the instability does not exist if ice flux is held constant and thickness allowed to vary.