ERUPTIONS OF MAGNETOHYDROSTATIC STRUCTURES AND ONSETS OF FLARES*

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Abstract: Two kinds of MHS equilibrium are studied; isothermal and nonisothermal. The onsets of two-ribbon flares and flare-loops are explained by this study.

I. Introduction

The characteristic of two-ribbon flares is the large flare-loop systems which appear at the onset of the flare and rise upward slowly into the corona. Actually, the flare and the loop may be different manifestations of a global loss of equilibrium. The magnetostatic equilibrium configurations above active regions and quiet regions have been investigated analytically and numerically. (For example, Priest et al., 1980; Melville et al., 1984, 1987; Jockers, 1978; Heavyvaerts et al., 1980; Sun et al., 1987). However, the actual number of solutions obtained so far is still small. New solutions of the equilibrium equations need to be found to explain new observational phenomena Although in many of those studies gravity had been considered, an isothermal simplifying assumption was still being used. The comparison between the theoretical models and the observational phenomena had been restricted by the assumption

In this paper two dimensionless equations of isothermal MHS equilibrium are studied. They are

$$\Delta F + aF^{3} \cdot \exp(-z/H) = 0, \qquad (1)$$

$$\Delta F + aF^{1.5} \cdot \exp(-z/H) = 0. \qquad (2)$$

To each equation a new similarity is derived. The properties of the solutions are examined. A nonisothermal MHS equilibrium equation is also studied and its numerical solution is obtained. In Section II, the similarity solutions of Equations (1) and (2) and the numerical solution of the nonisothermal equation are derived and discussed. The results are summarized in Section III.

II. Basic Equations and Equilibrium Solutions

In Cartesian coordinates, the magnetohydrostatic equilibrium equations are

$$(1/4\pi)\,(\nabla\times B)\times B - \nabla p - \rho g \hat{z} = 0 \qquad (3)$$

$$\nabla\cdot B = 0 \qquad (4)$$

They can be reduced to a single partial differential equation

$$\Delta F + aF^{b} \cdot \exp(-z/H) = 0, \qquad (5)$$

where a, b are two parameters. As to the meanings of the method which is similar to that expounded by Bluman and Cole (1974), we obtained a similarity variable

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$$v = \left[e^{z/2H} + \frac{AH(b-1)}{E}\sin(x/2H)\right] / \cos(x/2H) \qquad (6)$$

$(E \neq 0,\ b \neq 0)$, and a solution of Equation (5) may be expressed as

$$F(x,z) = f(v)\,(\exp(z/2H)/\cos(x/2H))^{(2/(b-1))} \qquad (7)$$

where f(v) must be a solution of the following equation,

$$\left(v^2 + \frac{A^2H^2(b-1)^2}{E^2}\right) f'' + \frac{2(b+1)}{b-1} vf' + \frac{2(b+1)}{(b-1)^2} f + 4aH^2 f^b = 0 \qquad (8)$$

(See Sun et al., 1987). Equation (8) may have a power series solution of the following form:

$$f(v) = a_0 + a_1 v^{-1} + a_2 v^{-2} + a_3 v^{-3} + \ldots\ldots a_n v^{-n} + \ldots\ldots \qquad (9)$$

After substituting for F from Equation (9) in Equation (8) we find that Equation (8) has solutions only if the parameter b equals one of some separated values. Therefore the equilibrium states corresponding respectively to the solutions of Equation (5) are "quantized". If b is a function of time, when it varies with time the equilibrium configurations cannot vary smoothly with time It follows that the equilibrium states of different values of b are disjoint sets in the sense that one state cannot evolve smoothly to another state of a different b; during the variation a global loss of equilibrium occurs. Now we only discuss the states of b=1.5 and b=3. When b=3, after substituting for f from relation (9) in Equation (8), we derive a recurrence formula of the coefficients of the power series

$$a_n = \frac{(n-1)(n-2)4A^2H^2/E^2}{n^2-3n+2} a_{n-2} - \left(4aH^2 \sum_{\substack{i,j,k=0 \\ i+j+k=n}}^{n-1} a_i a_j a_k\right) / (n^2-3n+2), \qquad (10)$$

$$\text{then,}\quad |a_n| \leq 4\frac{A^2H^2}{E^2}|a_{n-2}| + 4aH^2 \frac{1}{2}\frac{n+2}{n-2}|a_{max}(i \leq n-1)|^3.$$

It is easy to verify that for the equilibrium solution of b=3, a_0 must be equal to zero, and a_1, a_2 are arbitrary. Note that $|v^{-1}| \leq 1$, except for the origin, if

$$|a_{max}(i=0,1,2)| < \sqrt[3]{1/(20\ aH^2)} \qquad (11)$$

and

$$E^2 > 8A^2H^2 \qquad (12)$$

the series must be almost everywhere convergent. It means that there is an equilibrium configuration corresponding to the convergent series. If the parameters do not satisfy the conditions (11), (12), it is quite possible that the series is not convergent, and there is not an equilibrium configuration corresponding to it When any of the parameters (a, A/E, a_1, a_2) varies with time, the equilibrium configuration depending on them may become into nonequilibrium, and collapses. This is another cause of the violent bursts.

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The state of b=1.5 is similar to that of b=3. In this state te recurrence formula for the coefficients of the power series is as follows:

$$a_n = -\left[\frac{A^2H^2}{4E^2}\ (n-1)\ (n-2)\ a_{n-2} + 4aH^2\ d_n\right] / \left[(n-4)(n-5)\right] \qquad (13)$$

where

$$d_n = \left[\sum_{i,j,k=4,(i+j+k=n+b)}^{n-1} a_i\, a_j\, a_k \qquad - \sum_{i,j=b(i+j)=n+b}^{n-1} d_i\, d_j\right] / 2d_b \qquad (14)$$

It is easy to verify that if $a_0=a_1=a_2=d_0=d_1=d_2=d_3=d_4=d_5=0$, a_4 and a_5 are arbitrary (but they are all smaller than 1) and $A^2H^2/4E \ll 1$, $aH^2/a_4^{3/2} < 1/4$, the power series is almost everywhere convergent, and the equilibrium configuration exists. Otherwise, the latter may not exist. In order to compare the equiliurium configurations with the relevant observational phenomena, we consider nonisothermal cases. In this paper we only study a model of a temperature distribution which only depends on F. It is as follows

$$T' = T / \left[1 - \log\ \left[\alpha'(F-F_o) + 1\right]^{\nu}\right] \qquad (15)$$

where $\alpha' = \alpha/\ell$ Bo. $\alpha \ll 1$, $\nu \ll 1$, are dimensionless parameters. This model denotes that the highest temperature in the region considered occurs at the neutral sheet. Now Equation (5) becomes

$$\Delta F = -\left[\alpha(F-F_o)+1\right]^{(\frac{\nu z}{H} - 1)} \left[\alpha a\left(1 + \frac{\nu z}{(b+1)H}\right) F^{b+1} + (1-\alpha F_o)\ aF^b + \right.$$

$$\left. + \frac{1}{2}\beta_o C_o \frac{\nu\, \alpha}{H} z\right] e^{z/H} \equiv \phi\ (F) \qquad (16)$$

where Fo is the value of F at which the temperature is not affected by magnetic field. (The meaning of Co and βo are the same as those in Melville et al., 1984). After substituting for F in the right-hand side of Equation (16) from the isothermal solution of F, and solving numerically the Poisson equation, we derive a second approximate solution of Equation (16). (The first approximate solution is the corresponding isothermal solution). Then we perform a standard iteration, say

$$\Delta F_n = \phi\ (F_{n-1}) \qquad (17)$$

where n = 1, 2, 3, ..., assuming that solutions exist.

III. Summary

The equilibrium states of different values of b in Equation (5) are disjoint sets, a state cannot evolve smoothly to another state of a different b. For a given value of b, conditions (13) and (14) can help one to discriminate between the definite existence and the possibility thatno equilibrium exists. A set of parameters satisfying the conditions corresponds to an equilibrium configuration. Otherwise equilibrium may not exist. Variation of parameters may cause a global loss of equilibrium. This means that an eruption of MHS structures and a two-ribbon flares occur.

The numerical solution of Equation (16) (for b=1.5) was obtained successfully. Fig. 1 shows one of the equilibrium configurations

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for b=1.5, A=0.0. There is a neutral sheet there. The directions of the magnetic fields are denoted by arrows. Fig. 2 shows one of the equilibrium configurations for b=3, A=1.0 (See Sun and Liu 1989). When parameter A becomes zero, the configuration evolves to that shown in Fig. 1; it means that a neutral sheet occurs. Fig. 3 shows a nonisothermal equilibrium configuration. It is clear that, when the temperature of the plasma near the neutral sheet rises, a great part of the magnetic field rises upward, and a part of the weak field - originally located over the neutral sheet - becomes a magnetic island and the magnetic field lines curve upward. Because equilibrium still exists, the configuration can be seen for quite a long time. Following the rise of temperature at the neutral sheet, loops with strong magnetic fields occur successively at higher and higher heights, and the highest temperature layer is always located at the top of the loops. It is much more likely that nonisothermal equilibrium is responsible for the post-flare phenomena. In the case of b=3, a similar situation may also occur when A → 0.

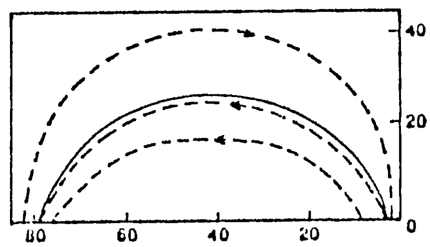

Fig. 1 Isothermal equilibrium for b=1.5 Solid line: neutral sheet; dashed line: magnetic field line. Relative coordinates

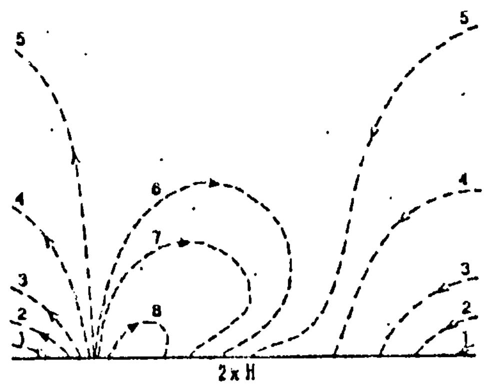

Fig. 2 Isothermal equilibrium for b = 3. Relative coordinates.

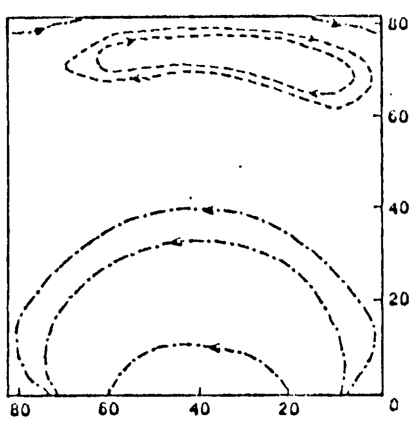

Fig. 3 Nonisothermal equilibrium of b=1.5. Dash-dotted line:magnetic field line; Dotted-line: magnetic island; relative coordinates.

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