THE RELATIVISTIC EFFECTS IN LOCALIZATION OF GAMMA-BURST SOURCES

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ABSTRACT. The paper discusses influence of relativistic effects in gamma-burst propagation and the time sync. It is shown that the phenomenon of gamma-source aberration, while using the localization algorithm based on the estimates of gamma-burst arrival time instants should be regarded as relativistic effect. Criterions are derived which determine whether it is necessary to account for the relativistic effects, depending on the localization accuracy.

1. INTRODUCTION

In recent space astrophysical experiments much attention was focussed on studying the phenomena of the so-called gamma-bursts, i.e. nonstationary space gamma-radiation [1]. The identification with the celestial bodies observed in other electromagnetic radiation bands is a major portion of these experiments. The basic identification technique is a comparison of gamma-burst source coordinates with the coordinates of the known celestial bodies. In determining gamma-burst sources coordinates (the so-called localization) one may consider gamma-burst sources as infinitely far removed, only estimating their celestial coordinates, that is the direction to the source $\bar{\ell}_\gamma$ in some given coordinates system C_o. Determination of $\bar{\ell}_\gamma$ in the system C_o is based on measuring gamma-burst arrival time instants $T_{\gamma i}$ at the point $\bar{r}_i(T_{\gamma i})$ where the detectors D_i are placed, moving in the system C_o along the trajectory $\bar{r}_i(T)$, i = = 1,2,..., N. To estimate $\bar{\ell}_\gamma$ the equations of type [2] are used

$$\bar{\ell}_\gamma^T (\bar{r}_{i_2} - \bar{r}_{i_1}) = - c \; (T_{\gamma i_2} - T_{\gamma i_1}), \tag{1}$$

where c is the light velocity. The gamma-burst arrival time instants is first determined in the coordinate time

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t_i of the system C_i associated with the detector D_i. The transformation $T \rightarrow t_i$ or $t_i \rightarrow T$ is called clock sync algorithm. Some geocentric coordinate system is commonly used as a system C_0, since there is a clock H to control the time. On the other hand, the coordinates of celestial bodies are usually catalogued in the heliocentric system. So, the transformation of the source coordinates from the geocentric to heliocentric system is a final stage of localization. Here the source parallax can be neglected, since the localization accuracy for the present time does not exceed the angular second units and the expected parallactic shift is sufficiently less than one angular second. Thus, only the gamma-burst source aberration should be taken into account.

According to the foregoing approach to localization the relativistic effects should be borne in mind in constructing algorithms for the following cases:

- in describing the gamma-burst propagation model;
- in constructing the sync. algorithms;
- with regard to gamma-burst source aberration.

2. DESCRIPTION OF THE GAMMA-BURST PROPAGATION MODEL

The signal propagation with the light velocity c in the arbitrary coordinate system is described by the differential equation $ds^2=0$. According to the Schwarzschild solution, for the coordinates system related to the center of masses of a Solar system planet, expression for the interval with an accuracy to the terms of the order of c^{-2} can be written as

$$ds^2=(1+2\varphi(\bar{r})/c^2)c^2dT^2-(1-2\varphi(\bar{r})/c^2)d\bar{r}^2, \tag{2}$$

where $\varphi(\bar{r})$ is the Newtonian gravitational at the point $\bar{r}$. Thus

$$|c\,dT| \approx (1-2\varphi(\bar{r})/c^2)\,|d\bar{r}|. \tag{3}$$

By integrating Eq. (3) along the burst propagation trajectory from the record point $\bar{r}_i$ to the point $\bar{r}_\gamma$ at which there is the source, we obtain that at the point $\bar{r}_i$ the burst generating at the time T_γ will arrive at the time

$$T_{\gamma i}=T_\gamma+L_{\gamma i}/c-2c^{-3}\int_0^{L_{\gamma i}}\varphi(\bar{r}_{\ell_i}(L))\,dL,$$

where $\bar{r}_{\ell_i}=\bar{r}_i+\bar{\ell}_\gamma L,$

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$$L_{\gamma i} = |\bar{r}_i - \bar{r}_\gamma| .$$

Since in the inertial reference frame the burst arrival time instant would be equal to

$$T_{\gamma i} = T_\gamma + L_{\gamma i}/C ,$$

the correction value to the time $T_{\gamma i}$ influenced by the relativistic effects, is equal to

$$\Delta T_{\gamma i} = -2C^{-3}\int_0^{L_{\gamma i}} \varphi(\bar{r}_{\ell_i}(L))dL .$$

If the gravitational potential of the planet fields is ignored, for $\Delta T_{\gamma i}$ we can derived the following expression

$$\Delta T_{\gamma i} = \frac{2K^2 M_\odot}{C^3} \ln \frac{\bar{r}_i^T \bar{\ell}_\gamma + r_i}{L_{\gamma i} + \bar{r}_i^T \bar{\ell}_\gamma + |\bar{\ell}_\gamma L_{\gamma i} + \bar{r}_i|} ,$$

where $M_\odot$ is mass of the Sun.
Since $L_{\gamma i} \gg r_i$, then

$$\Delta T_{\gamma i} \approx \frac{2K^2 M_\odot}{C^3} \ln \frac{\bar{r}_i^T \bar{\ell}_\gamma + r_i}{2L_{\gamma i}} .$$

According to (1) only the difference of $\Delta T_{\gamma i}$ is essential. As $L_{\gamma i_1} \approx L_{\gamma i_2}$, the system C_0 can be assumed as inertial in describing the burst propagation, if

$$\frac{2K^2 M_\odot}{C^3} \left| \ln \frac{\bar{r}_{i_1}^T \bar{\ell}_\gamma + r_{i_1}}{\bar{r}_{i_2}^T \bar{\ell}_\gamma + r_{i_2}} \right| \leqslant \Delta T_{sync} , \tag{4}$$

where ΔT_{sync} is required time sync accuracy.

For the spacecrafts being at about the same distance from the Sun, the expression (4) is written as

$$\frac{2K^2 M_\odot}{C^3} \left| \ln \frac{1-\cos\alpha_{i_1}}{1-\cos\alpha_{i_2}} \right| \leqslant \Delta T_{sync} ,$$

where α_{i_1}, α_{i_2} are angles between the direction to the center of the Sun and to the source at the points $\bar{r}_{i_1}$ and $\bar{r}_{i_2}$ respectively. As

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$$\kappa^2 M_\odot C^{-3} \approx .5\cdot 10^{-5} s,$$

then for extreme case (α_{i_1} = 90°, α_{i_2} = 15') the relativistic corrections should be treated if the sync. accuracy is 10^{-2} ms.

3. CLOCK SYNCHRONIZATION

Let the terrestrial clock is at the point r_H in the non-rotating system C_0. To determine the relativistic effects in clock sync. we use the expression (2) in the form

$$dS_i^2 = (1+2\varphi(\bar{r}_i)/C^2 - (1-2\varphi(\bar{r}_i)/C^2)V_i^2/C^2)C^2 dT^2,$$

where V_i is the velocity of the spacecraft with detector D_i. Since the increment of the proper time in the system C_i is equal to dS_i/C, then preserving only the terms of the order of C^{-2}, we find

$$dt_i^2 = (1+2\varphi(\bar{r}_i)/C^2 - V_i^2/C^2)\,dT^2. \tag{5}$$

Since $\varphi(\bar{r})/C^2 \ll 1$, $V_i^2/C^2 \ll 1$, then

$$\frac{dt_i}{dT} = 1 + \varphi(\bar{r}_i)/C^2 - V_i^2/2C^2. \tag{6}$$

For the terrestrial clock the similar equation can be written as

$$\frac{dT_H}{dT} = 1 + \varphi(\bar{r}_H)/C^2.$$

Eqs. (5) and (6) must be integrated together with the equations describing the motion of spacecraft and planets at the initial conditions T_H = 0, t_i = 0, T = 0. As a result the dependences $t_i(T)$ and $T_H(T)$ can be determined to reach the interrelation between t_i and T_H, T being ruled out. The dependence is designated as $t_i(T_H)$. Then in synchronizing the clock at the interval $[T_{H_1}, T_{H_2}]$ the relativistic corrections can be ignored, if

$$\max |t_i(T_H) - T_H| \leq \Delta T_{sync}, \quad T_{H_1} \leq T_H \leq T_{H_2}.$$

For example in the Signe-2 experiment the value $|t_i(T_H) - T_H|$ reached 10 ms [3].

4. ABERRATION OF THE GAMMA-BURST SOURCES

Aberration can be allowed for two ways. In the first place the coordinates of detectors and the burst arrival time in-

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stants can be transformed to the heliocentric system to localize the source. The second approach calls for transforming the coordinates of the source localized in geocentric system.

As is seen both approaches are identical only if the geocentric system motion relative to the heliocentric system is assumed to be uniform and rectiliner while the burst propagating between the detectors. The second approach is used for the photometrically measured celestial object coordinates. In this case the direction to the object in the heliocentric system $\bar{\ell}_\odot$ is linked with the direction of the some object in geocentric system $\bar{\ell}_\oplus$ at the same time through

$$\bar{\ell}_\odot = \bar{\ell}_\oplus - (E - \bar{\ell}_\oplus \bar{\ell}_\oplus^T) \bar{V}_\oplus / c, \tag{7}$$

where $\bar{V}_\oplus$ is the Earth velocity in the heliocentric system at the moment of $\bar{\ell}_\oplus$ determination, E is unit matrix.The typical feature of the experiment is that its duration may be arbitrary large. Thus if the Earth motion while the burst propagating can not be considered as uniform with regard to the localization accuracy, the relationship (7) cannot be used in treating the aberration. Here we must follow the first approach based on relativistic transformation of the burst arrival time instants and the detectors coordinates. We consider this approach in more detail to clear up the possibility of using the equation (7). The system C_0 is assumed to move relative to C_0' uniformly and rectilineary with velocity $\bar{V}$ coinciding with C_0' at $T = T' = 0$.

The 4-vector $\mathcal{R}_i$ is comparable with the radius-vector $\bar{r}_i$ and the time instant $T_{\gamma i}$ and the 4-vector $\mathcal{R}_i'$ is comparable with the radius-vector $\bar{r}_i'$ and the time instant $T_{\gamma i}'$ in the system C_0':

$$\mathcal{R}_i = \begin{bmatrix} \bar{r}_i \\ IcT_{\gamma i} \end{bmatrix}, \qquad \mathcal{R}_i' = \begin{bmatrix} \bar{r}_i' \\ IcT_{\gamma i}' \end{bmatrix}.$$

The source $\bar{r}_\gamma$ and $\bar{r}_\gamma'$ radius-vectors in the systems C_0 and C_0', respectively, along with the time instants T_γ and T_γ' at which the burst occurs, produce the following 4-vectors

$$\mathcal{R}_\gamma = \begin{bmatrix} \bar{r}_\gamma \\ IcT_\gamma \end{bmatrix}, \qquad \mathcal{R}_\gamma' = \begin{bmatrix} \bar{r}_\gamma' \\ IcT_\gamma' \end{bmatrix}.$$

As an origin of time T and T' we take the time instant at which the burst goes through the origin of coordinate system C_0 (or, that is the same, in the origin of coordinate system C_0', since at the time $T = T' = 0$ the systems coincide). In this case

$$cT_\gamma = -r_\gamma , \quad cT_\gamma{}' = -r_\gamma'$$

and hence

$$\mathcal{R}_\gamma = \begin{vmatrix} \bar{r}_\gamma \\ -I r_\gamma \end{vmatrix} , \quad \mathcal{R}_\gamma' = \begin{vmatrix} \bar{r}_\gamma{}' \\ -I r_\gamma' \end{vmatrix} . \tag{8}$$

By using $\mathcal{R}_\gamma$ and $\mathcal{R}_\gamma'$ we introduce the 4-vectors

$$\mathcal{X}_\gamma = \frac{1}{r_\gamma} \mathcal{R}_\gamma , \quad \mathcal{X}_\gamma' = \frac{1}{r_\gamma'} \mathcal{R}_\gamma' , \tag{9}$$

to write the Equation (1) in the system C_o as

$$\mathcal{X}_\gamma^T \left(\mathcal{R}_{i_1} - \mathcal{R}_{i_2} \right) = O. \tag{9a}$$

The vectors $\mathcal{R}$ and $\mathcal{R}'$ are linked through Lorentz's transformation

$$\mathcal{R}' = U \mathcal{R} ,$$

where

$$U = \frac{1}{\sqrt{1-\beta^2}} \begin{bmatrix} P + \sqrt{1-\beta^2} Q & -I\beta \frac{\bar{V}}{V} \\ I\beta \frac{\bar{V}^T}{V} & 1 \end{bmatrix} ,$$

$$\beta = \frac{V}{c} , \; P = \bar{V}\bar{V}^T / V^2 \quad Q = E - P .$$

Thus

$$\bar{r}_i{}' = \frac{1}{\sqrt{1-\beta^2}} \left((P + \sqrt{1-\beta^2} Q) \bar{r}_i + \bar{V} T_{\gamma i} \right) , \tag{10}$$

$$T_{\gamma i}' = \frac{1}{\sqrt{1-\beta^2}} \left(T_{\gamma_i} + \bar{V}_i^T \bar{r}_i / c^2 \right) , \tag{11}$$

$$\bar{r}_\gamma' = \frac{1}{\sqrt{1-\beta^2}} \left((P + \sqrt{1-\beta^2} Q) \bar{r}_\gamma - \bar{V} r_\gamma / c \right) , \tag{12}$$

$$r_\gamma' = \frac{1}{\sqrt{1-\beta^2}} \left(r_\gamma - \bar{V}^T \bar{r}_\gamma / c \right) . \tag{13}$$

By dividing (12) into (13) we find the relationship linking the directions to the source in the systems C_o and C_o' .

$$\bar{\ell}_\gamma' = \left((P + \sqrt{1-\beta^2} Q) \bar{\ell}_\gamma - \bar{V}/c \right) / \left(1 - \bar{V}^T \bar{\ell}_\gamma / c \right) . \tag{14}$$

Then to determine direction to the source in the system C_o' we can use either the equations of coordinates and time transformation (10) and (11), or the aberration relation (14). The result is the same. Indeed (8) and (9) imply that

$$r_\gamma' \mathcal{X}_\gamma' = U \mathcal{R}_\gamma = r_\gamma U \mathcal{X}_\gamma$$

and since $U^T U = E$, then

$$\mathcal{X}_\gamma^T (\mathcal{R}_{i_2} - \mathcal{R}_{i_1}) = \frac{r_\gamma'}{r_\gamma} \mathcal{X}_\gamma'^T (\mathcal{R}_{i_2}' - \mathcal{R}_{i_1}').$$

There if $\mathcal{X}_\gamma$ meets the equation (9a), $\mathcal{X}'_\gamma$ meets the appropriate equation written in the system C_0' . In particular, for $\beta^2 \ll 1$ we find from (14) the relationship (7), and from (10) and (11) the relationships

$$\bar{r}_i' = \bar{r}_i + \bar{V} T_{\gamma i}$$
$$T_{\gamma i}' = T_{\gamma i} + \bar{V}^T \bar{r}_i / C^2$$

As is seen the coordinate transformation is Galilean, and the time transformation involves the relativistic correction. If the system C_0 moves with variable velocity, it can be treated as the local Lorentz frame for each time instants with the constant velocity $\bar{V}(T_{\gamma i})$. Thus in this case the relationship (7) can be used if at the time ΔT of burst propagating between detectors $\bar{V}(T)$ changes so that

$$(\bar{V}(T+\Delta T) - \bar{V}(T))^T \bar{r}_i \leqslant C^2 \Delta T_{sync},$$

where, as earlier, ΔT_{sync} is the time sync. accuracy.

5. CONCLUSION

The relativistic effects in localization of gamma-burst sources may be sufficient for:

a) description of burst propagation model in coordinate system where the source is localized if the measurement accuracy is above 0.01 ms.

b) the aberration in localizing the gamma-burst source should be regarded as relativistic effects. This peculiarity is associated with that there is no time instant which could be used as a reference in determining the direction to the source. Particularly unlike photometric experiments the velocity of the detectors is insignificant when the aberration is accounted for.

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DISCUSSION

Kreinovich : the formulae coincide with the formulae for cosmic VLBI observation. Perhaps it will have other applications.