COMMENT ON THE DEFINITION OF THE NONROTATING ORIGIN

HEINRICH EICHHORN
Observatoire de Paris[1]
61, ave. de l'Observatoire
F-75014 Paris
France

ABSTRACT. The paper gives a rigorous and purely formal derivation for the relationship between the "nonrotating" origin and the x-axis of the Q_t system, i.e., the true equator system. Neglecting nutation, a nonrotating origin could also be achieved by putting m=0 in the formula for the time derivative of right ascension.

The condition defining the nonrotating origin is stated by Capitaine, Guinot and Souchay (1986) as "σ is kinematically defined in such away that, as *P* moves in the CRS, [*Oxyz*] has no component of instantaneous rotation with respect to the CRS around *Oz*." This obviously means that the axis of instantaneous rotation must lie in the *y-z* plane of [*Oxyz*].

We denote, for brevity, the CRS by K and the system [*Oxyz*] by k. It is therefore clear, that the matrix M(k,K), which transforms a given vector from k to K is

$$\mathbf{M}(\mathrm{k,K}) = \mathbf{R}_3(-E-90°)\mathbf{R}_1(-d)\mathbf{R}_3(S+90°) =$$

$$\begin{pmatrix} \sin E\sin S+\cos E\cos S\cos d & -\sin E\cos S+\cos E\sin S\cos d & \cos E\sin d \\ -\cos E\sin S+\sin E\cos S\cos d & \cos E\cos S+\sin E\sin S\cos d & \sin E\sin d \\ -\cos S\sin d & -\sin S\sin d & \cos d \end{pmatrix} = \begin{pmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{pmatrix}$$

In this expression, *E* and *d* are longitude and colatitude, respectively, of the z^k-axis with respect to K, and *S*, which replaces *E* + *s* of Capitaine, Guinot and Souchay, is the angle between the x^k-axis and the direction of the vector $(001)^T \times \hat{\mathfrak{x}}(E,d)^T$ with respect to K, i.e., that along the direction in which the x-y planes of K and k, respectively, intersect.

Since the matrix M(k, K) is orthogonal, we have

$$\mathbf{M}(\mathrm{K,k}) = \mathbf{M}^T(\mathrm{k,K}).$$

We therefore have

$$\boldsymbol{x}^k = \mathbf{M}^T(\mathrm{k, K})\boldsymbol{x}^K \quad \text{and} \quad \boldsymbol{x}^K = \mathbf{M}(\mathrm{k, K})\boldsymbol{x}^k.$$

Since we assume $\boldsymbol{x}^K$ not to vary with time, we have

[1] On leave from the Department of Astronomy, University of Florida, Gainesville, FL 32611

https://doi.org/10.1017/S0252921100063909 Published online by Cambridge University Press

$$\dot{x}^k = \left[\left(\frac{\partial}{\partial E} M(K,k)\right)\dot{E} + \left(\frac{\partial}{\partial S} M(K,k)\right)\dot{S} + \left(\frac{\partial}{\partial d} M(K,k)\right)\dot{d}\right] M(k,K)x^k,$$

which expresses the components of $\dot{x}^k$ in terms of x^k itself, as well as of E, S, d, $\dot{E}$, $\dot{S}$ and $\dot{d}$.

Routine calculations show that

$$\left(\frac{\partial}{\partial E} M(K,k)\right) M(k,K) = \begin{pmatrix} 0 & a_{33} & -a_{32} \\ -a_{33} & 0 & a_{31} \\ a_{32} & -a_{31} & 0 \end{pmatrix},$$

$$\left(\frac{\partial}{\partial S} M(K,k)\right) M(k,K) = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$\left(\frac{\partial}{\partial d} M(K,k)\right) M(k,K) = \begin{pmatrix} 0 & 0 & -\cos S \\ 0 & 0 & -\sin S \\ \cos S & \sin S & 0 \end{pmatrix}$$

One obtains the angular velocity vector ϖ (whose direction is in the axis of rotation) by taking the cross product of the vector with its velocity. Thus we get

$$\varpi^k = \begin{pmatrix} -(\dot{E}\sin S\sin d - \dot{d}\cos S)xy + (\dot{E}\cos S\sin d + \dot{d}\sin S)(y^2+z^2) + (\dot{E}\cos d - \dot{S})xz \\ (\dot{E}\sin S\sin d - \dot{d}\cos S)(x^2+z^2) - (\dot{E}\cos S\sin d + \dot{d}\sin S)xy + (\dot{E}\cos d - \dot{S})yz \\ -(\dot{E}\sin S\sin d - \dot{d}\cos S)yz - (\dot{E}\cos S\sin d + \dot{d}\sin S)xz - (\dot{E}\cos d - \dot{S})(x^2+y^2) \end{pmatrix}$$

This shows that ϖ depends on the vector; the requirement stated by Capitaine, Guinot and Souchay could therefore be changed to read:

"σ is kinematically defined in such a way that, as P (i.e., the z-axis of k) moves with respect to K, the equatorial plane of k has no component of instantaneous rotation with respect to the z-axis of k." Only for $z = 0$ will $\dot{E}\cos d = \dot{S}$ satisfy this requirement.

(Note that what I have done is to regard the motion of a vector (supposedly fixed in K) with respect to k, this mirrors the motion of the system with respect to the vector and is practically the same thing.)

There is a certain analogy of the whole situation with the precessional motion of the Q_m system with respect to the Q_0 system. The derivative of α with respect to time is given by $\dot{\alpha} = m + n \sin\alpha \tan\delta$. Even if we had a nonmoving origin for the right ascensions, which would be accomplished by setting the origin such that $m = 0$, we see that in general, $\dot{\alpha} = 0$ only on the instant equator, quite analogous to the situation we have described above.

Acknowledgement

I am indebted to my colleagues Nicole Capitaine and Suzanne Débarbat for comments and criticism.

Reference

Capitaine, N., Guinot, B. and Souchay, J. 1986. *Celest. Mech.* **39**, 283.