Towards a Theory of Theoretical Objects[1]

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Science is made possible by the introduction of theoretical objects. Why this should be so has never been made clear. Indeed, it has never been made clear how theoretical objects are rightly to be understood, or in what ways they differ from more ordinary sorts of physical objects. What follows is a sketch of a new theory. In my view, this theory becomes explicit on the so-called "Copenhagen interpretation" of quantum mechanics. But it has implicitly characterized scientific development since the revolution of the 16th and 17th centuries.

How are theoretical objects rightly to be understood? I say that they are objects whose existence is postulated by a theory is alright as far as it goes, but it doesn't go far enough. We also need to know what theories are. I think, in fact, that the notion of a theory is to be understood in terms of the concept of a theoretical object, and not the other way around. To say that they are imperceptible objects is more enlightening, but it is questionable (since many theories, those of Darwin and Skinner, for example, are expressly designed to restrict their objects to the clearly perceptible) and, despite a great deal of effort, still somewhat obscure (for the line between the perceptible and the imperceptible seems difficult if not also impossible to draw, even roughly). More implicit, neither postulation nor perceptibility explains the role that theoretical objects play in science, or why, whatever epistemological difficulties might attend them, they are indispensable.

In fact, if we look more closely at cases of theoretical objects we tend to take as paradigm, atoms for instance, their most striking features are

1) their similarity: atoms (at least in some of the older theories) are all alike - if you've, seen one of them, so to speak, you've seen all of them.

There is, one might also say, no difference worth mentioning between the type and the token. In this respect they are like species. This seems to be true, for a further example, of theories in the social sciences, particularly in the most developed, viz., economics and

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sociology: human beings, characterized with respect to a set of preferences or social relation are regarded as identical.

2) their underdetermination: atoms (on these same older theories) lack certain properties (color among them) that other sorts of physical objects routinely have.

The example suggests, what is in fact the case, that within individual theories what are otherwise perfectly respectable questions cannot be asked (think of the questions that cannot be asked within Skinner's behaviorist theory about human mental states and activities But at the outset I want to understand the notion of underdetermination more broadly than this. Thus according to a celebrated recent argument, our theories (e.g., concerning a native "gavegai"-speaker's ontology) are underdetermined by the evidence (e.g., the totality of the native's speech behavior) we have for them. Which is to say, on my reading, that the objects of theory are underdetermined, at any given moment, by the evidence on the basis of which we postulate them, however extensive that evidence is. Of course, it requires a separate argument, resting perhaps on narrowly empiricist assumptions, to draw the further conclusion that therefore the objects of theory are indeterminate.

An important analogy on this theory of theoretical objects is with mathematical objects (an analogy which helps to explain how the postulation of theoretical objects went hand in hand with the mathematization of nature in the 6th and 17th centuries). Theoretical objects are, in structure at least, very much like mathematical objects (although unlike mathematical objects, of course, in that they have causal and empirical properties).

Consider, for example, a typical proof in Euclid's *Elements*. We begin by postulating, e.g., a triangle having certain properties (that it is right-angled, that one side has length *1*, etc.). This particular triangle *ABC* is distinct from any other triangle not having the particular properties in question, but indistinct from other triangles which, however they might otherwise differ, have these properties. I.e., a variety of different triangles satisfy the conditions of postulation. Which is to say that in the case of mathematical objects introduced in the course of a typical proof, identity and order relations are often underdetermined. We are told to choose numbers specified simply with respect to some interval ("any number between one and ten", "any number greater than two", "any finite prime"), again without regard to order and identity. In the same way, theoretical objects are completely representative (hence the possibility of what amount to free-variable or one-step inductions to generalizations about them) and incompletely determined.

Let me use another analogy to broaden still further this notion of incomplete determination. Theoretical objects are often said to be like, or even to be, fictional entities in the superficial sense that they are imagined or imaginary (at which point the word "construct" comes into play). But in a deeper sense they are like fictional entities insofar as both are essentially incomplete. However much I would like to know more about Thea Kronberg, the heroine of Willa Cather's wonderful novel *The Song of the Lark* (what was her first appearance on the stage in Dresden like? Did she ever have any children?), there is nothing more to be known about her than is already set down. Many assertions about her, in contrast to her creator, are neither true nor false. Indeed, I don't think it misleading to say that theories tell stories about certain kinds of objects (adding at once that the stories are intended to be true and can, albeit it indirectly, be confirmed). In a sense, of course, the stories continue to unfold so long as there are tellers; new chapters are written. But at any given time there is an end to the story, and hence only so much we can learn, theoretically, about objects depicted in it.

The syntax of a theory reflects this fact. The syntax of a theory at any given time reflects a particular degree of determination of the object whose existence the theory postulates. From this point of view, theoretical objects can never be completely determined. Complete determination comes only at the level of classical semantics, which reflects a "platonic" point of view with respect to distributions of truth values. To the extent that theoretical objects are not completely determined, "in principle" to that extent we will have to modify the semantics for languages in which they figure.

Moreover, the underdetermination of theoretical objects is not a mere fact, nor does it rest on narrowly empiricist assumptions. It follows directly from our concept of what it is, at least in certain paradigm cases, to provide a theoretical explanation. In these paradigm cases, to provide a theoretical explanation is to provide a reductive explanation of the phenomena. Two principles characterize such reductive explanations:

P.1: the principle of micro-reduction: the properties of wholes are to be explained in terms of the properties of their parts.

P.2: the principle of property-reduction: the properties of these parts must differ from those of the wholes they are invoked to explain (for otherwise no genuine "reduction" has been carried out and the resulting explanations are vacuous).

This pattern is quite plausibly taken to be an important source of the persistent search for atomistic theories in science, and of the accompanying belief that the phenomena have not really been understood until such theories have been found to explain them. It dates, perhaps, from the time of Democritus. Heisenberg has more recently expressed the intuition involved as follows:

> It is impossible to explain...qualities of matter except by tracing these back to the behavior of entities which themselves no longer possess these qualities. If atoms are really to explain the origin of color and smell of visible bodies, then they cannot possess properties like color and smell...Atomic theory consistently denies the atom any such perceptible properties. (Heisenberg 1937, p. 119).[2]

There is a traditional reply to all of this. It is that theoretical objects (more properly, talk about them) are simply abstractions made for the purposes of systematization and explanation. All objects are perfectly determinate - it is just that in certain contexts and for certain purposes we consider, them to have certain properties, momentarily leaving to the side questions concerning other properties they might have (thus if I am asked to choose a number specified with respect to some interval as the basis for a proof, I choose a determinate number, no matter which indeterminately specified).

An example should help to clarify the issues. Uncontroversially, we come to understand natural phenomena by constructing a model of them. Minimally, a model is a representation of a physical system such that certain relational properties of the physical system are preserved in the representation (i.e., the model is structurally similar to the system), the representation is (ideally) quantitative, and predictions concerning the behavior of the physical system can be made on the basis of the representation and then checked out in the laboratory. Good models are confirmed in the course of such checking and, equally important, they suggest new and unexpected experiments as well. From my present point of view, two features of such models are crucial: they do not represent every aspect and element of the physical systems they model, nor could they if they were to play the (reductive) explanatory role assigned them; the properties of the parts of the model are not in general those of the model as a whole (the model as a whole has certain "systems" properties which the parts lack).

Now perhaps the most familiar of all physical models is the molecular model of a gas. In terms of this simple model we can explain such various phenomena as the mobility and mutual mixing of gases. We can also explain the observed relations between the pressure, temperature, and volume of a gas as codified in the Boyle-Charles Law. On this model, volumes of gas or ensembles of molecules have a temperature, measurable in familiar ways. Individual molecules do not. I want to say as a result that the objects of theory, the molecules, are underdetermined or indeterminate with respect to temperature, and that statements ascribing a temperature to them are neither true nor false. As indicated earlier, the principles of reductive explanation require as much. Others want to say that the model is no more than that, a model or idealization, and that all such statements are false, which in their view is just what "the model does not assign a temperature to individual molecules" means.

I think, however, that there are profound historical and philosophical reasons why the "mere abstraction" view is incorrect and why it would be a mistake to label all of the otherwise indeterminate assertions about the objects of theory false, reasons why theoretical objects are essentially underdetermined and hence why we should adjust our view of reality as a result.

First (for want of a better word) the "historical" reasons. I will advance them in the course of a very brief, somewhat breathtaking, radically unorthodox account of the origins of modern science.

The key name is that of Francois Vieta (1540-1603) and the crucial development is that of algebra and its application.[3] Vieta was the first to use a comprehensive symbolism in the development of his algebra (usually using consonants for known quantities and vowels for unknown quantities). He called his symbolic algebra *logistica speciosa* as against *logistica numerosa* (arithmetic). In his view, algebra (*logistica speciosa*) is a method of operating on "species" or forms of things. Thus, he understands that the general quadratic equation

$ax^2 + bx + c = 0$ (modern notation) defines not a single number but a class of numbers, and that general coefficients as well as unknowns can be symbolized. Two aspects of this development need to be underlined. One is that magnitudes, represented symbolically, are characterized by the equations into which they enter. The other is that insofar as these equations are general, the resulting magnitudes (indifferently arithmetical or geometrical) are themselves _general_ and therefore underdetermined.

This view contrasts very sharply with the standard _Greek_ view, according to which otherwise indeterminate magnitudes are simply abstractions of mathematical objects (e.g., _arithmoi_) which in themselves are completely determinate, and the "equations" (series of proportions) concerning them are similarly abstractions. Thus Aristotle (_Metaphysics_ M3, 1077b 17-20): "The general propositions in mathematics [namely the axioms, i.e., the common notations, but also all theorems of the Eudoxian theory of proportions] _are not about separate things_ which exist _outside of and alongside_ the [geometric] magnitudes and numbers, but are just about these; not, however, insofar as they are such as to have a magnitude or to be divisible [into discrete units]" and (_Metaphysics_ M2, 1077a 12) there cannot be a specific mathematical object which is "neither a [determinate] number [of monads] nor [indivisible, geometric] points, nor a [determinate period of] time."[4] But Aristotle's view parallels the contemporary reply under discussion that theoretical objects, generalizations, models are idealizations or abstractions of objects which _must be_ in all respects determined, whether or not _we_ are able to carry out the determination.

It is (primarily) Vieta's development, however, that made it possible to introduce "numbers" which could not be represented as geometrical magnitudes solely as the result of symbolic transformation or manipulation (e.g., "negative numbers" and "irrational numbers", both of which the Greeks proscribed), to use approximation methods (as Vieta does) in a free-swinging way, to make the _formula_ central, and to construe algebra as an "analytic art" (of calculation) and not a "science" (of description).[5]

Moreover, it is (primarily) this development (according to me if not also Klein) which is generalized and applied to the physical world (for the first time in an explicit way by Descartes) in what we call "modern science". Physical magnitudes are represented symbolically and often characterized purely in terms of symbolic operations. Relations between these magnitudes are represented by equations. These equations become the _laws_ of modern science; they are, in fact, means for calculating numbers. It is a crucial part of this development that the Greek view of causality, _this_ is a necessary and sufficient condition for _that_, is replaced by functional dependencies of the form $f(x) = y$, where "x" and "y" once again represent indeterminate magnitudes. It is in this way that magnitudes are _generalized_. But it is the generalization of the object (in contrast to a generalization of the method) which makes possible the characteristic features of modern science: the emphasis on prediction, the search for invariant laws and properties, the conception of science as a problem-solving rather than ontological activity. Vieta called the objects symbolized and then described in corresponding equations "species objects". We call them "theoretical objects" and in

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the process lose sight (as Vieta did not) of the deep ways in which they differ from ordinary sorts of objects.

In a nutshell: to say that all otherwise indeterminate statements in which reference is made to theoretical objects are "mere abstractions" or false is to presuppose that such objects are in fact determinate. But this is to ignore the tremendous conceptual revolution that took place in the 16th century when symbolic and algebraic methods were introduced and applied systematically, methods that both made "modern science" possible and involved the concept of a "general" and indeterminate magnitude, the concept of an underdetermined and theoretical object. In a smaller nutshell: theoretical objects are algebraic objects: algebraic objects are underdetermined; therefore, theoretical objects are underdetermined.

A more properly "philosophical" reason not wanting to identify the indeterminate with the abstract or the false depends on a more detailed description than so far given of the ways in which theoretical objects are indeterminate. At least five cases can be distinguished.

First case: objects which are at present underdetermined (with respect to some property), but whose further determination is possible given the resources of the theory. For example, certain theoretical values remain to be calculated or the appropriate experiments to be carried out; the determination of an electron's charge, say, before Millikan's oil-drop experiment.

Second case: objects whose further determination cannot <u>in fact</u> be carried out using the resources of the theory, although it is in principle possible. For example, objects whose further determination would involve the solution of <u>n-body</u> problems in classical mechanics.

Third case: objects whose underdetermination <u>follows from</u> the basic principles of the theory. For example, more controversially certain elementary quantum mechanical propositions, less controversially perhaps, free quarks. Quarks can stand a closer look.[6] Quarks are distinguished by a single quantum number, their isospin, playfully known as their "flavor". Different quarks have different degrees of isospin i.e. different "flavors", six at last count. They also have "color".[7] It is in principle possible to measure "flavor" but not "color" (since the three possible quark colors sum to a colorless white; this is so-called "color confinement"). As Kosso puts it,

> To say that color cannot be seen is just a sloppy way to claim that a colored particle cannot interact with another object which we regard as part of the observing apparatus in such a way that the apparatus object is left in a different state according to whether the interaction was with a red, green, or blue particle. The color of quarks is unobservable because physical interactions are colorblind. It is not a problem with our eyes or with our machines. The unobservability is dictated by the physical laws of interaction.

Fourth case: objects which are indeterminate insofar as the theory in which they figure does not assign them a particular property, although within the theory there is no reason in principle why the

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objects could not have this property. For example, individual molecules are in this way indeterminate with respect to temperature.

Fifth case: objects which are indeterminate insofar as certain otherwise legitimate propositions concerning them cannot be formulated within the vocabulary of the theory. For example, individual electrons on the classic quantum theory are in this way indeterminate with respect to (no scare-quotes) color.

I think we need to distinguish generally, in light of the above quick classification, between objects which are determinable with respect to a given property (but which we have not or cannot, in fact, determine) and objects which are essentially underdetermined and therefore, except on a question-begging "platonic" assumption, indeterminate. We also need to distinguish between propositions whose falsity can be shown on the basis of experimental procedures and those which are simply declared false in virtue of the fact that their truth cannot be shown. The quark case is particularly instructive in this connection. For in this case the robust or bivalent realist is going to have to assign truth values to propositions for which the theory itself, in different ways, precludes all empirical evidence.

The underdetermination of theoretical objects makes science possible by way of the role such objects play in explanation. So, too, does their similarity.[8] For it is only with respect to objects similar in the appropriate sense that unrestricted generalizations of the sort required by non-reductive scientific explanations, "laws of nature", are possible. This claim can be supported, very briefly, by two closely connected and familiar lines of argument.

One has to do with the classic problem of induction: for each generalization on our experience there is another supported by the same experience but incompatible with the first. Which generalization do we choose? So long as we are dealing with ordinary sorts of objects, like emeralds, the problem is insoluble; for every similarity we can find a dissimilarity, for every dissimilarity we can find a similarity. Only where we have theoretical objects, i.e., only where the similarity fixed from the outset, or where, we might also say, the theory-object match is perfect, are the appropriate generalizations indicated.[9]

The other line of argument has to do with the distinction between accidental and lawlike generalizations, e.g., between "all swans in France are white" and (Descartes' Second Law) "every body which moves tends to continue its movement in a straight line." How do we make the distinction? So long as we are dealing with ordinary sorts of physical objects the problem is insoluble: every non-trivial generalization with respect to such objects is contingent (they happen to share or not to share a particular property as the case may be). Only where we have theoretical objects, i.e., only where the generalizations are in the same way and in the same sense as necessary as they are in mathematics, can the appropriate distinction be made.[10]

We might ask, finally, whether theoretical objects as I have described them exist. The preceding discussion suggests two answers to this question. Insofar as theoretical objects are indispensable to scientific explanation, and insofar as science is an explanatory activity, they exist; no reductive account of them can be given. Insofar as objects are indispensable within the contexts of particular theories, i.e., insofar as there is no extra-theoretical criterion on the basis of which their existence can be judged, our realism must be entirely internal. The first answer preserves the objectivity of our theories, the second allows for their knowability. What more could we possibly want, or need? Of course, the world we face as a result is still strange, for all that we discovered it three to four hundred years ago.

Notes

[1]This is a shortened version of a paper read at Memphis State University. I am not yet able to answer all of the questions raised there. My position has grown out of conversations over the years with James Allard, Bas van Fraassen, Karel Lambert, Ralf Meerbote, Carl Posy, and John Winnie. Happily, I found myself located between the robust realism of Lambert, Meerbote and Winnie, on the one hand and the subtle idealism of Allard, van Fraassen, and Posy, on the other.

[2]See also his Physics and Philosophy (Harper 1958), chapter 4. According to Heisenberg, a completely consistent extension of this view, which he endorses, would require eventually that the physical world be explained in non-physical terms, in which case there would be no distinction between theoretical and purely mathematical objects.

[3]In what follows I merely highlight certain aspects of the detailed and penetrating discussion in Jacob Klein's neglected classic, Greek Mathematical Thought and the Rise of Algebra, first published in 1934-36. Those of us interested in the history and philosophy of science, incidentally, focus too much attention on the introduction of infinitesimal concepts and transcendental methods, neglecting the more fundamental role played by algebra.

[4]Quoted from Klein (1934-1936), p 161-162.

[5]Note that geometry and algebra are unalike in this precise respect: the drawn triangle ABC (what Kant, sensitive to the distinction, calls an "ostensive" construction) is both representative of a class of objects, triangles, and itself perfectly determinate; but the variables of algebra (used in what Kant calls a "symbolic" construction) are not in themselves perfectly determinate.

[6]What follows draws upon Peter Kosso's dissertation in progress at the University of Minnesota on the topic of unobservability.

[7]For a variety of technical reasons, the three quarks making up the baryon Δ^{++} must be further distinguished. One postulates the quantum number of color and distinguishes three values of "red", "green", and "blue".

[8]Underdetermination and similarity come together, once again, in the notion of a general or algebraic object, for while algebraic objects are partially specified they are also completely representative.

[9]In his classic discussion of the problem of induction, Mill asks: "Why is a single instance, in some cases, sufficient for a complete induction, while in others, myriads of concurring instances, without a single exception known or presumed, go such a very little way toward establishing a universal proposition." A System of Logic (Mill 1881, p. 186). The examples he gives (chemical elements for single-instance inductions, crows for many-instance inductions), however, suggest strongly that the difference lies in the kind of object (theoretical as against ordinary) under investigation. Since theoretical objects are completely representative, what holds for one will hold for all.

[10]We could put it this way. Lawlike generalizations are generalizations about species or kinds, and all such kind-generalizations are either necessary or false.

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