closed algebra generated by S can be identified with the space $P^\infty(\mu)$, the weak* closure in $L^\infty(\mu)$ of the polynomials. As a result, an understanding of $P^\infty(\mu)$ yields information about S. These ideas were used to great effect in 1978 by S. W. Brown, who showed that subnormal operators always have non-trivial invariant subspaces, thereby settling what was at the time the main open question in the subject.

The theory is now at the stage that a substantial amount of progress has been made, yet there is still much to be understood and many problems remain. The aim of the present book is to give a comprehensive account of the theory to date. It is something of a mixture between a textbook and a research monograph in that the author includes much of the necessary background material from function theory. After some preliminaries and an account of the structure of normal operators, the main part of the book divides roughly in two. The first half consists of an account of the basic theory of subnormal operators, a detailed analysis of the unilateral shift and a discussion of hyponormal operators. The latter half is devoted to the more recent developments involving uniform algebras and rational approximation. An account is given of Sarason's characterisation of $P^\infty(\mu)$ and this is then put to work in the final chapter, where Brown's invariant subspace theorem is proved and a functional calculus for subnormal operators due to the author and R. Olin investigated.

The book has been carefully written, with plenty of examples to illustrate the theory and helpful exercises for the conscientious reader. It will be of great use, both for those wanting to learn about subnormal operators for the first time and for specialists wanting an up-to-date account of the subject.

T. A. GILLESPIE

LYNDON, ROGER C. *Groups and geometry* (London Mathematical Society Lecture Note Series 101, Cambridge University Press, 1985), 217 pp. £11.95.

Several books could have been written with this comprehensive title, so let prospective readers first be informed of the scope and aim of this one. It culminates with an account of Fuchsian groups, this following a penultimate chapter on hyperbolic groups (mapping the upper half of the complex plane onto itself) which, in turn, follows a chapter on inversive geometry. The logical sequence of these chapters is patent and the strategy of their unfolding is well conceived. So much should suffice to warn any reader seeking other groups or geometries.

The opening of the first chapter, on symmetries and groups, presents the elementary facts about groups of order not exceeding 6 and dihedral groups in a way that any expositor would do well to follow; an account (p. 7) of Cayley's theorem and an excursus (pp. 13–14) on free groups provide a contrast. The author's intention is to "describe groups in two ways: either a geometric description or a presentation by generators and relations." Tietze transformations of finite presentations of the same group into one another are described, and the chapter ends with the caution that the word problem is undecidable.

The second chapter (isometries of the Euclidean plane) and the third (subgroups of the group of isometries of the plane) deal with those matters expounded by Coxeter in the third chapter of his *Introduction to geometry*. The group of isometries has (p. 29) a chain of normal subgroups with successive quotients all abelian. Coordinates, matrices, complex numbers are called in aid and the affine group noticed. The three regular tesselations of the Euclidean plane afford the opportunity of introducing their fundamental regions, and presentations, involving three generators, of their groups of symmetries are given. The finite symmetry of the regular polyhedra, operating also on their circumscribing sphere S^2, are found, and hyperbolic triangle groups are mentioned with their presentations in terms of involutory generators—these last acting as inversions in Euclidean circles mapping lines of the hyperbolic plane.

The fourth chapter studies the crystallographic groups; any that are conjugate in the affine group are isomorphic and there are, to within isomorphism, 17 types. The first move (p. 63) is to establish the crystallographic restriction (cf. p. 61 of Coxeter's book) and some very closely argued details fill the next 10 pages. It is interesting to contrast these with the declaration (p. 82) in the next chapter: "our general goal of avoiding metric considerations as far as possible". All 17 groups are presented in terms of generators and relations; the chapter ends with a complete list and 17 diagrams (p. 75) illustrating the actions of the groups.

Chapter five describes the tesselations in Euclidean space E^n of n dimensions and S^n, the n-sphere in E^{n+1}; an inductive process steps up into E^{n+1} from established geometry in E^n. A reader could perhaps best ponder the procedure (pp. 94–97) of how to construct a 24-cell in E^4 by building upon a cube in E^3; less complicated examples precede this and a decidely more involved one follows it. No claim is made that polytopes assembled by these procedures are regular. One is again afforded a tantalizing foretaste (p. 93) of what could happen in hyperbolic space.

Two shorter chapters follow: "incidence geometry of the affine plane" and "projective geometry". There is nothing wrong with these, quite the contrary; one cannot have everything in a single monograph. But to one to whom "groups and geometry" immediately conjures up, say, a long vista of the multitude of results in projective geometry over finite fields these chapters would be a reminder of so much that is not in them.

Immediately following these topics one is launched on the engrossing trilogy alluded to above. Under the group M of inversions in E^2 the aggregate composed of all lines and circles is closed; its subgroup M^+, of index 2, that preserves orientation is isomorphic to the group of linear fractional transformations of a complex variable z and so is sharply triply transitive on the points of E^2: its operations are parabolic or loxodromic according as they have one or two fixed points, and its finite subgroups are isomorphic to the finite groups of rotations of S^2. The whole of M is realized when not only z but also its complex conjugate are used.

M^+ is transitive on the aggregate of lines and circles, so that the stabilisers H of these are conjugate subgroups: any line or circle may be used, two likely choices being (a) the real axis $y=0$, (b) the unit circle $|z|=1$. So use those bilinear transformations with *real* coefficients. These map $y=0$ onto itself, and do not transpose the two half planes $y>0$ and $y<0$ when, as has been prescribed, they preserve orientation (p. 161). So $y>0$ is a model of the hyperbolic plane, its *lines* being those circles orthogonal to and those lines perpendicular to $y=0$. A metric is introduced first (p. 173) on $x=0$ and then extended to the whole hyperbolic plane by using the necessary invariance under H; this leads to formulae for lengths and areas. The other model $|z|<1$ for the hyperbolic plane can also be used (p. 168). The last theorem of this penultimate chapter plays, we are warned, an important role in the final chapter: it concerns those points at equal (hyperbolic) distance from two separate points.

Fuchsian groups G are *discontinuous* subgroups of H; their study is based on their fundamental regions, and more particularly on Dirichlet regions which are not only fundamental but convex. Their treatment is illustrated by the modular group, triangle groups and Schottky groups. The Dirichlet regions provide a tesselation of the hyperbolic plane; the dual to this is a Cayley tesselation which is used to obtain a presentation for G. One can thence derive Poincaré's polygon theorem and, eventually, the classification of all G with compact fundamental region by their generators and relations (p. 205).

The book's value is enhanced by judiciously chosen selections of problems, and pertinent references occur at the ends of chapters. There is a bibliography of 35 entries.

W. L. EDGE

HALLENBECK, D. J. and MACGREGOR, T. H. *Linear problems and convexity techniques in geometric function theory* (Monographs and studies in mathematics, Vol. 22, Pitman, 1984), 182 pp. £26.50.

Let S denote the set of all functions $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$, analytic and univalent in the unit disk $D=\{z:|z|<1\}$. Great interest has been shown for many years in extremal problems for S. But S is not a linear space and results from convexity theory, topological vector spaces and functional analysis are difficult to apply. If $A(D)$ denotes the functions analytic in D with the topology of local uniform convergence then S is a compact subset of $A(D)$. It is reasonable, therefore, to determine *extreme points* of S considered as a subset of $A(D)$, namely those $u\in S$ so that the equation $u=tx+(1-t)y$, $0<t<1$, has as its only solution in S the solution $x=y$, and the *support points* of S, namely those $v\in S$ which maximise the real part of some linear functional on S which is continuous with respect to the topology of $A(D)$.