ON THE FINE STRUCTURE OF THE MAGNETIC FIELD IN THE UNDISTURBED PHOTOSPHERE

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1. Some authors have recently proposed a set of fine structure models (Severny, 1968; Stenflo, 1966; Grigoryev, 1969) to explain the reaction of the solar magnetograph to the structure of the magnetic fields on the Sun. We tried to make a more sophisticated model in order to obtain more realistic results. Let the magnetic fields observed in the undisturbed photosphere be described as a totality of basic elements with equal area and with different magnetic field strengths. They cover the solar surface independently and closely. Let us consider a case of three kinds of elements. The first kind of elements has a magnetic field strength H_1 and an occurrence ξ, the second kind of elements has a magnetic field strength H_2 and the occurrence η and the third kind of elements without magnetic field has the occurrence $1-\xi-\eta$. If an ideal magnetograph has an entrance slit of area equal to the area of the η elements, then the recorded magnetic field is a mean field of these η elements and its values have a trinomial distribution. We assume that over large areas a balance of opposite polarity fluxes exists

$$\xi H_1 + \eta H_2 = 0 .$$

Further let us assume that deviations from this distribution, atmospheric scintillations, instrumental optical errors, electronic noise, etc. as a whole lead to distortions in the distribution which have a distribution close to a Gaussian law. Then the distribution of observed values of the magnetic field must be a composite of the normal and of the trinomial distributions. The parameters of the proposed model are connected with the moments of the observed distribution. The system of equations of the 9th order may be solved.

Using scans in an undisturbed region of the photosphere made with different resolutions we obtained a large number of observed distributions. The determination of the model parameters leads to the best agreement of different scan treatments if the magnetic field in the basic elements is from tens up to hundreds of gauss, the size of the elements is close to 2″, and the occurrence of them does not exceed some percent. In rare cases the opposite polarity magnetic fields may differ up to 10 times. The detailed results of this study will be published soon.

2. Statistical analysis of weak magnetic field using the autocorrelation function method was done first by Howard (1962) and later this method was used to analyze the fine structure of weak magnetic fields by Vasiljeva (1964), Severny (1968), Beckers and

Howard (ed.), Solar Magnetic Fields, 252–259.

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Schröter (1968), and Livingston (1968). In these papers autocorrelation functions were used mainly to determine the characteristic scale of the magnetic field fine structure. We intend to study periodic structure of the magnetic field spatial distribution in the undisturbed photosphere in high latitude regions of the Sun.

Conformity of the calcium network and supergranular cells on the one hand and a correlation between magnetic field and brightness in K Ca II on the other hand permit us to make the indirect conclusion that the strongest magnetic fields are concentrated on the borders of supergranular cells (Simon and Leighton, 1964). Direct evidence of the existence of the magnetic field regular structures with a scale corresponding to the supergranular cells may be obtained analyzing records of the magnetic field fluctuations in undisturbed regions with the help of the autocorrelation function method.

To detect the periodic component in a signal $H(t)$ one may use the autocorrelation function

$$B(\tau) = \langle H(t)\, H(t+\tau)\rangle_t .$$

If we assume that the magnetic field is concentrated on the borders of the supergranulation cells and the magnetic field polarity is the same one at the opposite sides of cells then $B(\tau)$ will be similar to a curve in Figure 1 (a). The distance between maxima must be of the order of the regular component period C, and the width of the regular component peaks must influence the value of the correlation interval $\tau_{0.5}$

$$B(\tau_{0.5}) = \tfrac{1}{2} B(0) .$$

But if the magnetic field polarity at the opposite sides of the cells is opposite, then the form of $H(t)$ may be of such a kind as is shown in Figure 1(b). The period of the regular component is two times more than in the first case. Unlike the first case $B(\tau)$ must have a minimum at $\tau = c, 3c, 5c, \ldots$. If we used absolute values of $H(t)$, then the autocorrelation function would be of the same kind as in the first case, that means the maxima located at $\tau = c, 2c, 3c, \ldots$ but the width of the maxima would be narrower.

In the general case the function $H(t)$ really may be a superposition of both kinds of curves considered, and therefore it is useful to analyze separate possibilities for $H(t)$ computing autocorrelation functions of $H(t)$ and of $|H(t)|$. Comparing them one may obtain more exact information about the magnetic field spatial structure.

We used records of magnetic field in high-latitude regions of the Sun with different resolutions. The scan length was of the order 280″–290″. About 150 autocorrelation functions for the northern hemisphere and 80 for the southern hemisphere were computed. The atmospheric oscillations, the finite aperture and the time constant of the magnetograph were not taken into consideration.

The correlation interval may be used as a size of field structural elements. In Table I the average values of the correlation intervals $\tau_{0.5}$ are presented for both the hemispheres and for the different resolutions.

At the resolution 1″.8 × 4″.2 the average value of $\tau_{0.5}$ is equal to 3″.3 or 2300 km. A smaller value was obtained by Howard (1962) with the resolution 2″ × 2″.5. A similar

TABLE I

Resolution	N-hemisphere		S-hemisphere	
	ACF of $H(t)$	ACF of $\lvert H(t)\rvert$	ACF of $H(t)$	ACF of $\lvert H(t)\rvert$
4″	3″.4	2″.0	3″.2	2″.3
8″	5.0	3.9	4.9	–
17″	5.3	3.4	6.2	–
32″	–	–	5.3	–

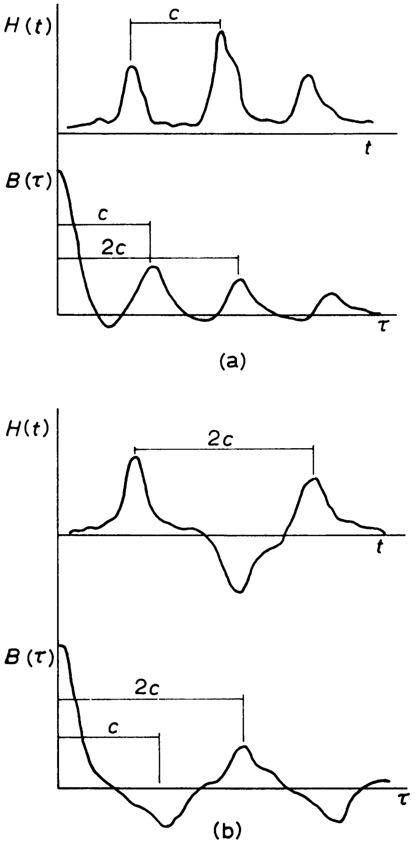

Fig. 1. (a) The form of the signal $H(t)$ with the magnetic fields of the same signs at the edges of a supergranular cell and its auto-correlation function; (b) the form of the signal $H(t)$ with magnetic fields of opposite signs at the edges of a supergranular cell and its auto-correlation function.

result was obtained by Vasiljeva (1964) with the resolution 2″.4 × 6″. A sufficiently greater correlation interval 7″.7 was obtained by Severny (1968) with the resolution 2″.5 × 9″. The data of Table I show that the mentioned discrepancies may be caused by different resolutions. However such deviations in the structural element sizes may be real and depend on the time and element location.

The other peculiarity of the autocorrelation functions is the presence of secondary maxima and minima within ranges 16–27″ and 34–47″. Looking through all the autocorrelation functions one may find that minima occur in the same range as maxima. In Table II the summary of maxima and minima locations is given. The secondary maxima near 40″ occur 2–3 times more often than the minima. Usually in scans the

TABLE II

Total number of ACF	131
Number of ACF having maximum in the range of 34 – 47″	83
Number of ACF having minimum in the range of 34 – 47″	34
Number of ACF having maximum in the range of 16 – 23″	30
Number of ACF having minimum in the range of 16 – 23″	62

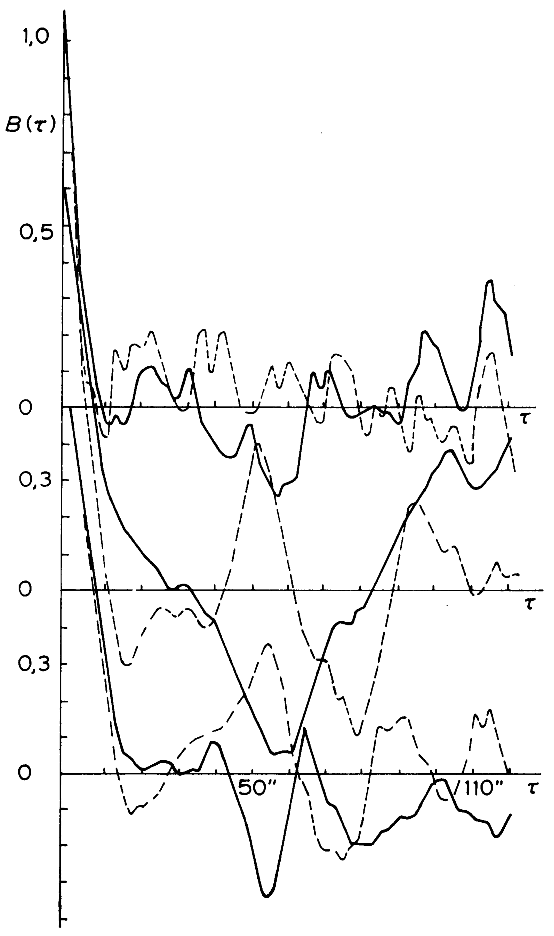

Fig. 2. Examples of ACF of some scans. The solid line presents ACF of the original signal $H(t)$ and the dashed line presents ACF of $|H(t)|$.

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minimum near 40″ in the case of $H(t)$ corresponds to the maximum in the case of $|H(t)|$ (Figure 2). This means that sometimes a regular structure of the magnetic field exists which corresponds to the supergranulation cell size and has strong magnetic fields of opposite polarity on the borders of cells.

It is useful to note that this fact may explain why the autocorrelation function computed by averaging data of different scans has no evident extrema close to 40″. Moreover the differences of autocorrelation functions of neighboring scans reflect the spatial distribution of such regular structures. It is interesting that the sharpest maxima near 40″ belong to scans located at distances of about 40″, and the autocorrelation functions of more close scans may be extremely different even without extrema.

The next peculiarity of computed autocorrelation functions is the increased fine structure in them when the resolution is better. It is evidence of the other characteristic size present in the spatial structure of the magnetic fields.

3. In order to study the fine structure of the magnetic field we tried to estimate the power spectra. Using the original scan records we computed periodograms for each scan and further averaged them. One may make an approximate estimation of the power spectra without use of the autocorrelation functions but determined within wider range of wave numbers $K = 2\pi/\lambda$.

The true power spectrum of the magnetic field spatial distribution is distorted by the atmosphere turbulence, by the instrument, and after adding the instrumental noise by the time constant. We tried to exclude these effects. After the reduction for the time constant the periodogram became as is shown in Figure 3. We assumed the instrumental noise to be of the Poisson type – an additive one. Then the smooth curve

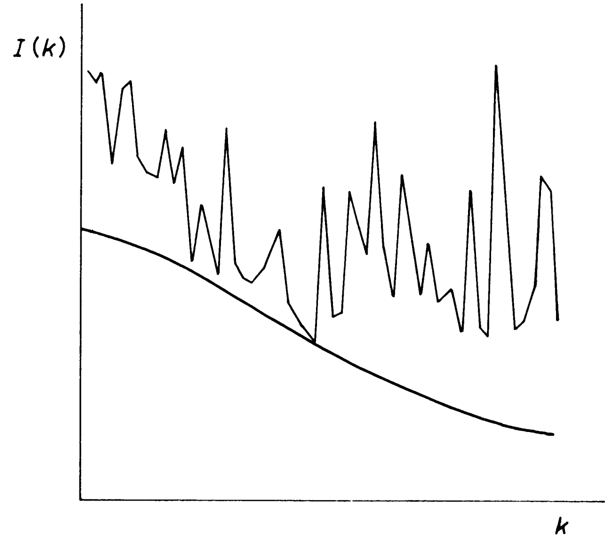

Fig. 3. The mean power spectrum corrected for the time constant. The solid line corresponds to the estimation of the noise power spectrum.

below the periodogram fitting only the deepest minima may be adopted as an estimation of the noise power spectrum. Unfortunately it is difficult to determine exactly the instrumental noise during the scans because the separate records of the instrumental noise give power spectra which agree poorly with the presented periodogram. This is the weakest point of our investigation.

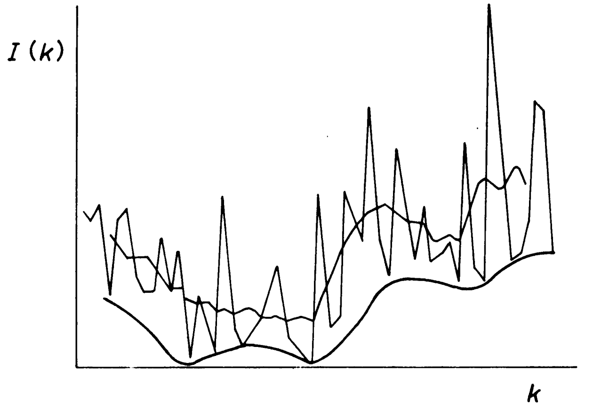

Fig. 4. The mean power spectrum corrected for the time constant without noise. The solid curve corresponds to a 'background' spectrum, the thin curve corresponds to a greatly smoothed spectrum.

Subtracting the noise power spectrum from the periodogram we obtain an estimation of the structure power spectrum (Figure 4). It is interesting that this spectrum is a superposition of a sharp spectral line system and of a smoothed spectrum. One may detect among the lines a set of equidistant maxima. Let the lower smooth curve be the estimation of the smoothed 'quasi-continuum'. If we determine this quasi-continuum as a background spectrum then the corresponding amplitudes of the equidistant peaks form a set of Fourier coefficients of a certain regular structure. So we can synthesize the estimation of the autocorrelation function of this regular structure. Using various weighting procedures (for example Gibbs' factors etc.) we obtain similar curves (Figures 5). As the main period of such structure is close to 50″, we presume to identify

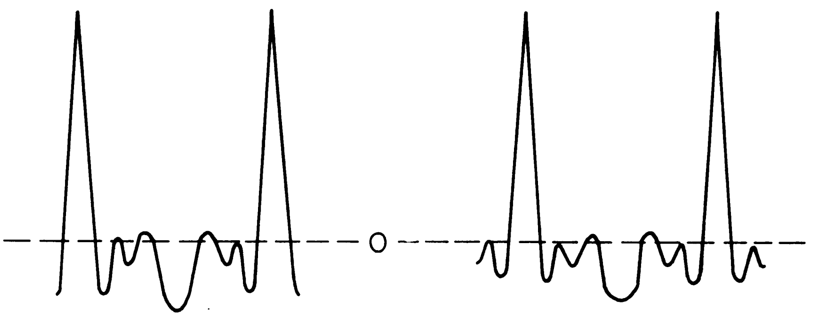

Fig. 5. Possible ACF of the internal magnetic field distribution in the supergranular cell.

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this structure with the fine structure of the magnetic field inside the supergranular cell.

One may see a strong concentration of the magnetic fields on the periphery of the cell and the presence of a magnetic field of opposite polarity in the center of the cell. Remembering the data given in Table II we may say that the higher frequency of the autocorrelation minima near 20″ is connected with this structure. It is obvious that the averaging of periodograms reduced the amplitudes of extrema (both the central minima and the periphery maxima) from 2.4 to 3 times, but the ratio of these amplitudes is only slightly influenced.

Considering the background spectrum one may see two distinct maxima within the large values of K. If we smooth very much the spectrum (a thin curve in Figure 4) then its form confirms also the presence of these maxima. These maxima may be caused by groups of basic elements of the magnetic field spatial structure and correspond to the values of λ 4 and 3 times larger than the basic element size. Therefore the basic element size is equal to 1″.8 or 1300 km. This estimation is in accordance with the results by Livingston (1968).

4. Discussing the results above we may make the following conclusions. As the magnetic fields at the supergranulation cell periphery occupy an area which is about some tens of per cent of the whole cell area, so in this part of the cell according to the average occurrence of the basic elements obtained by us, the density of those elements is approximately ten times higher than in other parts of the supergranular cell. This ring consists of closely packed elements.

The most optimistic estimation of fluxes leads us to a flux of the ring 14 times larger than the flux of the central part with approximately the same density of the elements. This means that about 93% of the supergranular cell flux is linked with the magnetic fields of other cells. Such great interdependence of the supergranulation magnetic fields is remarkable. Therefore the large-scale magnetic fields are not only a result of the chance cluster of the supergranular cells but on the contrary the system of these cells is formed and controlled by the large-scale magnetic field which is a more fundamental formation. Probably this is the reason why the interplanetary magnetic fields correlate better with the background magnetic fields, because they form together the fundamental slow varying large scale supersystem of magnetic fields.

References

Beckers, J. M. and Schröter, E. H.: 1968, *Solar Phys.* **4**, 303.
Grigoryev, V. M.: 1969, *Soln. Dann. (U.S.S.R.)* No. **6**, 77.
Howard, R.: 1962, *Astrophys. J.* **136**, 211.
Livingston. W. C.: 1968, *Astrophys. J.* **153**, 929.
Severny, A. B.: 1968, *Izv. Krymsk. Astrofiz. Obs.* **38**, 3.
Simon, G. W. and Leighton, R. B.: 1964, *Astrophys. J.* **140**, 1120.
Stenflo, J. O.: 1966, *Arkiv Astron.* **4**, 2.
Vasiljeva, G. Ya.: 1964, *Izv. Glavnoj Astron. Obs. (Pulkovo)*, No. **175**, 28.

Discussion

Lamb: I would like to ask two questions concerning the magnetic field autocorrelation function which you obtained. First, for what time scales do the features in the spectrum which you have described persist? Second, how have you defined the so-called 'background' autocorrelation function used in your analysis?

Kuklin: In the case of the smallest height of the entrance slit equal to 4″.2 the scanning of the whole region (more than 50 scans) requires 60–80 min, with a scanning speed equal to 2″ per s.

We may consider this point as one of the weak points of our study because it was made in some sense arbitrarily. We select the 'background' power spectrum as a bottom of the linear spectrum minima (a curve tangent to those minima).