

DETERMINATION OF THE INCLINATION OF ROTATIONAL AXES AND  
ROTATIONAL VELOCITY FROM THE LINE PROFILES OF ROTATING STARS

M. RUUSALEPP

Tartu Astrophysical Observatory, Estonia, USSR

For a grid of rotating hot stars the computed line widths at the half maximum depth for the lines HeI  $\lambda$  4471 and MgII  $\lambda$  4481 are given. Based on the correspondent observational data the inclination of the rotational axes  $i$  and the reduced rotational velocity  $w$  have been found for 19 B and Be stars.

Key words: stellar rotation - line profiles.

To make it possible to specify the rotational state of a star we must start by calculating some line profiles for a grid of stars, which are differently oriented and rotate at different angular velocities. As the fit features for such a task we have chosen the HeI line at  $\lambda$  4471 Å and MgII line at 4481 Å.

Calculated model line profiles for those lines we have taken from a paper by Stoeckley and Mihalas (1973), where a grid of intensity profiles for nonrotating stars is given for six effective temperatures,  $T_{\text{eff}}$ , from 15000 K to 27500 K and for ten values of the surface projection cosines,  $\mu$ , from 1.0 to 0.1. Taking the limb darkening into account a grid of rotationally broadened profiles for these two lines was computed assuming that the shape of the stellar surface is given by the Roche model, the von Zeipel theorem for the effective temperature distribution holds and the star rotates as a solid body.

For a nonrotating star we have taken the surface gravity,  $g$ , to be  $10^4$  cm/s<sup>2</sup>, and the effective temperature values 27500 K and 22500 K. Model profiles were calculated for 4 values of the inclination of the rotational axes  $i$  and for 8 values of the reduced angular velocity  $w$ , defined

as the ratio of the angular velocity to the critical angular velocity. The calculated half-widths (the line width at half maximum depth) are given in Table 1 and Fig. 1.

For both spectral lines used by us each value of the half-width measured from observations gives us a dependence of  $i$  versus  $w$  as a curve and an intersection of these curves defines the values of  $i$  and  $w$ . Such intersecting curves are given in Fig. 2 for nine stars. The observed values for half-widths have been taken from a paper by A. Slettebak et al. (1975). The error box for  $i$  and  $w$  has been singled out by taking the error of the half-widths to be  $\pm 0.1 \text{ \AA}$ . The extreme values for  $i$  and  $w$  thus obtained are given in Table 2, where we have compiled the main results of our study for 19 B and Be stars. An example of finding the error box for  $\beta$  Lup is given in Fig. 3.

In Fig. 4 the mean values of  $i$  and  $w$  are given for the stars under study. These values do not show any clustering or any peculiar distribution which might indicate systematic errors in the measurements analysis.

In our study we have followed the paper by Hutchings et al. (1977), where it was shown that  $i$  and  $w$  can be determined for rotating stars by finding the ratio of the half-widths of the UV and the visual spectral lines. Among those 14 stars, unfortunately, none coincided with the stars studied by us.

If  $i$ ,  $w$  and  $V_C$  (the critical velocity at the stellar equations) are known, we can determine  $V \sin i$  by  $V \sin i = V_C w \sin i$ . The value obtained depends on the adopted value of  $V_C$ . The values of  $V_C$  used by us were taken from a handbook by Allen (1973). The calculated values of  $V \sin i$  are given in Table 2, where also the values of  $V \sin i$ , found by Boyarchuk and Kopylov (1964), by Uesugi and Fukuda (1970) and by Slettebak et al. (1975) are presented. The results of the last two catalogues are compared with our results on Fig. 5. There is a tendency for our values to run slightly higher than those of Slettebak et al. In addition, the results of Stoeckley (1968), namely that for  $\eta$  U Ma holds  $i = 50^\circ \div 90^\circ$  and for 48 Per holds  $i = 30^\circ \div 50^\circ$  overlap with our results. To use in full the rotationally distorted spectral lines for finding the values of  $i$ ,  $w$  and  $V_C$ , more spectral line profiles, including those of UV region, are to be used.

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Table 1: Computed half-widths of MgII  $\lambda$  4481 and HeI  $\lambda$  4471 lines

w	Lines	i			
		0°	30°	60°	90°
$T_{\text{eff}} = 27500^{\circ}$ $\log g = 4.0$					
0.2	HeI	1.40	1.50	1.80	2.00
	MgII	0.45	0.65	1.00	1.25
0.3	HeI	1.40	2.40	3.00	3.20
	MgII	0.45	1.10	1.50	1.75
0.4	HeI	1.40	3.00	3.60	4.00
	MgII	0.45	1.42	2.10	2.50
0.5	HeI	1.40	3.40	4.25	4.80
	MgII	0.45	1.80	2.75	3.35
0.6	HeI	1.40	3.70	4.70	5.40
	MgII	0.45	2.30	3.65	4.25
0.7	HeI	1.30	4.00	5.20	6.00
	MgII	0.50	2.75	4.35	5.00
0.8	HeI	1.30	4.20	5.60	6.40
	MgII	0.50	3.35	5.00	5.60
0.9	HeI	1.30	4.40	5.80	6.60
	MgII	0.50	3.95	5.50	6.10
$T_{\text{eff}} = 22500^{\circ}$ $\log g = 4.0$					
0.2	HeI	1.00	1.20	1.80	2.25
	MgII	0.50	0.60	0.95	1.20
0.3	HeI	1.00	1.60	2.80	3.20
	MgII	0.50	0.90	1.35	1.80
0.4	HeI	1.00	2.20	3.40	3.85
	MgII	0.50	1.30	2.10	2.50
0.5	HeI	1.00	2.80	3.80	4.25
	MgII	0.50	1.85	2.90	3.45
0.6	HeI	1.00	3.20	4.20	4.75
	MgII	0.50	2.45	3.70	4.30
0.7	HeI	1.00	3.40	4.60	5.20
	MgII	0.50	3.30	4.50	5.10
0.8	HeI	1.00	3.60	4.80	5.60
	MgII	0.50	4.25	5.30	5.90
0.9	HeI	1.00	3.90	5.10	5.90
	MgII	0.50	5.10	6.00	6.60

Table 2: *i*, *w* and *V sin i* values for program stars

N <sup>o</sup>	Star	HD	Sp.	HeI (Å)	MgII (Å)	<i>V sin i</i> (km/s)*			<i>i</i>	<i>w</i>	
						S.	B.	U.			
1.	HR 3237	68980	B1.5IIIe	3.37	2.10	115	166	167	90-113	14 <sup>o</sup> -25 <sup>o</sup>	0.58-0.85
2.	β <sup>2</sup> Sco	144218	B2 V	2.62	1.21	50	85	84	81- 96	21 <sup>o</sup> -28 <sup>o</sup>	0.36-0.40
3.	β Lup	132058	B2 V	3.34	3.00	100	130	130	167-200	27 <sup>o</sup> -34 <sup>o</sup>	0.63-0.70
4.	ν Cen	120307	B2 V	2.70	1.75	70	170	94	130-155	27 <sup>o</sup> -40 <sup>o</sup>	0.40-0.50
5.	χ Car	65575	B2 V	1.85	1.31	50	100	98	38- 63	5 <sup>o</sup> -10 <sup>o</sup>	0.60-0.76
6.	α Mus	109668	B2 IV-V	4.14	3.51	110	215	198	191-233	22 <sup>o</sup> -30 <sup>o</sup>	0.80-0.90
7.	ν <sup>2</sup> Cen	121790	B2 IV-V	4.18	3.26	125	150	153	203-231	25 <sup>o</sup> -35 <sup>o</sup>	0.70-0.85
8.	φ Cen	121743	B2 IV-V	3.08	2.03	80	180	115	154-183	40 <sup>o</sup> -54 <sup>o</sup>	0.41-0.47
9.	ω Ori	37490	B2 IIIe	4.43	4.14	160	204	195	192-206	54 <sup>o</sup> -71 <sup>o</sup>	0.62-0.68
10.	γ Ori	35468	B2 III	2.27	1.30	50	61	64	64- 78	23 <sup>o</sup> -39 <sup>o</sup>	0.35-0.47
11.	HR 6143	148703	B2 III	3.23	1.55	75	77	83	93:	50 <sup>o</sup>	0.35
12.	ν Ori	41753	B3 V	1.99	1.03	40	40	42	94-104	30 <sup>o</sup> -34 <sup>o</sup>	0.35-0.36
13.	η Hya	74280	B3 V	3.55	2.33	100	130	132	134-146	20 <sup>o</sup> -23 <sup>o</sup>	0.67-0.80
14.	HR 3206	68243	B3 V	3.82	3.72	90	158	160	215-243	34 <sup>o</sup> -42 <sup>o</sup>	0.68-0.72
15.	η Uma	120315	B3 V	4.85	4.00	150	215	216	261-300	41 <sup>o</sup> -60 <sup>o</sup>	0.60-0.81
16.	HR 6087	116087	B3 V	5.75	4.89	190	-	-	366-389	66 <sup>o</sup> -76 <sup>o</sup>	0.71-0.77
17.	HR 4537	102776	B3 Ve	5.93	5.43	205	-	270	412-436	60 <sup>o</sup> -73 <sup>o</sup>	0.82-0.90
18.	48 Per	25940	B3 Vpe	4.71	4.03	250	210	217	254-277	34 <sup>o</sup> -40 <sup>o</sup>	0.76-0.89
19.	λ CMA	45813	B4 V	3.60	2.86	110	-	138	183-215	36 <sup>o</sup> -49 <sup>o</sup>	0.52-0.60

\*S.- Slettebak et al., B.- Boyarchuk and Kopylov, U.- Uesugi and Fukuda, R.- Ruusalepp

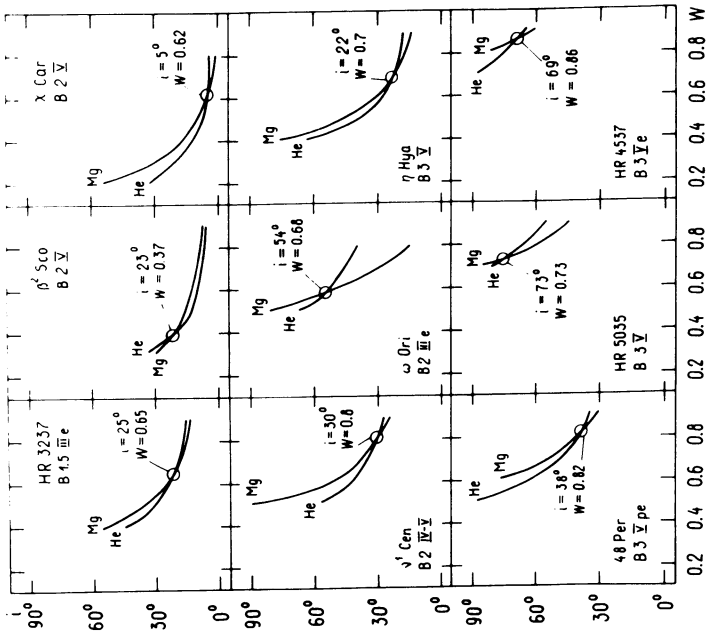


Fig.2. Illustration of procedures for determining  $i$  and  $w$  from measured line half-widths for a sample of program stars.

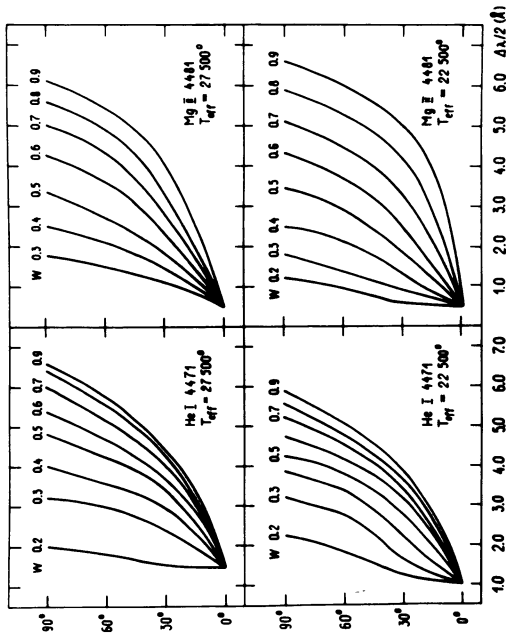


Fig.1. Computed half-widths for MgII and He I lines as functions of  $i$  and  $w$ . Other parameters are  $T_{\text{eff}}=27500^{\circ}\text{K}$  and  $22500^{\circ}\text{K}$ ;  $\log g=4.0$

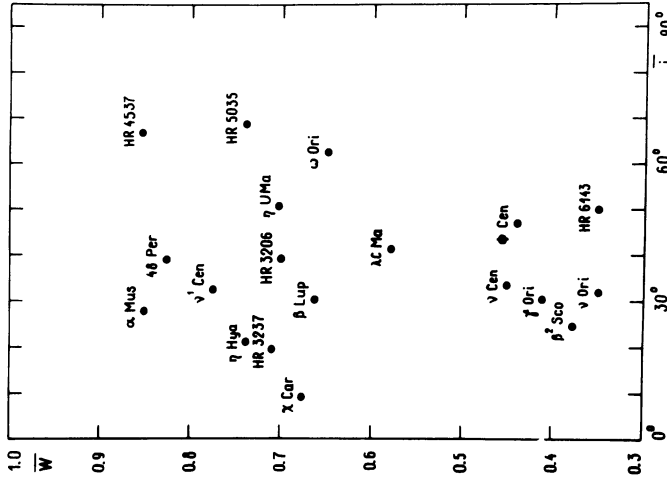


Fig. 4. Distribution of  $\bar{i}$  and  $\bar{W}$  values of program stars.

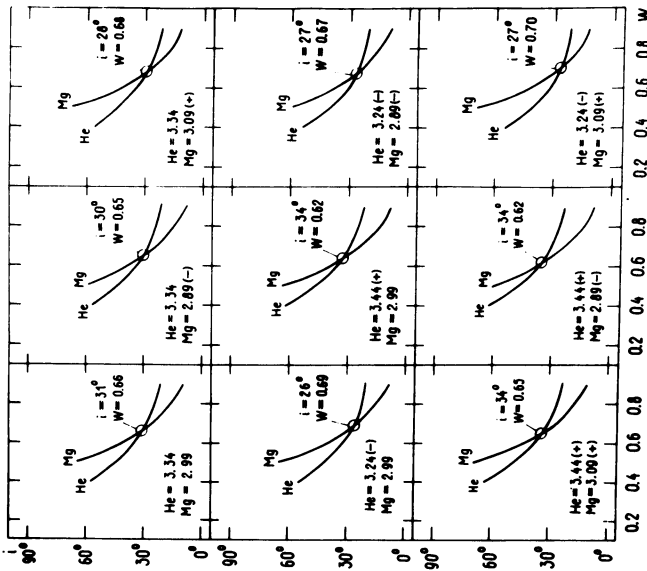


Fig. 3. Defining of  $\bar{i}$  and  $\bar{W}$  values for  $\beta$  Lup when measured half-widths have been changed  $\pm 0.1 \text{ \AA}$ . (+) indicates increase and (-) indicates decrease of line half-widths.

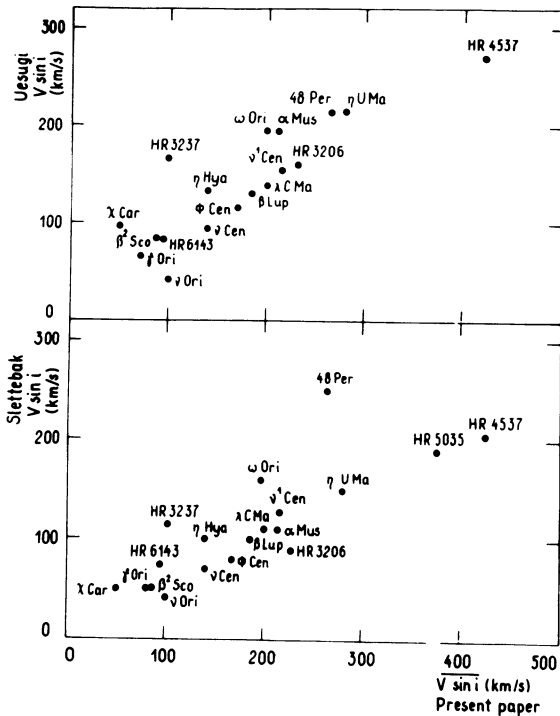


Fig.5. A comparison of rotational velocities given by Slettebak and by Uesugi with those given in the present paper.

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## DISCUSSION

Sonneborn: What are the errors in your determinations of  $i$  and  $\omega$ ?

Ruusalepp: The error is dependent on the star and on the observed half-widths of the spectral lines. The characteristic values of the errors are  $\pm 10^\circ$  for  $i$  and  $\pm .1$  for  $\omega/\omega_c$ .

Endal: From the viewpoint of someone who models rotating stars, the indeterminacy of the  $\sin i$  factor in observations is very aggravating. I believe that any investigation into measuring  $v$  and  $\sin i$  separately should be strongly encouraged.

Marlborough: Have you compared your results for  $i$  and  $\omega/\omega_c$  with those published by Hutchings and collaborators?

Ruusalepp: Among the sample of stars under examination there were no stars which coincide with stars studied in that paper.