CORRESPONDENCE.

MAXIMUM MORTALITY PERCENTAGES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I trust that I will not be considered presumptuous if I venture to offer a few words of friendly comment on a portion of a paper which has deservedly taken its place as one of the classics of actuarial science, and which has as its author one of the most honoured leaders of the profession. I refer to the essay by Dr. T. B. Sprague, "On the rate of mortality prevailing among assured lives, as influenced by the length of time for which they have been assured" (*J.I.A.*, xv, 328).

In that paper Dr. Sprague pointed out that in the experience of the Twenty Offices (H^M) a maximum mortality percentage is apparently reached between the eleventh and fifteenth years of assurance, and that this supposed fact may be accounted for by the operation of two factors, (1) the gradual wearing out of the beneficial effect of the medical examination at entry, and (2) the effect produced by the withdrawal of healthy lives What I wish to suggest now is, whether the peculiarity in question is not capable of an entirely different explanation, and due to an entirely different cause.

In an investigation into the effects of selection, the plan of comparing the actual number of deaths with those predicted by a standard table is certainly an excellent one. It is, however, essential that the table selected as a standard be one which is in every way suitable and reliable. If the gauge by which we measure be one which is not applicable, the results arrived at by its use will, in all probability, be incorrect. Dr. Sprague himself recognized this when he said, "It occurred to me that the fact of grouping together persons of all ages at entry might have a disturbing effect on the results, as would clearly be the case if the table of mortality from which the probable deaths are computed gave much too high a mortality at the early or middle ages." A careful examination will, I think, convince us that this is exactly what has happened, although the predicted mortality is too low, instead of too high, at the early and middle ages. Any table deduced in the ordinary way from the experience of life companies is itself influenced by selection, and is therefore from its very nature an improper standard. During early and middle adult life, an addition is made at each age of a

Percentage of Actual Deaths to those computed by the H^M Table.

Years of Assurance	Present Ages															All Ages
	15–20	21–25	26–30	31–35	36–40	41–45	46–50	51–55	56–60	61–65	66–70	71–75	76–80	81–85	86–90	91–96	
0	52·9	40·6	41·4	63·5	40·4	37·5	44·9	44·0	46·6	25·1	49·9	16·1	...	...	...	...	44·6
1	...	88·2	79·0	72·3	63·1	70·9	68·8	51·4	...	62·4	63·3	79·9	...	...	...	...	71·5
2	118·7	...	104·2	91·5	85·3	91·0	86·3	83·6	79·4	73·2	69·8	...	...	...	...	...	88·9
3	...	105·9	109·6	105·2	100·2	96·0	99·4	93·4	...	...	...	89·9	...	...	...	...	98·6
4	...	138·8	133·7	...	...	...	100·1	...	88·4	...	...	...	51·5	...	...	...	108·4
5	141·9	162·8	...	108·8	106·5	100·8	101·1	96·7	94·8	99·2	92·3	99·8	85·4	...	...	...	101·3
6–10	...	186·3	135·6	116·6	114·1	106·2	104·9	99·8	101·4	101·7	104·6	101·3	92·7	...	...	...	105·1
11–15	...	...	140·3	126·7	120·2	107·5	108·8	108·9	108·7	104·6	105·2	105·1	96·7	93·3	...	...	108·7
16–20	...	...	...	...	118·9	124·9	109·2	109·7	108·2	106·4	105·8	105·2	100·1	93·5	89·8	...	105·2
21–25	...	...	...	...	...	131·4	109·7	113·6	102·3	103·3	101·1	...	101·3	95·8	...	...	104·5
26–30	...	...	...	...	...	...	106·8	107·7	...	103·1	...	102·5	104·4	105·9	97·6	...	103·5
31–63	...	...	...	...	...	...	...	...	93·6	100·0	99·7	90·7	103·1	109·3	107·9	108·2	102·1
Total	104·6	95·5	103·0	98·6	100·9	99·7	100·3	99·8	99·5	100·2	100·0	100·8	99·5	101·2	99·1	108·2	100·07

large number of newly-selected risks. At ages 20 or 25 the lives exposed are all practically fresh from the examiners' hands.

As age advances the proportion of these new cases to the total becomes steadily less, and after 65 the lives exposed may be considered as, for practical purposes, containing none which have been recently admitted. The consequence is that the mortality shown by the table during the earlier years of life is unduly depressed by this artificial condition, and that the table as a standard becomes increasingly severe with advancing age. It may seem out of place to refer to such a well-known fact, but I do so because I think it is the key to the whole problem.

Let us now examine the percentages brought out by Dr. Sprague. I regret that it is impossible to arrange them under uniform groupings, but I have endeavoured to do this as nearly as possible. In each set I give in their exact order all the percentages. Those who wish to examine the slight variations which have been made in the groupings are referred to the original paper.

[*See preceding page.*]

In the above table the maximum in each set is printed in heavier type, while the percentages for the years 11 to 15 are enclosed within parallel rules to assist the eye. If we glance from left to right along the line opposite to any year of assurance we will be struck by the marked and steady reduction which takes place in the percentages. Taking, for example, the years 11 to 15, we find a gradual fall from 140·3 for ages 26 to 30, to 108·9 for ages 51 to 55, and to 93·3 for ages 81 to 85. This is, of course, what we would expect. It does not necessarily mean that the mortality at the higher ages is more favourable than at the younger, but rather that the mortality table by which the expected deaths are computed provides for a greater number of deaths proportionately at the older ages than at the younger.

Possibly, however, it may appear to some that this statement is inconsistent with the fact that the percentages for all durations combined as given at the foot of each column are all very close to 100, and are about equally heavy at all ages. It is merely necessary to point out that this is because the majority of deaths at the younger ages occur under policies of short duration, while at the higher ages the majority are under policies of long duration.

We now note several points brought out by the above table:

(1) Discarding ages over 90, seven of the 15 sets show an uninterrupted progression from lower to higher percentages, the highest percentage of all being in each case the last one in the group.

(2) Four of the remaining sets show an uninterrupted progression until the last percentage but one. They show a falling off in only the last item. The numbers of deaths on which the last percentages in these four sets are based are respectively 36, 23, 221, and 119, making in all only 399 deaths out of a total of 7,344 at those ages. Whether a uniform grouping of the durations or a very

slight re-arrangement of them might not cause the supposed maxima to disappear entirely from these groups is a matter about which no definite opinion can at present be expressed. At any rate they are evidently of but slight importance.

(3) Of the remaining four sets one only shows a maximum falling between the eleventh and fifteenth years of assurance, the other three falling later.

It is therefore clear apparently that so far as the subdivided ages are concerned, they do not, on the whole, confirm the maximum mortality theory, but on the contrary, tend to show that no such maximum exists—certainly not between durations 11 and 15.

How then does the summary for all ages combined show a maximum between years 11 and 15? Simply because by the uniting of all the divided sets the peculiarity of the mortality table comes into play, and the percentages for the longer durations of policies become more favourable, because these are the percentages which belong also to the older ages of life, where they are, as we have already shown, naturally and of necessity lower than at the younger ages.

Perhaps a practical illustration will be the most satisfactory way of explaining the matter. Let us take two hypothetical groups and then combine them.

Hypothetical Illustration.

Years of Assurance	PRESENT AGES 36-40			PRESENT AGES 66-70			COMBINED AGES		
	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-cent
1–10	1,000	800	80	100	62	62	1,100	862	78·4
11–15	1,000	1,150	115	500	500	100	1,500	1,650	110·0
16–20	200	250	125	1,000	1,020	102	1,200	1,270	105·8
21–30	...	...	...	600	618	103	600	618	103·0
TOTAL	2,200	2,200	100	2,200	2,200	100	4,400	4,400	100·0

In this illustration we have two separate groups of lives, each having a mortality which agrees exactly with the table. Neither shows any maximum, but on the contrary both get progressively worse. When, however, they are combined they produce percentages which on their face would indicate a maximum, although such does not exist in reality at all.

It may, however, be objected that as this combination is a mere supposition, it may perhaps have no bearing on Dr. Sprague's tables at all. To settle this doubt, I have picked out and combined the seven sets of lives which admittedly show no maximum (ages 15–20, 21–25, 26–30, 31–35, 41–45, 81–85, 86–90). As the groupings are

https://doi.org/10.1017/S002026810000559X Published online by Cambridge University Press

not identical I have had as before to place all irregular durations into the group to which they most nearly belong. The results are as follows:

Percentage of Actual to Computed Deaths.

Years of Assurance	Expected Deaths	Actual Deaths	Percentages	
0	345·90	164	47·4	47·4
1	603·53	459	76	76
2	488·38	469	96·1	96·1
3	509·63	528	103·5	103·5
4– 5	1,156·16	1,247	107·9	107·9
6–10	920·44	1,053	114·5	114·5
11–15	364·72	399	109·4	110·7
16–20	232·49	262	112·7	
21–25	127·01	139	109·4	109·4
26–30	117·34	121	103·2	106·5
31–63	175·27	191	108·9	
TOTAL	5,040·87	5,032	99·8	99·8

We here see in actual operation the principle referred to. Not one of these seven groups shows any maximum when considered by itself, and yet, when they are combined, there is a very marked maximum observable between the sixth and tenth years. This is conclusive proof that so far at least as this section of the experience is concerned, the supposed maximum is due solely to the nature of the table used as a standard. The addition of one other set (36–40) would have been sufficient to bring the maximum forward to the 11th–15th years of duration.

If it be supposed that as the Peerage Table is not founded on assurance experience it therefore is a proper standard for use in such a case as the present, and that the objections which we have been urging cannot apply to it, I would point out (1) that the Peerage Table is based solely on one particular stratum of society and is therefore inapplicable, and (2) that its mortality curve follows a peculiar course, being high at ages under 30 and over 70, and low between these ages. It would naturally therefore give results very similar to those of assurance tables, and like them must be discarded.

From a consideration of these facts, I think we must come to the conclusion that, much as we must all respect Dr. Sprague's opinion on this or any other point, the statistics which we have been examining contain no satisfactory evidence of the existence of such a maximum mortality epoch as has been supposed.

Yours truly,

T. B. MACAULAY.

Montreal,
10 *April* 1895.