# EMPIRICAL SCALING LAWS FOR CORONAL HEATING

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## ABSTRACT

We review the origins and uses of scaling laws in studies of stellar outer atmospheres, with particular emphasis on the properties of coronal loops. The evidence for a fundamental structuring of the Solar corona is reviewed and a discussion of thermodynamic scaling laws is presented. In order to intercompare different theories for coronal formation and heating, it is necessary to recast the theories in terms of observable quantities. As an example, we present a discussion of magnetic field-related heating and scaling laws which can be obtained relating coronal pressure, temperature and magnetic field strength; available data are shown to be consistent with scaling laws obtained in this way. However, some parameters of the theory must be treated as adjustable at the present time and it is necessary to examine data from other stars in order to determine whether these are true parameters in coronal heating. We examine some of the difficulties involved in using unresolved stellar data when dealing with loop atmospheres, by first treating the Sun as an unresolved source. Using the detailed observations now available we examine the limits of applicability of single-loop models. The possibilities and limits of stellar data are then discussed.

## 1. THE USES OF SCALING LAWS

The outstanding problem in coronal physics for over four decades has been the explanation of the corona's high temperature. The realization that some flux of mechanical energy is needed to maintain the million degree plasma has led in recent years to a plethora of theories, whose number is limited primarily by the ability of observations to disagree with specific predictions of the various theories.

It is clearly recognized now (see Vaiana and Rosner 1978) that a major difficulty which arises in attempting to specify the coronal heating mechanism is that the fraction of the Solar luminosity which needs to go into maintaining the corona is extremely small, of order $10^{-6}$. Thus there are many mechanisms which can be proposed and for each one a serious evaluation would involve a detailed and sometimes difficult search for observable signatures which could then be compared with available data, or which may necessitate new instrumental developments.

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The other major difficulty in studying coronal heating is that there may be more than one mechanism operating, depending on which part of the corona is under consideration. That is, the closed regions of strong magnetic fields in an active region core may be heated by an entirely different process than that responsible for heating the large scale, weak field regions or the open field regions with their large energy losses to the escaping Solar wind. It is also unclear at this time whether the more sudden and violent release of energy in a Solar flare occurs by the same process that provides the quasi-steady heating of the more quiescent corona; the possibility also exists that in the two different types of flare, the compact flare and the two-ribbon flare (Pallivicini, Serio and Vaiana 1977), there are again different processes responsible for heating the coronal plasma.

Although the difficulties just listed may appear to represent an unsatisfactory situation, the exact opposite is the case. The complexities of present work on coronal heating are a sign that our approach is more realistic than in previous studies and that we are likely to be approaching a better understanding of coronal formation and heating. In the following, I will attempt to provide a small piece of the story by reviewing briefly some of the steps which have led us away from the simple acoustic theories, with particular emphasis on the role played by magnetic fields. This discussion will be followed by an examination of some ways in which we can use scaling laws in combination with the available data for critical comparison with heating theories; we will then conclude with some discussion of the uses of stellar observations in extending the Solar results which have thus far been obtained. The present discussion covers only a small part of the total picture and should be viewed as complementary to the presentations by Vaiana, Rosner, Ionson and Chiuderi in these Proceedings.

## a) Coronal Structure

The structuring of the corona is a fact of fundamental importance in understanding the physics of coronal formation and heating. The existence of some structure and also the importance of understanding the role of magnetic fields in active regions was recognized before high resolution x-ray images were available (Billings and Saito 1964). However, the availibility of on-disk observations of the corona in x-rays revealed the ubiquitous loop structuring of the emitting regions and the close connection with regions of strong surface magnetic fields (Vaiana Krieger and Van Speybroeck 1970). A series of rocket flights led to semi-empirical modelling of coronal loop structures and coronal magnetic fields (Vaiana, Krieger and Timothy 1973) and the long duration *Skylab* missions permitted study of the temporal evolution of the emission regions in the corona and the quantitative link between magnetic field strength and coronal emission, as we will discuss below.

Examples of the observed loop structure in the corona are shown in figure 1, which illustrates a typical view of the corona-as seen from *Skylab*. In this image we find most of the typical non-flare coronal structures and we also see the boundary of a large N-S oriented coronal hole which represents an open magnetic field topology. The key point to notice, one which has been well documented by detailed studies (Van Speybroeck, Krieger and Vaiana 1970; Vaiana, Krieger and Timothy 1973; McIntosh *et al.* 1976), is that all of the coronal emission comes from closed structures (cf. however Maxson and Vaiana 1977). In all of the following discussion we will assume that high temperature plasma, i.e. material hot enough that its primary radiation is x-ray emission, on solar-type stars is predominantly confined in closed loop structures as is the case in the Solar corona.
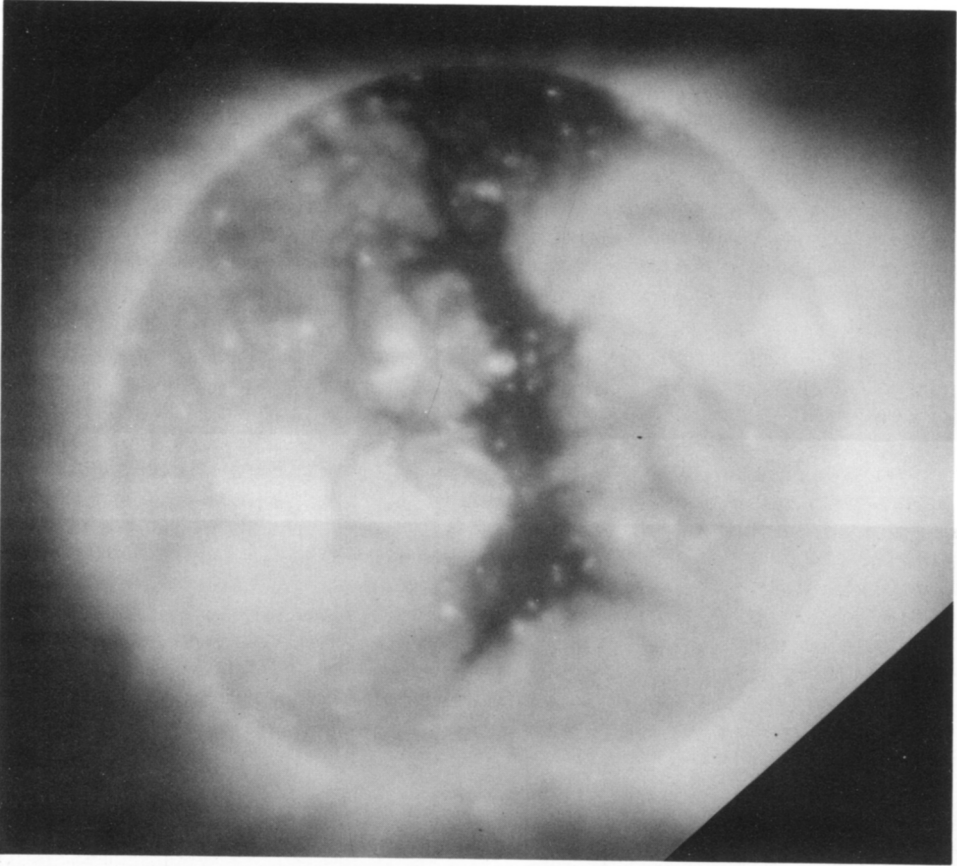
Fig. 1. The Solar x-ray corona as seen from Skylab on 1 June 1973. This photo shows all of the typical loop structures seen in the corona outside of flares and it illustrates the dominance of closed loops in controlling the emission. A large open structure (coronal hole) is clearly visible near disk center as a region of low emission.

The connection between photospheric magnetic fields and coronal emission regions is illustrated in figure 2, which shows a series of xray exposures of increasing duration and a standard photospheric magnetogram. The xray images show successively larger and fainter loop structures, ranging from the bright inner core loops of the active reions to the large-scale loops interconnecting separated regions. The diffusion of the magnetic fields across the surface after they have emerged is also evident by comparing the small, newly emerged region to the older, larger one.
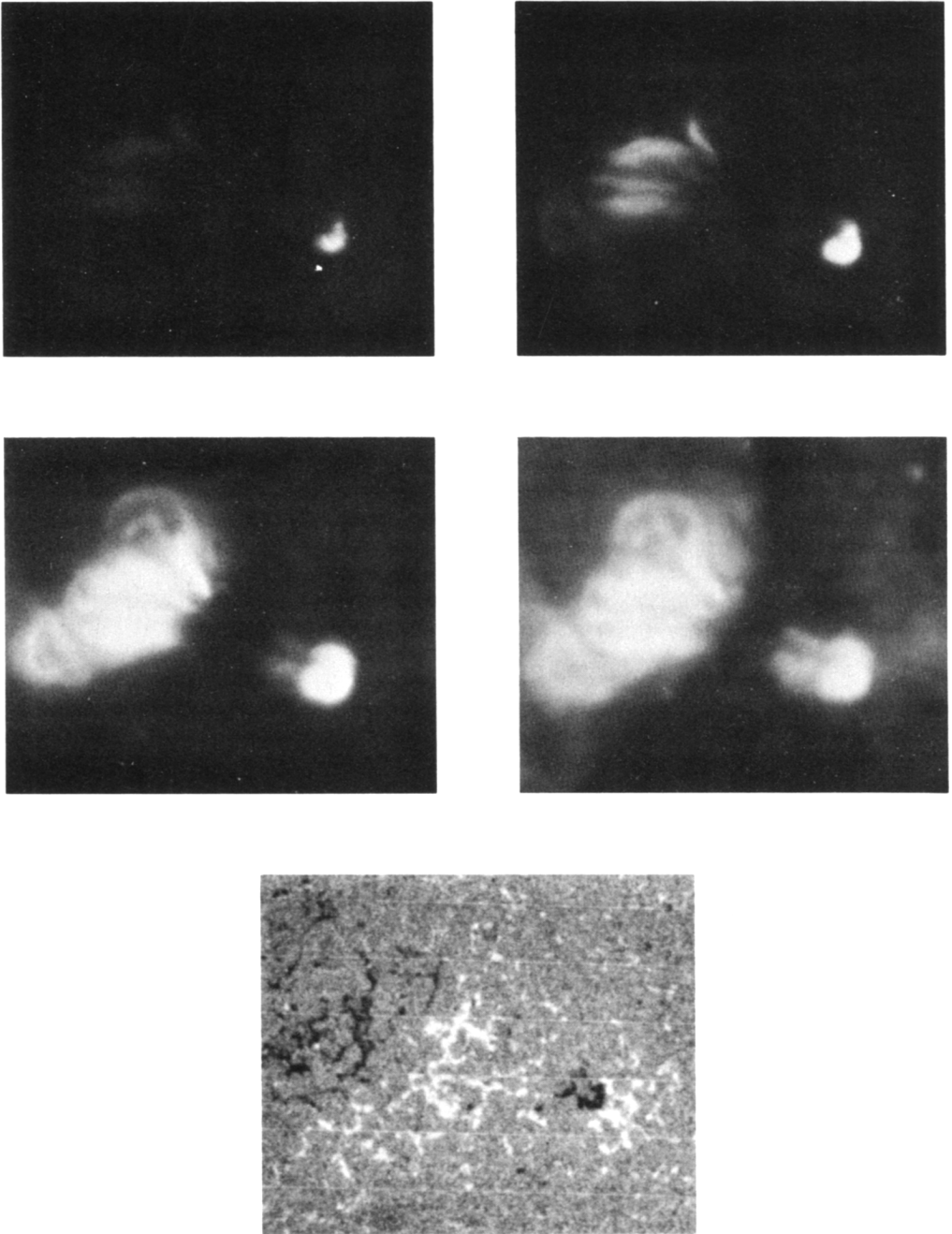
FIG. 2—Four X-ray photographs of a pair of active regions; the larger is one rotation old, and the smaller is 4 days old at the time of these photographs (1973 June 20, 0900 UT). X-ray exposure durations increase by successive factors of 4 from top left to bottom right, showing successively larger and fainter loop structures surrounding the bright cores. Note also that the energy density in the younger region is more than an order of magnitude greater than that of the older region.

The rapid variability and possible complexity of active region loop structures are illustrated in figure 3, which shows ~ 3 days in the life of an active region. The large number of loops and the interaction between different active regions is evident and indicative of the xray corona during times of moderately high activity.

The importance of structuring in coronal physics was put on a firm theoretical foundation by Rosner, Tucker and Vaiana (1978) and by Rosner and Vaiana (1978), who showed that each individual loop structure could be considered as a relatively independent "mini-corona". The true Solar corona, as observed at any given time, is to be viewed as an ensemble of these building blocks having a mix of statistical properties which are determined by the stochastic eruption of magnetic flux from the Solar interior. This work led directly to a *testable* scaling relation involving observable quantities and it forms the basis for most of the subsequent efforts at formulating scaling relations by which coronal heating theories may be tested.
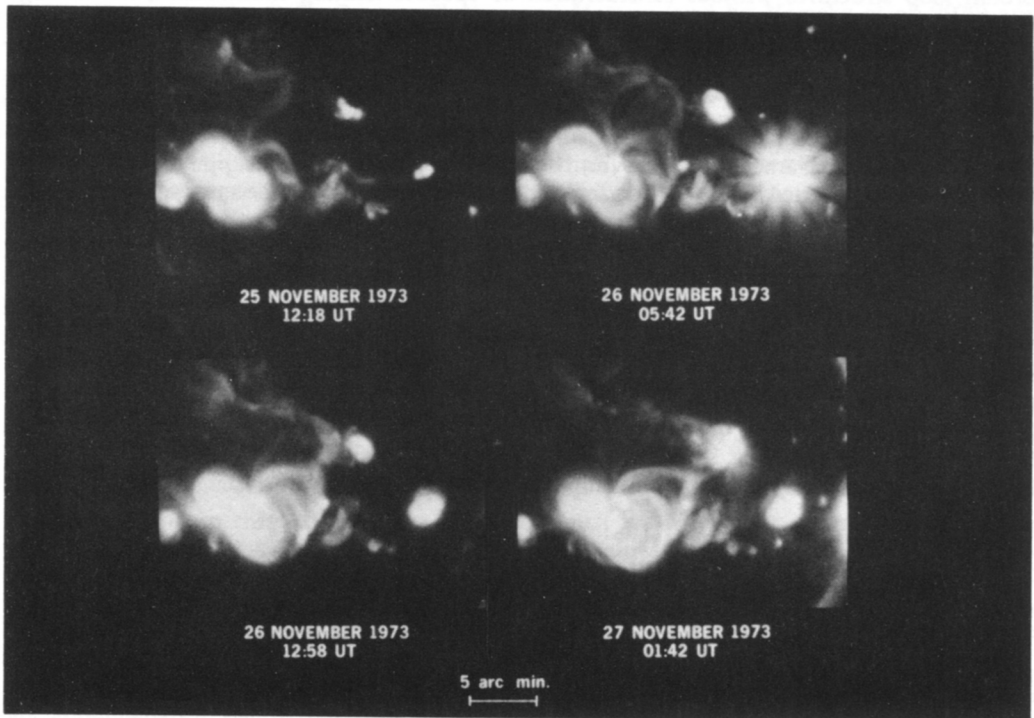


Fig. 3. The rapid and complex development of active regions loops seen in x-rays, as shown by two days' history of a large well-developed region and two newly emerging regions.

## b) Testing the Role of Magnetic Fields

In the present discussion we will concentrate on observational tests of magnetic field-related heating theories. This work has been motivated by the types of observations described in the preceding section, which argue for an active role for magnetic fields in coronal physics. That is, the Solar surface fields play a passive role in determining the coronal topology as we have shown, but they may also play an active role by providing the direct means for dissipation of mechanical energy flux to heat the coronal plasma *in situ*.

Given the conditions under which astronomical observations are made, it appears unlikely that any particular theory can ever be *proven*. The best that we can hope to do is to search for observational implications of an hypothesis and determine whether the data are or are not consistent with the theory's predictions. We have begun this type of work for the general case of magnetic heating, in which the energy for coronal heating is supplied by stressing of the magnetic field and subsequent dissipation of this nonpotential field energy.

We will start by examining the types of scaling laws which have been proposed for coronal loop structures without consideration of specific heating mechanisms, i.e. scalings of the thermodynamic kind. However, we will then concentrate on those scalings which are more directly related to a specific mechanism for coronal heating. In particular, we will describe a simplified but general loop model which specifically incorporates the magnetic field in an active role, leading to scaling laws relating observable quantities to the magnetic field strength, finding that significant tests can be performed over a wide range of the observable parameters. However, there are parameters which appear in the theory which cannot be varied when observing only the Sun and it is necessary to use stellar observations in order to provide more stringent tests of the theory.

We note also that this review will concentrate primarily on heating via the dissipation of magnetic stresses in the corona. There have of course been proposed numerous other mechanisms for heating the Solar outer atmosphere, including heating by acoustic processes (*a la* Biermann 1946 and Schwarzschild 1948), by MHD body waves (Osterbrock 1961), by MHD surface waves (Ionson 1978) and by *in situ* dissipation of currents (Rosner *et al.* 1978). Most of these theoretical papers have not directly addressed the question of scalings among observable quantities by which the theories may be tested. A summary of the present overall state of heating theories and an up to date bibliography of work in these various areas may be found in Kuperus, Ionson and Spicer (1981).

## 2. SOLAR LOOP ATMOSPHERES

### a) The T, p, L Scaling Laws

The first theoretical investigation which explicitly took into account the fundamental nature of closed loop structures as building blocks for the Solar corona was that Rosner, Tucker and Vaiana (1978); similar calculations were also performed independently by Craig, McClymont and Underwood (1978). The Rosner *et al.* study adopted the view that coronal heating as a whole must be understood by first studying the energy balance of the individual loop structures, which are to be considered as relatively isolated mini-coronae.

The work considered the energy balance relation for the loop as a whole

$$\int_V (E_H + f \cdot v) \, d^3r = -\int_V E_R \, d^3r + L_{foot} + L_{sides} , \qquad (2.1)$$

where the $L$'s represent losses across the boundaries of the loop, $E_H$ is the local mechanical heating function, $f$ is the net volume force exerted on the fluid, $v$ the bulk plasma velocity and $E_R$ the total radiative loss. The boundary conditions are treated by noted that cross-field effects are expected to be small, so that $L_{sides}$ is negligible and eq 2.1 becomes one-dimensional; in addition, they assumed $v=0$ at the boundaries and then argued that the conductive flux at the base of the loop must be small. In that case, conduction serves only as an energy redistribution mechanism and radiation is the only remaining loss mechanism for a closed, quiescent (quasi-static) loop.

The major result of this model, for the case in which the heating function is constant in space, is a relation involving the *observable* quantities temperature, pressure and loop length:

$$T_{max} = 1.4 \times 10^3 \, (pL)^{1/3} \qquad\qquad (2.2).$$

This relation contains no free parameters and depends, in addition to the above assumptions, only on the form assumed for the radiative loss function $P(T)$, or equivalently on equality between the radiative and conductive loses out of the "coronal" portion of the loop, i.e. the segment above the height at which the divergence of the conductive flux vanishes. As pointed out by Rosner, Tucker and Vaiana (and by Vesecky et al. (1979) and Galeev et al. (1981)), this equality holds over a surprisingly large range of coronal loop size, temperature and pressure values so that for essentially all Solar coronal loops the arguments leading to eq 2.2 hold.

The RTV paper treated loops for which the pressure scale height of the coronal plasma is large compared to the loop length, so that the pressure could be considered constant throughout the coornal portion of the loop. Extension of this work to larger loops, such as those making up the large scale structures in the corona, requires numerical simulations. these have been peformed by Serio et al. (1981) who used numerical computations of stationary loop atmospheres to calculate: i)loops having $L$ larger than $s_p$ and also, ii) loops having a local temperature minimum at the top. For the first class they found a simple extension of the previous scaling law:

$$T = 1.4 \times 10^3 \, (p_o L)^{1/3} \exp[-.04L \, (2/s_H + 1/s_p)], \qquad\qquad (2.3)$$

where $p_o$ is now the coronal base pressure, $s_H$ is the heating deposition scale length and $s_p$ is the pressure scale height.

In addition to this extended scaling law, which we will adopt in the following, they also found that over a broad range of parameter values loops having $L > \sim 3 \, s_p$ do not have stable solutions unless they have a local temperature *minimum* at the top. They discussed this result within the context of prominence formation and stability, as part of the overall evolutionary history of magnetic field structures in the corona.

In the remainder of this paper, when we investigate other scaling relations involving, e.g. the magnetic field strength we will assume that scaling relations 2.2 or 2.3 are generally valid for Solar-type coronae. In the case of the TRV scaling law, which contains no adjustable parameters, we use it directly. The Serio et al. extension contains the pressure scale height and the heating deposition scale height as parameters. The former is obtained directly by measurement of the coronal temperature and knowledge of the stellar surface gravity and the latter quantity will be assumed to be large (usually infinite).

## b) Models of Single Loops

A simple but general model of the energy balance in a single loop may be presented as shown in figure 4; the present approach follows that of Golub *et al.* 1980. The energy available for heating the coronal portion of this loop is ultimately supplied by shearing the longitudinal magnetic field in the region of the solar atmosphere for which $\beta > 1$, i.e., the photosphere or below. The energy available for heating is

$$W_m = B_j^2 V/8\pi , \qquad (2.4)$$

where $B_j$ is the nonpotential component of the magnetic field and $V$ is the volume over which the stresses are induced. The azimuthal field is generated from the longitudinal field by

$$\partial B_j/\partial t = B_z \, \partial v_\phi/\partial z. \qquad (2.5)$$

If we approximate the derivative by $v_{\phi_o}/D$, where $D$ is the depth of the shear zone, then since $V = \pi a^2 D$, we have

$$\partial B_j/\partial t = B_z \, \pi a^2 \, v_{\phi_o}/V. \qquad (2.6)$$

Note that the quantity <u>a</u> in these equations represents the cross-sectional radius of the magnetic flux tube in the high-$\beta$ portion of the atmosphere and the subscript on $\phi$ indicates that the velocity below the photosphere is to be considered.
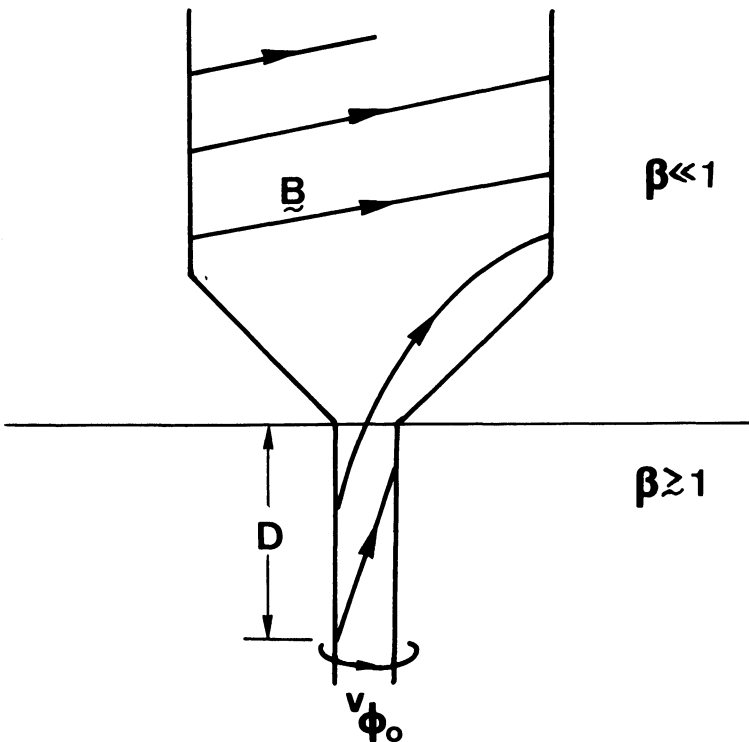


Fig. 4. Schematic representation of the loop model parameters.

The coronal extension of such a loop will have a larger cross-section due to the drop in the external gas pressure with increasing height above the photosphere. Because of this expansion, any twist which is present in the high $\beta$ region will be transmitted upward and amplified, as shown by Parker (1974). As viewed from the corona, this transmition of stresses will be an effective twisting velocity at the base of the loop, which we label $v_\phi$. The energy available for heating the corona is again given by eq. 2.1, but with $V = 2\pi R^2 L$, where R is the minor radius of the loop and L is the loop semi-length (i.e. distance along the loop from its base to its apex). The energy generation rate is then

$$dW_m/dt = 0.25 \, B_j \, B_z \, v_\phi \, R^2. \tag{2.7}$$

Combining this result with the scaling law of Rosner, Tucker and Vaiana (1978) and assuming that *all of the available energy goes into heating*, we find that the coronal gas pressure is related to the magnetic field strength

$$p = 2.6 \, B_2^{12/7} \, L_9^{-1/7} \, (\alpha \, v_4)^{6/7}, \tag{2.8}$$

where the subscripts indicate division by the indicated powers of ten and $\alpha \equiv B_j/B_z$.

Unfortunately, the parameter $\alpha$ involves the azimuthal field $B_j$, which is not a measureable quantity, while the effective twisting velocity $v_\phi$ may be treated as an adjustable normalization parameter for our purposes. In dealing with $\alpha$ several approaches are possible: Golub *et al.* (1980) simply assumed that all coronal loops have $\alpha$ constant, so that the normalization constant becomes $\alpha v_\phi$ instead of just $v_\phi$. Subsequently, Galeev *et al.* (1981), in a theoretical study of coronal heating via collisionless tearing modes, arrived at $\beta \ (\equiv 8\pi \, p/B^2) = 0.3$ for all loops, leading to $B_j = 8.8 \, p^{1/2}$, which may be substituted directly into eq. 2.5 in order to eliminate $B_j$. These two approaches lead respectively to the following scalings of coronal pressure with other measureable quantities:

$$p \propto B_z^{12/7} \, L^{-1/7} \, (av_\phi)^{6/7}, \qquad \text{(constant } \alpha) \tag{2.9a}$$

$$p \propto B^{3/2} \, L^{-1/4} \, v_\phi^{3/2}, \qquad \text{(constant } \beta) \tag{2.9b}.$$

A comparison of these two theoretical predictions with Solar data is shown in figure 5, for which we have integrated over an active region in each of the five observations. We note that the example shown here is of a single active region which was followed throughout its entire evolutionary history, from initial emergence through its blending into the large scale quiet corona.

In the present discussion we have treated $v_\phi$ as an adjustable parameter; its value is found to be $\sim 1$ km s$^{-1}$, which is in accord with the observed magnitude of the turbulent velocities in the Solar outer convective zone. However, this agreement can be further tested if we turn to the stellar observations, examining stars having very different values of the surface turbulent velocities. If the scaling laws can be reformulated in a suitable manner and the relevant observations performed, then these additional tests can be applied. We will look into this question in some detail in the remainder of this paper.
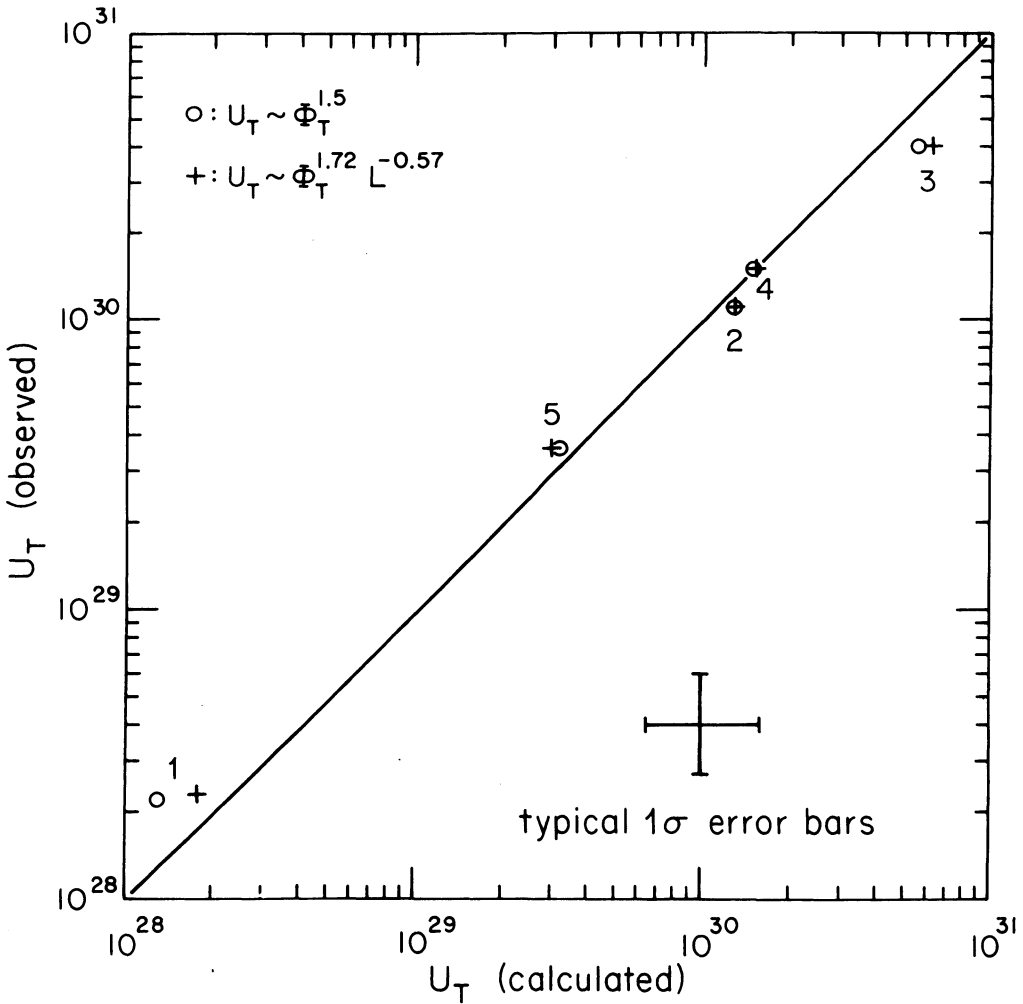
**Fig. 5.** Comparison of observed x-ray emission *vs.* calculated, based on the measured values of magnetic flux and region size. Data are for a single Solar active region followed throughout its entire lifetime, from initial emergence through evolution into large scale corona. Data points are based on x-ray and magnetic field (KPNO) measurements, using the predictions of equs. 2.9a,b.
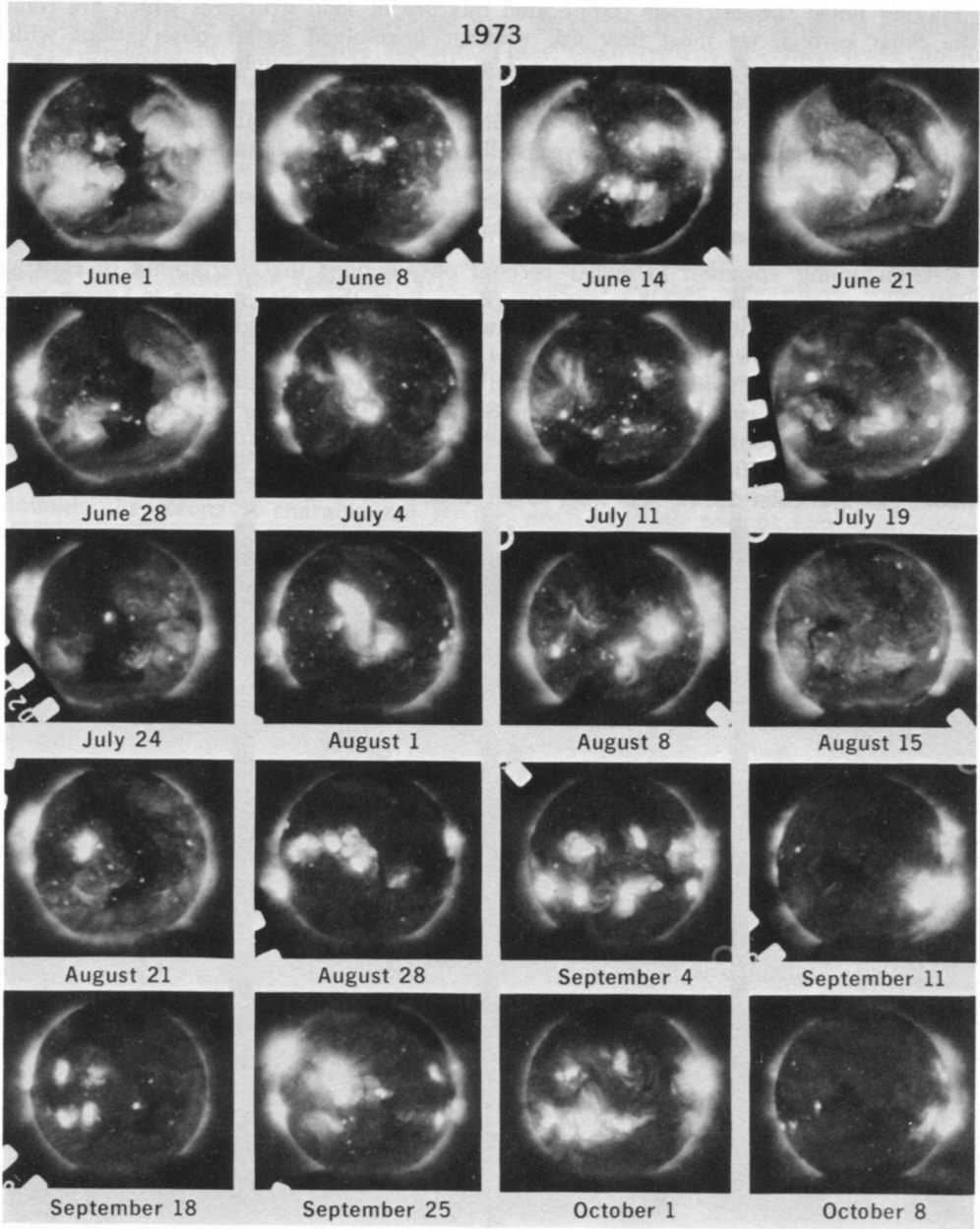
Fig. 6. Five rotations of the X-ray corona as seen from Skylab, illustrating the major changes in coronal structure and emission occurring on relatively short time scales.

## 3. OBSERVATIONS OF UNRESOLVED ATMOSPHERES

Having noted the extremely complicated mixture of loop structures which are found in the Solar corona, we must now ask whether unresolved stellar observations which necessarily average over the entire ensemble of structures in a stellar atmosphere can be of any real use in studying the nature of the coronal heating mechanism. We have addressed this question in some detail (Golub 1982) and find that within certain limits there is a good possibility that unresolved stellar data can answer some questions initiated within the Solar context.

### a) The Sun as an Unresolved Source

It is becoming apparent in stellar coronal observations that variability in emission properties is quite common. This is not surprising, since the Solar corona is known to be variable on relatively short time scales, even when flares are excluded. This is illustrated in figure 6, which shows five Solar rotations observed from *Skylab*; in this figure, each neighboring image in a row is ~90° separated from the adjacent images. Thus each vertical column shows the evolution of the same Solar longitude during the observing period. It is clear that the corona is highly variable, even from one week to the next. In particular, during the fourth and fifth rotations the corona varied from a solar maximum to a minimum configuration during less than seven days.



Fig. 7. The Solar x-ray corona at minimum; note that the structure is dominated by the evolved remnants of the numerous small emerging flux regions.

The variation in coronal composition during different portions of the Solar cycle can be even more extreme than those illustrated above. For example, x-ray data obtained near Solar minimum show an entirely different kind of corona (figure 7) which is dominated by large numbers of small active regions. These small regions are short-lived, do not develop sunspots and have relatively low x-ray emission measure but they are found to contain as much magnetic flux as the larger, longer-lived active regions found at times of higher Solar activity (Golub 1980).

In order to produce a quantitative representation of coronal emission in terms of the various known atmospheric components we must take into account the Solar cycle variation of each component and the evolutionary histories of the different types of emerging field regions. A first attempt at modelling this complicated mix of structures is shown in figure 8, which shows the fractional area coverage by various atmospheric components (top) and the relative percentage contribution to the total coronal emission (bottom), all as functions of the Solar cycle.

From quantitive studies of this kind we may draw the following preliminary conclusions:

The average Solar corona is dominated by a single type of structure approximately half of the time. At Solar maximum, the entire coronal emission may be characterized by the properties of large active regions, with $T \sim 10^{6.6}$ K and $EM \sim 10^{50.5}$. At Solar minimum the corona is characterized by the small emission regions and their diffused states, with $T \sim 10^{6.3}$ K and $EM \sim 10^{49}$.

## b) Model Predictions and Comparison to Data

We may combine the scaling laws eq. 2.2 (2.3) and eq. 2.9 discussed above in order to eliminate one of the vairables. For the case of stellar observations, the coronal pressure is not directly observable as it is on the Sun, so that the choose the new scaling law to be of the form

$$T = 1.2 \times 10^3 \, B^{1/2} \, L^{1/4} \text{ (cgs)}, \tag{3.1}$$

where we have omitted the twisting velocity $v_\phi$ as a parameter and will restrict our attention to stars having turbulent velocities close to that of the Sun.

Following our discussion in §III.a we will assume that all of the coronal emission in each stellar source comes from a single type of loop structure. Since we are dealing with quiescent emission levels from these stars, we may take the loops to be as large as the pressure scale height

$$L = 5 \times 10^3 \, T \, (g/g_\odot)^{-1}. \tag{3.2}$$

The reason for this is that the activity and rapid fluctuations in emission will derive predominantly from newly emerging active region loops. These will rapidly grow in size until they become comparable in size to the pressure scale height, at which time the level of activity will have decreased substantially (see Galeev et al. 1981). Further growth in loop size will not change eq. 3.2, since the emission scale parameter will thereafter remain equal to the pressure scale height.
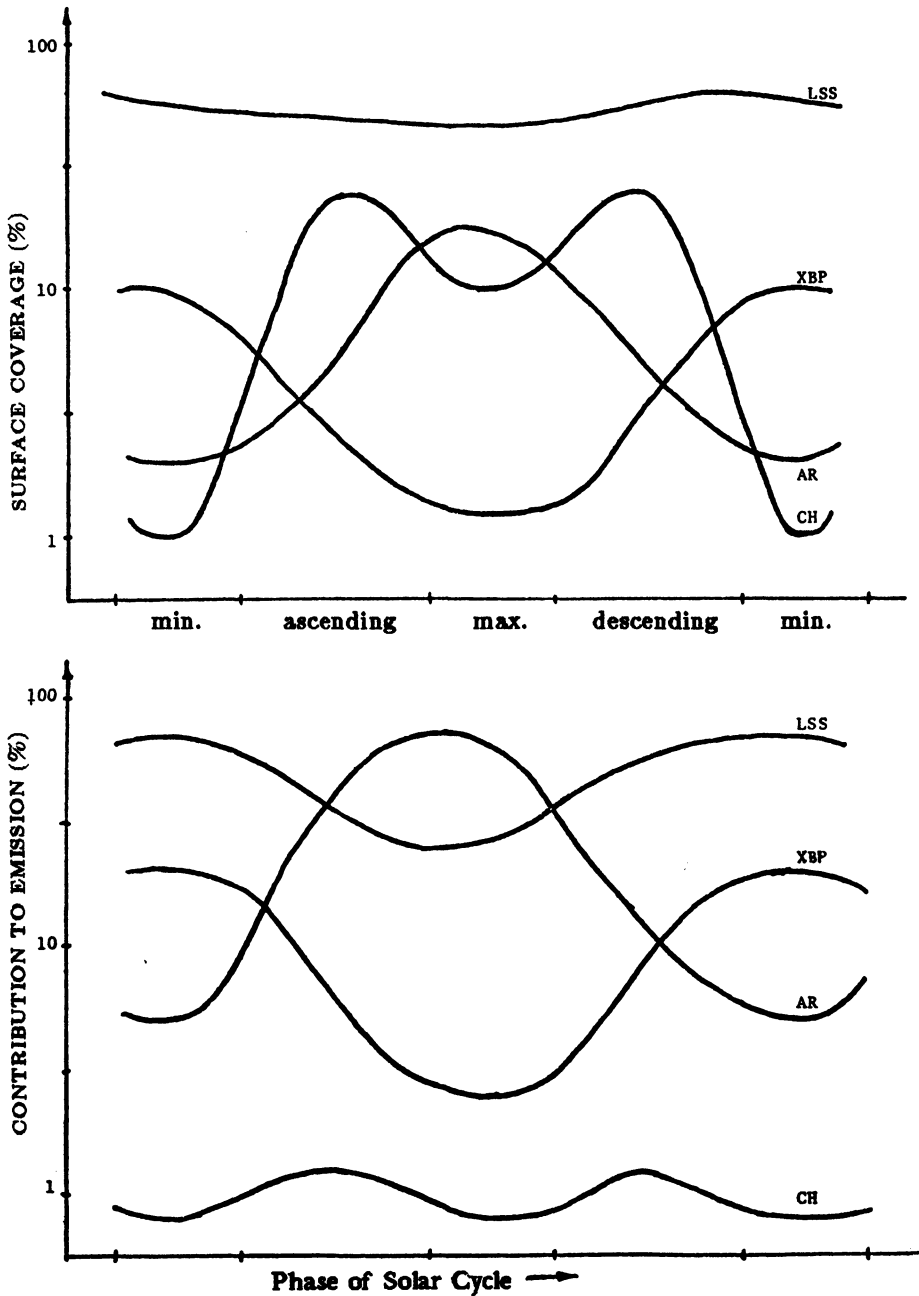
Fig. 8. Fractional area coverage factors (top) and relative contributions to coronal luminosity (bottom) for the various atmospheric components of the Solar corona, all as a function of phase in the Solar cycle.

For an atmosphere of such loops, we have shown elsewhere (Golub *et al.* 1982) that measurement of the coronal temperature and xray luminosity are sufficient to specify the atmosphere. The quantities of interest in the present discussion are the coronal filling factor $f$ and the magnetic field strength $B_{em}$ in the emitting regions. These are given by

$$f = [3.4 \times 10^{-9}/ P(T)] \; F_x \; [T^3/(g/g_\odot)]^{-1} , \qquad (3.3a)$$

$$B_{em} = 1.2 \times 10^{-8} [ T^3/(g/g_\odot)]^{1/2} , \qquad (3.3b)$$

where $P(T)$ is the plasma emissivity and $F_x$ is the average surface flux of xrays at the stellar surface; the average magnetic field on the star is given by $<B> = f \, B_{em}$. We have calculated these quantities and list them in table 1 for some Solar-type stars. It will be quite informative in the next few years to find out whether magnetic field values measured on some of these objects agree with these predictions.

Table 1. Predicted Stellar B Values.

| Star | $F_x$ (erg cm$^{-2}$ s$^{-1}$) | $T_{cor}$ (K) | $f$ | $B_{em}$ (g) | $<B>$ (g) |
|---|---|---|---|---|---|
| Quiet Sun | $3 \times 10^4$ | $1.8 \times 10^6$ | 0.15 | 30 | 5 |
| Active Sun | $1 \times 10^5$ | $3.5 \times 10^6$ | 0.07 | 80 | 6 |
| $\alpha$ Cen A | $1.3 \times 10^4$ | $2.1 \times 10^6$ | 0.06 | 90 | 5 |
| $\alpha$ Cen B | $6.1 \times 10^4$ | $2.1 \times 10^6$ | 0.18 | 60 | 11 |
| $\pi^1$ UMa | $1.2 \times 10^6$ | $4.0 \times 10^6$ | 0.22 | 100 | 20 |
| $\lambda$ And | $1.5 \times 10^6$ | $5.0 \times 10^6$ | >1 | 670 | 670 |

## 4. ACKNOWLEDGEMENTS

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## DISCUSSION

WALTER:  I have got a picky question about the temperature of $\lambda$ And. Swank and White's SSS (Solid State Spectrometer) observations indicate a two-component corona, with a temperature of $\sim 8 \times 10^6$ K in the cool component, and a hot component of a few tens of $10^6$ K. What about the hot component, and how would the higher temperature of the cooler component affect your estimates of $f$ and $B$ ?

GOLUB:  First, I have neglected the hot component, since the work I presented applies to the large-scale quiet corona, which I equate with the cooler and steadier component of $\lambda$ And. The temperature I used was the one that we (CFA) obtained from the IPC. If we use the $8 \times 10^6$ K value, then the agreement with the Giampapa and Worden value of $B$ is even better than the one I quoted.

ROBERTS:  Since scaling laws are increasingly being used in stellar work it is very important that their form is properly checked and understood for the Sun, where observations can be made in greatest detail. Now, as you mentioned in your review, the simplest available scaling law, due to Rosner et. al, has no free parameters (such as the heating rate). If I understand it correctly, this is because at the base of a loop the conductive flux vanishes. But it should be remembered that the $L$ in the scaling relation is not the same as the measured $L$, because a small part of the loop lies hidden in the chromosphere (where, incidentally, much of the important and complex physics of loops occurs). If we avoid this problem by letting $L$ correspond to higher levels in the loop (measuring $L$ from a point in the corona say), then we no longer get a simple scaling relation but instead find that the explicit form of the heating enters into the determination of $T_{max}$. It would, of course, be nicer to stick to the simpler relation. But to do this — indeed, to use scaling laws at all — we must be certain that such relationships are well borne out by solar observations.

My question, then, is: Do we get adequate agreement with observations, especially if we compare directly $T_{max}$ observed with $T_{max}$ calculated (rather than comparing some integrated quantity, where errors may well be masked), and what in fact is the extent of disagreement in any one active region? What, too, are the error bars involved?

GOLUB: A full answer to this question would be quite involved. First, the loop length $L$ as used in the scaling law is entirely insensitive to the thickness of the transition zone, since it is so thin. Of course, there is much physics there, but as a parameter in the scaling law, $L$ is much more influenced by measurement problems such as projection effects and curvature of field lines. Concerning the agreement with data, as Dr. Rosner points out it is quite remarkable that such a simple theory agrees so well with such a wide variety of observed loop parameters. If we view the scaling as an empirical result, then it may be more acceptable to you.

ROSNER: On the whole, I agree with Robert's point. However, a more significant difficulty is that the energy equation may need to include effects such as enthalpy fluxes. Nevertheless, the $T = T(p, L)$ scaling does provide an adequate description of the data. Thus, although one might argue with details, the main point of having temperature, pressure, and size scale of emitting structures correlated in a very particular functional way is a very useful device for interpreting stellar observations, as shown by Dr. Golub.

IONSON: For solar-type stars you have set the emission scale height equal to the loop height. This seems reasonable. What is your proposal with regard to non-solar stars?

GOLUB: Since I am modelling solar-type stars and restricting attention to quiescent emission, I believe that it is reasonable to say that we are looking at the evolved, large-scale loops. The more compact, higher-pressure loops are likely to be more active, as they are on the Sun, and will represent a variability on top of a quiescent baseline emission level.

CRAM: Would you comment on the possible magnetic structure in a star like λ And, which has a coronal magnetic field about equal to its photospheric magnetic field?

GOLUB: You are asking what a star would look like if we squeezed the magnetic fields together to an extent not observed on the Sun. We have no experience with this question, and so it is difficult to guess. Perhaps we could use field extrapolation programs to do some modelling.

KUIN: I have a comment on the large filling factor in λ And and the assumption of loop lengths of the same magnitude as the scale height. If one assumes smaller loop lengths, one obtains the same results for smaller filling factors, because the loops have higher pressures. Two components may be present with different filling factors and lengths. An example of this kind of analysis is the work of Mewe et al. (1982, *Astrophys. J.* **260**, p. 233). In this case one of course has to assume something for the filling factors.

GOLUB: Certainly it is true that the X-ray measurements do not fully constrain the model atmosphere. For instance, we can define a locus of points in the $f$ - $B$ plane, with different points along this curve representing different loop lengths. Therefore we need an additional constraint on any one of these parameters in order to specify the other two. Since I want to estimate $B$, I have chosen to present an argument on what the loop length $L$ should be. The value of $f$ also comes out of this calculation, but was not my main concern. It only becomes important if unreasonably high values of $f$ come out of the calculation.