IS CHANDLER FREQUENCY CONSTANT?

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ABSTRACT. The observed polar motion in the period 1860-1985 is analyzed in order to decide whether Chandler frequency was constant. It is shown that while the phase of annual wobble was very stable throughout the interval in question, Chandler wobble phase was subject to sometimes very rapid changes. The most pronounced negative phase changes were always accompanied by extremely low amplitudes, and a significant correlation was found between Chandler wobble phase and its integrated amplitude. The most probable explanation is that the frequency of Chandler wobble is variable and amplitude-dependent, which might be caused by non-equilibrium response of the ocean.

1. INTRODUCTION

Solution of rotational motion both for the rigid and elastic Earth, as well as for more sophisticated Earth models with elastic mantle, fluid outer and rigid inner core allow only a constant frequency of free motion of its axis of rotation within the Earth's body. If the Earth is assumed rigid, the theoretical period of free motion is equal to only 305 days (Euler period), for the oceanless Earth with fluid outer and rigid inner core and an elastic mantle it is equal to 400 days, and if we further account for an equilibrium ocean, the period ammounts to 435 days, which is very close to the observed value (Lambeck,1980). Since it is free motion, only its frequency is uniquely given by the respective theory, its amplitude and initial phase, being integration constants, can be determined only by observations. However, the observations show substantial discrepancies from any of the above mentioned theories. Many authors, including Chandler himself, found in the spectral analysis of the results two or more frequencies close to basic Chandler frequency (see e.g. Chandler,1901, Colombo and Shapiro,1968, Gaposchkin,1972, Wu Shou-xian et al,1979 or Pejović,1983). Some others (e.g. Proverbio et al.,1972 or Carter,1981 and 1982) are of the opinion that there is only a single free frequency that changes with time, and there are also some (e.g. Guinot,1972

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and 1982 or Wilson and Vicente,1981) arguing that there is no evidence for temporal variation in the Chandler frequency. When studying the long-period behaviour of polar motion, the author of the present paper (Vondrák,1985) found non-linear relationship between frequency of Chandler wobble and its amplitude in the interval 1900-1984. Here we shall study much longer interval of observations (1860-1985) in order to decide whether Chandler frequency is constant or not.

2.DATA ANALYZED AND THE ANALYSIS

The data set to be analyzed consists of the following subsets:

i) Polar motion in the interval 1960-1890 as derived by Rykhlova (1970) from the observations of absolute declinations at Pulkovo, Washington and Greenwich.

ii) In the interval 1890-1962 the coordinates of the pole as derived by Fedorov et al.(1972) from all available latitude observations all over the world.

iii) In the interval 1962-1972 the coordinates of the pole as determined by the Bureau International de l´Heure from optical astrometry.

iv) In the interval 1973-1985 the coordinates of the pole as determined by the BIH from the combination of optical astrometry and modern space techniques.

It is well known that the second most significant component of polar motion to Chandler wobble is the forced annual wobble, most probably caused by atmospheric excitation. Therefore the following simple model of analysis is used; the coordinates of instantaneous pole in a six-year interval is expressed by formulas

$$x=x_o+C_{xc}\cos 2\pi f_o t+S_{xc}\sin 2\pi f_o t+C_{xa}\cos 2\pi t+S_{xa}\sin 2\pi t \qquad (1)$$

$$y=y_o+C_{yc}\cos 2\pi f_o t+S_{yc}\sin 2\pi f_o t+C_{ya}\cos 2\pi t+S_{ya}\sin 2\pi t \quad ,$$

where $f_o = 1/1.19$ is the adopted provisional value of the Chandler frequency, x_o and y_o express the long-periodic part of polar motion and the quantities C, S denote the parameters of Chandler (subscript c) or annual (subscript a) wobble. Time t is reckoned in years from 1900.0. All these ten unknowns were solved for in six-year long running intervals by the method of least squares. The adjusted values of the coefficients C, S were further used to determine both semi-minor and semi-major axes of the Chandler (b_c, a_c) and annual (b_a, a_a) wobbles, the direction of semi-major axes of these ellipses (ψ_c, ψ_a) and phases of both wobbles (φ_c, φ_a) from the equations

$$(a - b)\cos(\varphi - 2\psi) = C_x + S_y \qquad (2)$$

$$(a - b)\sin(\varphi - 2\psi) = S_x - C_y$$

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$$(a + b) \cos \varphi = C_x - S_y$$

$$(a + b) \sin \varphi = S_x + C_y \ .$$

The phase is defined as west longitude of the instantaneous pole (with respect to its mean position) at the instant 1900 + 1.19n (Chandler wobble) or at the instant 1900 + n (annual wobble), where n is an integer. Consequently, if the motion has exactly the assumed period (i.e. 1.19 y and 1 y, respectively) both phases should be constant. Their actual values, derived from eqs (2), are displayed, together with semi-major axes of both wobbles, in Fig. 1. It is clear that the phase of annual wobble is relatively stable; it oscillates around a constant mean value. The corresponding amplitude is also more stable than that of the Chandler wobble. On the other hand, the phase of Chandler wobble exhibits sometimes very rapid changes, with no return

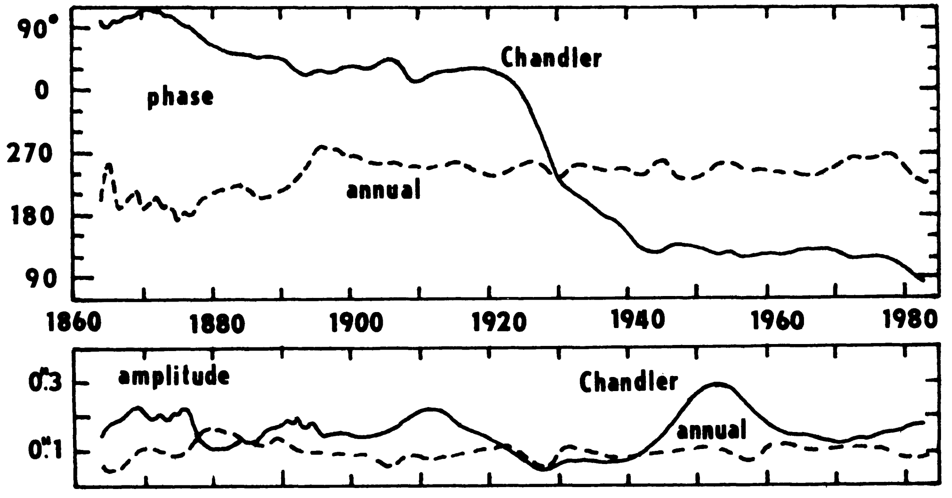

Figure 1. Chandler and annual wobble phase and amplitude. Notice the correlation between rapid change of phase and minima of Chandler wobble amplitude.

back to original values (1870-1890 or 1920-1940). Remarkable feature is the coincidence with the minima of amplitude. It seems that there is a certain dependence between the two quantities. If we adopt the hypothesis pronounced recently by Carter (1981, 1982) that Chandler wobble is frequency-modulated, there should exist a correlation between the observed phase φ_c and the integral $\int(a-a_o)\,dt$, where a is the total amplitude of polar motion and a_o its mean value, to which the adopted Chandler frequency correspond. Fig. 2 displays the cross-correlation between the two quantities. Broken line connects the points

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for each six-year interval. It is evident that there must be a very strong correlation between Chandler wobble phase and integrated amplitude; the coefficient of correlation ammounts to 0.986. This means that the relation is practically functional. Nevertheless, it does not

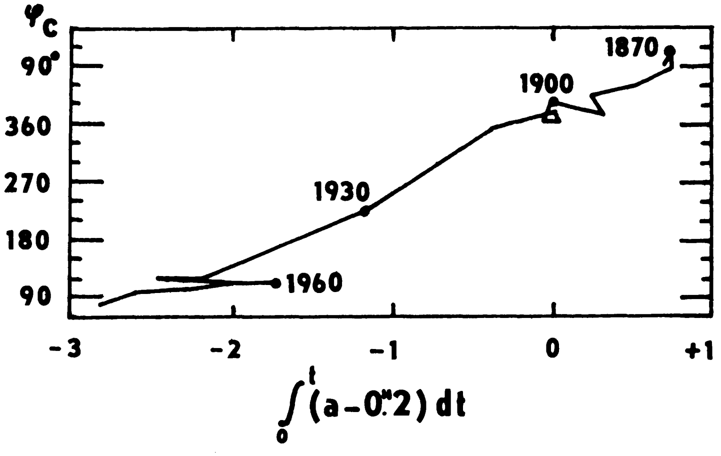

Figure 2. Correlation between integrated amplitude of polar motion and Chandler wobble phase.

seem to be linear. We can see from Fig. 1 that positive deviations of the amplitude from its mean value (as in the years 1870-1875, 1910-1915 or 1950-1960) cause incomparably smaller change in phase than the negative deviations (e.g. in 1880-1885 or 1925-1940). In order to demonstrate this more clearly, the formula

$$f = f_o - \partial \varphi_c / \partial t \qquad (3)$$

was used to calculate the actual value of Chandler frequency between two consecutive six-year intervals; the resulting values are plotted against the average amplitude in the same intervals (Fig.3). Though there are not many results for extremely low amplitudes, it seems to be evident that there is non-linear functional dependence between the two quantities.

3.DISCUSSION

It is obvious that Chandler frequency varied substantially during the last century - from 0.830 to 0.900 cpy. There is also strong evidence that its value is amplitude-dependent. A non-linear relation between the two quantities seems to fit the observations better than a linear one (see Fig. 3). The most probable explanation is the one offered by Carter (1981) - by nonequilibrium

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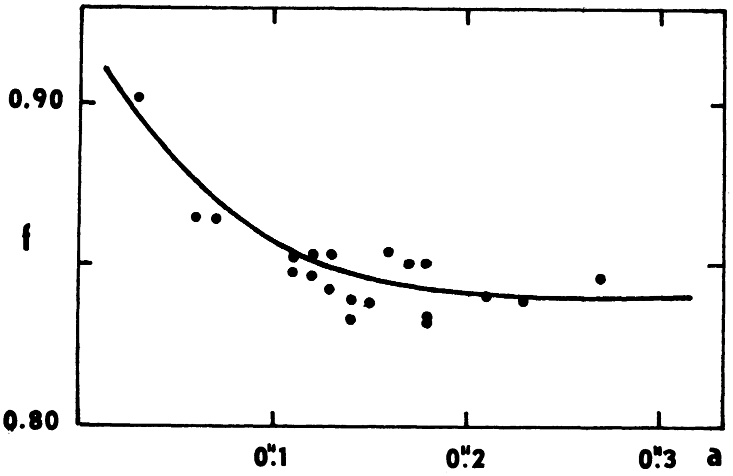

Figure 3. Non-linear dependence between polar motion amplitude a and Chandler frequency f.

response of the oceans to polar motion. Notice that the intersection of empirically drawn curve with zero-amplitude axis is close to f=0.912, i.e. to the free frequency of the oceanless Earth with elastic mantle and a fluid core. It means that the ocean reacts somehow "lazily" to polar motion; for very small amplitudes of polar motion it seem not to respond at all (no pole tide raised in the ocean) and only for larger and larger amplitudes its response is closer and closer to equilibrium. In other words, if the oceanic excitation function is not linear with respect to polar motion, but hyberbolic of the type

$$\psi_i = A\left(\sqrt{m_i^2 + B^2} - B\right), \quad i=1,\ 2 \qquad (4)$$

where ψ_i and m_i are excitation function and polar motion components in x and y coordinate, respectively, and A , B are certain constants, the observed effect can be fully explained by the response of the ocean.

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DISCUSSION

Sekiguchi: Dr. Naito of ILOM confirmed that there is no correlation between the amplitude of the pole tide and polar motion during the first half of the 20th century, including 1920–1940.

Reply by Vondrak: I have not read this article.

Lambeck: Can you reconcile your conclusion with the analysis by Okubo, 1982 (*Geophysical Journal*) in which he concludes that the damped oscillator model, driven by a non-stationary excitation mechanism, adequately describes the data?

Reply by Vondrak: I cannot imagine how any excitation could cause such a steady process as the Chandler wobble phase change between 1920–1940. I think it would be rather erratic.