# 15
# Polarization in deep-inelastic scattering

Suppose that the initial lepton beam is longitudinally polarized. If the target nucleon is unpolarized and unobserved, there is no effect on the cross section because parity is conserved in the strong and electromagnetic interactions. We consider parity violation induced by the weak interaction in the next chapter.

Suppose, however, that the target nucleon is polarized along the incident electron direction so that it is *also* longitudinally polarized in the infinite-momentum frame. There is then additional information on the strong-interaction spin structure of the nucleon in deep-inelastic scattering (DIS) experiments carried out under these conditions. Many such $\vec{\mathrm{N}}(\vec{\mathrm{e}}, \mathrm{e}')_{\mathrm{DIS}}$ experiments have now been performed, starting with the work of Vernon Hughes and collaborators at SLAC [Hu83]. A theoretical analysis of such experiments follows immediately from our discussions of the quark–parton model in chapter 14 and of the polarization of spin-1/2 fermions in appendix D.

In the extreme relativistic limit (ERL) one can simply insert the appropriate helicity projection operator for massless fermions in the lepton trace. For helicity $h = \pm 1$ one uses[1]

$$P_h = \frac{1}{2}(1 - h\gamma_5) \tag{15.1}$$

The result is that the lepton trace now takes the form

$$\begin{aligned}
\eta^h_{\mu\nu} &= -2\varepsilon_1\varepsilon_2 \frac{1}{2}\sum_{s_1}\sum_{s_2} \bar{u}(k_1)\gamma_\nu u(k_2)\bar{u}(k_2)\gamma_\mu(1 - h\gamma_5)u(k_1) \\
&= -\varepsilon_1\varepsilon_2 \text{ trace}\left[\gamma_\nu \left(\frac{-i\gamma_\lambda k_{2\lambda}}{2\varepsilon_2}\right)\gamma_\mu(1 - h\gamma_5)\left(\frac{-i\gamma_\rho k_{1\rho}}{2\varepsilon_1}\right)\right]
\end{aligned}$$

[1] Note that the $h$ used here is twice the spin projection.



https://doi.org/10.1017/9781009290616.018 Published online by Cambridge University Press

$$
\begin{aligned}
&= \frac{1}{4}4\left[k_{2\nu}k_{1\mu} + k_{1\nu}k_{2\mu} - (k_1 \cdot k_2)\delta_{\mu\nu} + h\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}\right] \\
&= \eta_{\mu\nu} + h\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}
\end{aligned} \tag{15.2}
$$

Here, as before

$$
\eta_{\mu\nu} = k_{2\nu}k_{1\mu} + k_{1\nu}k_{2\mu} - (k_1 \cdot k_2)\delta_{\mu\nu} \tag{15.3}
$$

Since the helicity is a pseudoscalar under the parity transformation (witness the $\gamma_5$ in the helicity projection operator), there can now be a pseudotensor contribution $h\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}$ to the response tensor.

Assume the target is longitudinally polarized and has helicity aligned ($\uparrow$) along **p**. Now carry out exactly the same impulse approximation calculation in the quark–parton model as in the previous section only now insert a projection operator for the relativistic *quarks* of helicity $h_i = \pm 1$

$$
P_{h_i} = \frac{1}{2}(1 - h_i\gamma_5) \tag{15.4}
$$

Evidently $h_i$ measures the helicity of the quarks relative to the helicity of the nucleon (here $\uparrow$). An inspection of the arguments leading to Eq. (14.34), and the above analysis, indicate that one should make the following replacement in Eq. (14.34)

$$
\begin{aligned}
&\left\{p'_\mu p_\nu + p'_\nu p_\mu - (p \cdot p')\delta_{\mu\nu}\right\} \to \\
&\qquad \left\{p'_\mu p_\nu + p'_\nu p_\mu - (p \cdot p')\delta_{\mu\nu} + h_i\,\varepsilon_{\mu\nu\lambda\rho}p'_\lambda p_\rho\right\}
\end{aligned} \tag{15.5}
$$

One then proceeds in exactly the same manner to Eq. (14.36) with the result

$$
W^{(i)}_{\mu\nu} \doteq \frac{Q_i^2}{2}\delta(\eta_i - x)\left[\delta_{\mu\nu} + \frac{2\eta_i}{m\nu}p_\mu p_\nu + \frac{h_i}{m\nu}\varepsilon_{\mu\nu\lambda\rho}p_\lambda q_\rho\right] \tag{15.6}
$$

An incoherent sum over all quarks implies that there is an additional Lorentz covariant contribution to the DIS response tensor for this nucleon with positive helicity of the form

$$
\delta W^{\uparrow}_{\mu\nu} = W^{\uparrow}\frac{1}{m^2}\varepsilon_{\mu\nu\lambda\rho}p_\lambda q_\rho \tag{15.7}
$$

The quark–parton model identifies

$$
\begin{aligned}
\frac{2\nu}{m}W^{\uparrow} &= \sum_i Q_i^2 h_i f_i(x) \\
&= \sum_i Q_i^2[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)]
\end{aligned} \tag{15.8}
$$

https://doi.org/10.1017/9781009290616.018 Published online by Cambridge University Press

Here an obvious notation has been introduced to denote the helicity of the quark relative to the helicity of the nucleon. The quark–parton model *predicts* that the combination on the left side of Eq. (15.8) obeys Bjorken scaling in DIS, and furthermore, that it measures the helicity distribution of the quarks inside the nucleon in the infinite momentum frame

$$\begin{aligned}
\frac{2\nu}{m}W^{\uparrow} &\rightarrow G_1(x) &&; \text{DIS} \\
G_1(x) &= \sum_i Q_i^2[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)] && \qquad (15.9)
\end{aligned}$$

It remains to investigate the physical consequences of the additional terms in the lepton and target response tensors in Eqs. (15.2) and (15.7) in the polarized case.

These results can be used to compute the asymmetry for scattering of the lepton by the target in the case when the helicities are both aligned or antialigned. Define this asymmetry by

$$\mathscr{A} \equiv \frac{d\sigma_{\uparrow\uparrow} - d\sigma_{\downarrow\uparrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\downarrow\uparrow}} \qquad (15.10)$$

The subscripts refer to the particle *helicities*, the convention here being that the first subscript is that of the electron and the second that of the nucleon. Parity invariance of the strong and electromagnetic interactions implies that $\mathscr{A}$ will be unchanged under a reversal of *both* helicities, as the reader can readily verify explicitly from the preceeding arguments.[2]

First note that when two tensors are contracted, they must both be even or odd in the interchange of the indices $\mu$ and $\nu$ to get a non-zero result. Then, since all common factors *cancel in the ratio*, the problem reduces to the evaluation of the following expression

$$\mathscr{A} = W^{\uparrow}\frac{1}{m^2}\frac{\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}\,\varepsilon_{\mu\nu\sigma\tau}p_\sigma q_\tau}{\eta_{\mu\nu}W_{\mu\nu}} \qquad (15.11)$$

The denominator is evaluated in Eq. (11.32)

$$\begin{aligned}
\eta_{\mu\nu}W_{\mu\nu} &= (k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - k_1\cdot k_2\,\delta_{\mu\nu})\left(W_1\delta_{\mu\nu} + W_2\frac{p_\mu p_\nu}{m^2}\right) \\
&= W_1(-2k_1\cdot k_2) + W_2\frac{1}{m^2}(2\,p\cdot k_1\,p\cdot k_2 - p^2\,k_1\cdot k_2) \\
&= W_1 q^2 + W_2\frac{1}{m^2}\left(2\,p\cdot k_1\,p\cdot k_2 - \frac{1}{2}m^2q^2\right) \qquad (15.12)
\end{aligned}$$

[2] Compare with Eq. (13.55).

https://doi.org/10.1017/9781009290616.018 Published online by Cambridge University Press

Recall $q = k_2 - k_1$ and $q^2 = -2k_1 \cdot k_2$ in the ERL. The numerator is evaluated using Eq. (D.18)

$$\begin{aligned} W^{\uparrow}\frac{1}{m^2}\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}\,\varepsilon_{\mu\nu\sigma\tau}p_\sigma q_\tau &= 2W^{\uparrow}\frac{1}{m^2}(k_2 \cdot p\, k_1 \cdot q - k_1 \cdot p\, k_2 \cdot q) \\ &= -W^{\uparrow}\frac{q^2}{m^2}\, p \cdot (k_1 + k_2) \end{aligned} \tag{15.13}$$

In the quark–parton model in DIS with $x = q^2/2m\nu$ one has from before

$$\begin{aligned} 2W_1 &= F_1(x) = \sum_i Q_i^2[f_i^{\uparrow}(x) + f_i^{\downarrow}(x)] \\ \frac{\nu}{m}W_2 &= F_2(x) = xF_1(x) \\ \frac{2\nu}{m}W^{\uparrow} &= G_1(x) = \sum_i Q_i^2[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)] \end{aligned} \tag{15.14}$$

Hence one can write the asymmetry $\mathscr{A}$ as

$$\begin{aligned} \mathscr{A} &\equiv \frac{N}{D} \\ N &= -G_1(x)\, m\,\nu\ [p \cdot (k_1 + k_2)] \\ D &= F_1(x)\left[m^2\nu^2 + \left(2\,p \cdot k_1\, p \cdot k_2 - \frac{1}{2}m^2q^2\right)\right] \end{aligned} \tag{15.15}$$

An equivalent expression is

$$\begin{aligned} \mathscr{A} &= \frac{G_1(x)}{F_1(x)}\mathscr{D} \\ \mathscr{D} &= -\frac{m\,\nu\ [p \cdot (k_1 + k_2)]}{[m^2\nu^2 + (2\,p \cdot k_1\, p \cdot k_2 - m^2q^2/2)]} \end{aligned} \tag{15.16}$$

These two expressions give the quark–parton result for $\mathscr{A}$ in DIS written in Lorentz invariant form. The first factor shows that what is being measured in these experiments is the ratio $G_1(x)/F_1(x)$. The second *depolarization* factor (of the virtual photon which must transmit the spin information) is purely kinematic.

In the laboratory frame where $p = (\mathbf{0}, im)$, one has in the ERL

$$\mathscr{D} = \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1^2 + \varepsilon_2^2 - 2\varepsilon_1\varepsilon_2 \sin^2\theta/2} \tag{15.17}$$

This reproduces the result in [Hu95].

If one retains correction terms of $O(m/\varepsilon_1)$, and correspondingly considers other directions of the polarization of the target, then the expression

https://doi.org/10.1017/9781009290616.018 Published online by Cambridge University Press

for the polarization asymmetry becomes more complicated, and one can, in fact, measure an additional spin structure function $G_2(x)$, whose interpretation in the quark–parton model is more ambiguous. The full response for arbitrary target polarization is given in [Vo92], where experimental results from the scattering of very-high-energy polarized muons from polarized nucleon targets is also discussed.