THE STUDY OF LIGHT CURVES WITH THE AID OF SMOOTHING SPLINE-FUNCTIONS

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Abstract. By means of smoothing spline functions an attempt is made to approximate cepheid-type light curves. As an example the light curve of XZ Cyg is used.

Given the times of observations $t_i \in [a, b]$, $i = 1, \ldots, n$, and the corresponding magnitudes m_i of a variable star, the analytic expression of the function $m(t)$ (which represents the light curve of the variable star) may be determined. Since the magnitudes m_i are affected by the observational errors, a reasonable compromise between the approximation and the observational data is given by

$$E(m) = \sum_{i=1}^{n} [m(t_i) - m_i]^2$$

and the smoothness of the solution by

$$L(m) = \int_a^b [m''(t)]^2 \, \mathrm{d}t,$$

where m'' denotes the second derivative of the function $m(t)$.

The solution of the problem will be given by the function $s(t)$ which minimizes the functional

$$\Phi(m) = L(m) + \varrho E(m)$$

in the Hilbert space of the real function $m(t)$, having an absolutely continuous first derivative and square integrable second derivative in the interval $[a, b]$. The coefficient ϱ is a positive constant and denotes the degree of compromise between the approximation and the smoothness of the solution.

In a more general case, Anselone and Laurent (1968) have demonstrated that the function $s(t)$, called the smoothing spline-function, exists and is unique. In our case, if $n > 2$, the function $s(t)$ is composed of segments of polynomials of degree 3 which at $t = t_i$, as well as their first derivatives, are equal.

The smoothing spline-function $s(t)$ is the solution of the differential equation

$$s''(t) = \sum_{j=1}^{n-2} \mu_j \varphi_j(t)$$

which satisfies the initial conditions

$$s(t_i) = m_i - \frac{1}{\varrho} \sum_{j=1}^{n-2} \mu_j b_j^i, \quad i = 1, \ldots, n$$

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where $\varphi_j(t)$ is the nucleus of the divided differences built on the knots t_j, t_{j+1}, t_{j+2}; $j=1,\ldots,n-2$ and b_j is the vector of the components, $b_j=(0.\ldots 0, b_j^j, b_j^{j+1}, b_j^{j+2}, 0, \ldots 0)$. The components b_j^j, b_j^{j+1}, b_j^{j+2}, are the coefficients of the corresponding divided difference. $\mu_j\,(j=1,\ldots,n-2)$ are the solutions of the linear algebric system

$$\sum_{j=1}^{n-2} \mu_j \left[\langle \varphi_i, \varphi_j \rangle_{\mathrm{H}} + \frac{1}{\varrho}\langle b_i, b_j \rangle_E\right] = \sum_{j=1}^{n} m_j b_i^j, \quad i = 1, \ldots, n-2$$

where

$$\langle \varphi_i, \varphi_j \rangle_{\mathrm{H}} = \int_a^b \varphi_i(t)\,\varphi_j(t)\,\mathrm{d}t$$

and

$$\langle b_i, b_j \rangle_E = \sum_{k=1}^{n} b_i^k b_j^k .$$

The actual numerical construction of the smoothing spline-function is very laborious and it is advisable to use it only with the aid of a computer. For this purpose we developed a FORTRAN programme for the FELIX C-256 computer.

For any set of photometric or spectrophotometric observations we can construct, according to the above described method (which will be called the method of the smoothing spline-functions) an analytic function which approximates the observations. This is very important for the determination and the study of the characteristics of the light and radial velocity curves.

With the method of the smoothing spline-functions we can determine:

(1) The analytic expression of the light and radial velocity curves.

(2) The maxima and minima of the light or radial velocity curves from the first derivative equal to zero condition.

(3) The amplitude of the light or radial velocity curve $A=s(t_{\max})-s(t_{\min})$.

(4) The asymmetries of the light curve and their variation during the Blažko effect.

(5) The study of the humps on the rising branches of the light curves of some pulsating stars with Blažko effect (SW Andromedae for instance), or the humps present in the light curve of some pulsating stars of RRc type.

(6) The slope of the rising branch of the light curve and its variation during the Blažko effect.

(7) The fundamental period and the beat period for the pulsating stars with Blažko effect.

(8) Other fine characteristics of the light or radial velocity curves.

As an example, we have considered the photoelectric observations of XZ Cygni made on 13 September 1973 at the Cluj Astronomical Observatory. A sequence of these observations and the smoothing spline-curves for $\varrho=10^6$, 10^8, 10^9, are given in Figure 1. The mean observational error of quantities Δv is $\varepsilon_0=0.01$ mag. The mean error of the smoothing spline-curve with respect to the observational data is

$$\varepsilon_c = \left[\sum_{i=1}^{n} (s(t_i) - m_i)^2/(n-3)\right]^{1/2},$$

where n is the number of observations. The most favorable value of ϱ is determined by the condition $\varepsilon_c = \varepsilon_0$. For our example this amounts to $\varrho = 10^8$. The moment of the maximum of the light curve, obtained from the first derivative equal to zero condition, is Max. hel. = J.D. 2441939.4503.

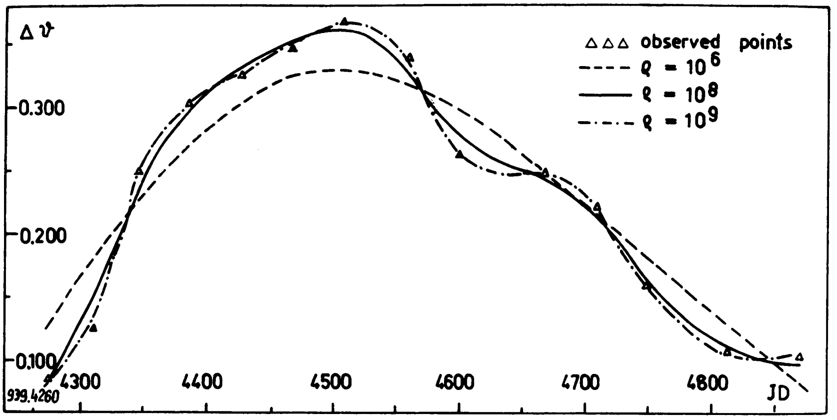

Fig. 1. Photoelectric observations of XZ Cyg made on 13 September 1973 and the smoothing spline-curves for $\varrho = 10^6$, 10^8 and 10^9.

Reference

Anselone, P. M. and Laurent, P. J.: 1968, *Num. Math.* **12**, 66–82.