J. Aust. Math. Soc. **102** (2017), 446–447
doi:10.1017/S1446788716000227

REPLY TO COMMENT

'POSITIVELY HOMOGENOUS LATTICE HOMOMORPHISMS BETWEEN RIESZ SPACES NEED NOT BE LINEAR'

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(Received 14 September 2015; accepted 13 April 2016; first published online 8 July 2016)

In [6], the principal aim is to generalize, under the same hypotheses, earlier results obtained by Mena and Roth [4], Thanh [5], Lochan and Strauss [3] and Ercan and Wickstead [2]. More precisely, it is shown that the assumption in the earlier papers on the uniform completeness is not necessary. Moreover, we give a response to [1].

Unfortunately, some typographical errors were introduced.

(1) The statement of Theorem 4, page 270, is incorrect as given. So, we replace in the statement of Theorem 4, page 270 the condition '$T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}_+$' by '$T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}$'.

No changes need to be made in the proof.

(2) The same thing for the statement of Corollary 6, page 270. So, we replace in the statement of Corollary 6, page 270 the condition '$T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}_+$' by '$T(\lambda e) = \lambda T(e)$ for each $\lambda \in \mathbb{R}$'.

This has no effect on its proof. We only just add in page 271, line 7 the following paragraph:

Since $\beta \in \mathbb{R}_+$, it follows that

$$\begin{aligned} -T(-\beta g) &= \sup((-T(-\beta g)) \wedge nT(e)) \\ &= \sup(-[(T(-\beta g)) \vee (-nT(e))]) \\ &= \sup(-[(T((-\beta g) \vee (-ne))]) \\ &= \sup(-[(T(-((\beta g) \wedge (ne)))]). \end{aligned}$$

Since $(\beta g) \wedge (ne) \in A_e$ and since T is linear on A_e, it follows that

$$\begin{aligned} -T(-\beta g) &= \sup(-[(T(-((\beta g) \wedge (ne)))]) \\ &= \sup(- - [(T(((\beta g) \wedge (ne)))]) \\ &= \sup([(T(((\beta g) \wedge (ne)))]) \\ &= T(\beta g). \end{aligned}$$

Consequently, $T(-\beta g) = -T(\beta g) = -\beta T(g)$.

(3) The same thing for the statement of Theorem 8, page 271. So, we replace in the statement of Theorem 8, page 271 the condition '$T(\lambda u_i) = \lambda T(u_i)$ for each $\lambda \in \mathbb{R}_+$' by '$T(\lambda u_i) = \lambda T(u_i)$ for each $\lambda \in \mathbb{R}$'.

This has no effect on its proof. We only just add in page 272, line 10 the following paragraph:

Since $\lambda \in \mathbb{R}_+$, it follows that

$$\begin{aligned}
-T(-\lambda x) &= \sup_H \Big(\sum_{i \in H} ((-T(-\lambda x_i)) \wedge nT(u_i)) \Big) \\
&= \sup_H \Big(\sum_{i \in H} (-(T(-\lambda x_i)) \vee (-nT(u_i))) \Big) \\
&= \sup_H \Big(\sum_{i \in H} -T((-\lambda x_i) \vee (-nu_i)) \Big) \\
&= \sup_H \Big(\sum_{i \in H} -T(-((\lambda x_i) \wedge (nu_i))) \Big).
\end{aligned}$$

Since $(-\lambda x_i) \wedge (nu_i) \in B_{u_i}$ and since T is linear on B_{u_i}, it follows that

$$\begin{aligned}
-T(-\lambda x) &= \sup_H \Big(\sum_{i \in H} - - T(((\lambda x_i) \wedge (nu_i))) \Big) \\
&= \sup_H \Big(\sum_{i \in H} T(((\lambda x_i) \wedge (nu_i))) \Big) \\
&= T(\lambda x).
\end{aligned}$$

Consequently, $T(-\lambda x) = -T(\lambda x) = -\lambda T(x)$.

To end this note, the author apologizes for any inconvenience caused.

References

[1] F. Ben Amor, 'Comment on "Positively homogenous lattice homomorphisms between Riesz spaces need not be linear"', *J. Aust. Math. Soc.* to appear.

[2] Z. Ercan and A. W. Wickstead, 'When a lattice homomorphism is a Riesz homomorphism', *Math. Nachr.* **279** (2006), 1024–1027.

[3] R. Lochan and D. Strauss, 'Lattice homomorphisms of spaces of continuous functions', *J. Lond. Math. Soc.* **25** (1982), 379–384.

[4] R. Mena and B. Roth, 'Homomorphisms of lattices of continuous functions', *Proc. Amer. Math. Soc. (2)* **71** (1978), 11–12.

[5] D. T. Thanh, 'A generalization of a theorem of R. Mena and R. Roth', *Ann. Univ. Sci. Budapest. Eötvös Sect. Math.* **34** (1992), 167–171.

[6] M. A. Toumi, 'When lattice homomorphisms of Archimedean vector lattices are Riesz homomorphisms', *J. Aust. Math. Soc.* **87** (2009), 263–273.

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