Cosmic Magnetic Fields:
From Planets, to Stars and Galaxies
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Low-dimensional models of stellar and galactic dynamos

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Abstract. A regular method how to simplify dynamo model for a particular celestial body up to a dynamical system is suggested. Dynamical system obtained occurs specific for a thin galactic disc, a fully convective star and a thin convective shell.

Keywords. Magnetic fields – convection – stars: low-mass

1. Introduction

Traditional dynamo models for magnetic field generation in various celestial bodies including Sun, stars, planets and galaxies presume a substantial knowledge concerning spatial distribution of magnetic field generators, i.e. differential rotation and so-called α-effect connected with mirror asymmetry of turbulence. Solar differential rotation is known quite in details due to impressive progress of helioseismolody, rotation curves of many galaxies come directly from observations however we still know some integral quantities for differential rotation of many stars and planets. Even less complete are our knowledge concerning α-effect which come from order of magnitude estimates and are supported by current helicity observations in solar active regions Kleeorin, Kuzanyan, Moss *et al.*, 1983). Because of that, it looks reasonable to simplify dynamo model for a celestial bodies with poorly known hydrodynamics up to dynamical system with coefficients depending on integral quantities representing hydrodynamical flow driving dynamo.

Dynamo models in form of a dynamical system was considered in several papers, e.g. Ruzmaikin (1981) suggested to reduce solar dynamo model to the Lorentz attractor. Kitiashvili & Kosovichev (2008) used a dynamo model in form of a dynamical system for a predictions of properties for the further solar activity cycle. A choice of a particular dynamical system in such papers is motivated by rather general similarity of terms in the full set of equations and that ones participating in the dynamical system so a more regular method to obtain low-dimensional dynamo models in form of dynamical systems looks desirable. Here we present such a method.

2. Constructing dynamical system

We follow here an old idea of Rädler & Wiedemann (1989) who suggested to present the desired solution as a combination of several first free decay modes. The number of the modes exploited is chosen as a minimal one which gives an adequate dynamo action (steady growth for a galactic disc and oscillatory one for spherical models) in a kinematic (linear) case. The desired solution of the dynamo system is a set of Fourier coefficient as functions of time. Coefficients of the system are calculated as various integrals which involves these free decay modes and rotation curve or spatial distribution of α. Rädler & Wiedemann (1989) stressed that the number of the free decay modes required for the dynamo self-excitation occurs to be unexpectedly large and the Fourier coefficients obtained are rather unstable. We suggest that this happens because quite a lot of first free decay modes do not contribute much into the dynamo action. We identify such useless modes taking the free decay modes one after the other from the set used to construct the solution. If the excitation occurs to be independent on the mode selected we exclude it from the set. The procedure of selection is described in details by Sokoloff & Nefyodov (2007).

Spectrum of free decay modes depends of geometrical shape of dynamo region. Correspondingly, dynamical system obtained depends on the geometry of dynamo region. In particular, dynamical system for a dynamo generation in thin galactic disc nearby a given radius (local disc dynamo problem) requires two modes only what determines a remarkable robustness of galactic dynamos known from experiences with more detailed galactic dynamo models. The low-dimensional model of galactic dynamo is presented by Moss, Shukurov & Sokoloff (1999). Dynamical system for a fully convective star (Sokoloff, Nefedov, Ermash, *et al.*, 2008) requires 5 free decay modes. We present below the dynamical system obtained for a star with a thin convective shell from classical Parker migratory dynamo model:

$$\frac{da_1}{dt} = \frac{R_\alpha b_1}{2} - a_1 - \xi^2 \frac{3R_\alpha b_1}{8}(b_1^2 + 2b_2^2)\,,$$

$$\frac{da_2}{dt} = \frac{R_\alpha}{2}(b_1 + b_2) - 9a_2 - \xi^2 \frac{3R_\alpha (b_1 + b_2)}{8}(b_1^2 + b_1 b_2 + b_2^2)$$

$$\frac{db_1}{dt} = \frac{R_\omega}{2}(a_1 - 3a_2) - 4b_1\,, \qquad \frac{db_2}{dt} = \frac{3R_\omega a_2}{2} - 16b_2\,.$$

Here b_1 and b_2 are Fourier amplitudes of first toriodal free decay modes, a_1 and a_2 are first poloidal modes. A simple algebraic α-quenching in form $\alpha = \alpha_0(1 - B^2/B_0^2)$ is supposed (B_0 is the equipartition magnetic field strength and toroidal magnetic field B is measured in units of B_0). Dimensionless numbers R_α and R_ω represent normalized intensities of α-effect and differential rotation correspondingly.

The dynamical system obtained is quite remote from naive expectation based on usual cartoon for Parker migratory dynamos which includes toroidal and poloidal magnetic fields represented by first free decay modes. Nevertheless, the model demonstrates a dynamo self-excitation in form of traveling wave of toroidal magnetic field and realistic nonlinear behaviour in form of nonlinear activity waves.

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