IBM 7090. The substitution involves the repeated use of two programs. The first run of the first program gives all second-order product series of the lunar co-ordinates and the left-hand sides of equations (1) - (3); subsequent runs give successively the product series of orders 3, 4, ... The other program computes the Ω terms from the lunar product series. We have used the programs to compute the LHS and the Ω_2 terms.

The new program gives:

(i) an independent check of the IBM 650 substitution;

(ii) a complete substitution in a few days;

(iii) greater over-all accuracy;

(iv) increased accuracy for arguments with small divisors.

The present coding aims at an accuracy of $1'' \times 10^{-5}$ for terms without small divisors, and for an omitted term to be as large as $0''\cdot001$ the period would be greater than 2000 years. If the solution to this precision appears to warrant higher precision the coding could be easily modified and the question would be a simple one of economics.

Among the more interesting parts of this work is the determination of the motions of the perigee and node. In view of the fact that the IBM 7090 substitution is almost ready we have decided not to prepare semi-definitive values of these quantities even though we feel that this could be done with a reasonable amount of work from the results at hand. Present indications are that the motion of the node given by Brown in the Memoirs (**1**) will be changed very little while the motion of the perigee may be increased by thirty seconds per century. Terms not included in the Memoirs and discussed later by Brown (**2**) account for about half the expected change.

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3. OUTLINE OF AN APPLICATION OF VON ZEIPEL'S METHOD TO THE LUNAR THEORY

Dirk Brouwer

The main problem of the lunar theory has been solved by many investigators by various methods. Yet the problem remains of sufficient interest to encourage further efforts. An investigation in progress at the Yale Observatory has as its principal aims: (*a*) evaluating the effectiveness of the von Zeipel method for obtaining the solution; (*b*) examining the causes of the slow convergence according to powers of n'/n in Delaunay's theory and, if possible, finding a remedy for this defect.

Applications of von Zeipel's method have been presented in several recent papers (**1, 2**); hence only a minimum of detailed explanation will be given here. The principle of the method is to eliminate the periodic terms in a single transformation or a sequence of transformations with the ultimate object of obtaining a final Hamiltonian that is a function of the variables corresponding to Delaunay's L, G, H.

In dealing with the lunar theory it may be advisable to start with the problem in which the inclination of the Moon's orbit and the eccentricity of the Sun's orbit are ignored, and in which the disturbing function is reduced to its principal part. The third limitation amounts to putting the ratio of a/a' equal to zero. The solution of the problem so simplified should yield those

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terms in the Moon's motion that are independent of Delaunay's $\gamma = \sin \frac{1}{2} I$, of e', and of a/a'. The equations may then be put in the form

$$\frac{dx_j}{dt} = \frac{\partial F}{\partial y_j}, \qquad \frac{dy_j}{dt} = -\frac{\partial F}{\partial x_j}, \quad j = 1, 2$$

with

$$x_1 = L = (\mu\, a)^{1/2}, \qquad y_1 = l,$$
$$x_2 = G = L\,(1 - e^2)^{1/2}, \quad y_2 = g + h - \lambda',$$

in which L, G, l, g, h are the well-known Delaunay variables, while

$$\lambda' = \nu t + \lambda_0'$$

is the Sun's mean longitude, ν and λ_0' being constants. The designation ν is used for the Sun's mean motion, usually designated by n'.

The Hamiltonian F is

$$F = F_0 + F_1.$$

$$F_0 = \frac{\mu^2}{2x_1^2} + \nu x_2, \qquad F_1 = \frac{1}{4}\frac{\nu^2}{n^2}\frac{\mu^2}{x_1^2}\left\{\frac{r^2}{a^2} + \frac{3r^2}{a^2}\cos(2f + 2y_2)\right\},$$

in which f is the true anomaly and r the radius vector. Throughout the symbol n will stand for μ^2/x_1^3.

F_1 can be expressed explicitly in terms of y_1, y_2 by expansions in elliptic motion,

$$F_1 = F_{1a} + F_{1b},$$

$$F_{1a} = \frac{1}{4}\frac{\nu^2}{n^2}\frac{\mu^2}{x_1^2}\left\{1 + \frac{3}{2}e^2 + \frac{15}{2}e^2\cos 2y_2\right\},$$

$$F_{1b} = \frac{1}{4}\frac{\nu^2}{n^2}\frac{\mu^2}{x_1^2}\left\{\sum_1^{\infty} C_j \cos jy_1 + \sum_{-\infty}^{\infty}{}' D_j \cos(jy_1 + 2y_2)\right\}.$$

The symbol Σ' indicates that the term with $j = 0$ is to be omitted; this term is included in F_{1a}.

It would be possible to eliminate in a single von Zeipel transformation all the periodic terms having y_1 and y_2 in the arguments. However, terms with y_2 only in the argument will produce a development in powers of ν/n, while terms having y_1 in the argument lead to a development in powers of ν^2/n^2. For this reason it appears desirable to proceed in two steps. In the first transformation only terms with y_1 in the arguments will be included in the determining function S.

S will thus be developed in powers of ν^2/n^2,

$$S = S_0 + S_1 + S_2 + \dots$$
$$S_0 = x_1' y_1 + x_2' y_2.$$

In the following the primes with x_1, x_2 may be omitted, provided it is understood that in all developments the x-variables are to be considered primed. The expression for S_1 is

$$S_1 = \frac{1}{4}\frac{\nu^2}{n^2}\frac{\mu^2}{x_1^2}\left\{\Sigma \frac{C_j}{jn}\sin jy_1 + \Sigma' \frac{D_j}{jn}\left(1 - \frac{2\nu}{jn}\right)^{-1}\sin(jy_1 + 2y_2)\right\}$$

In Delaunay's theory the factors $(1 - 2\nu/jn)^{-1}$ are developed in powers of ν/n by binomial expansions. In order to examine the possibility that this feature is the primary cause of the

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slow convergence in powers of ν/n, it appears desirable to avoid the binomial expansions, even at the cost of considerable complication of the development.

The part S_2 of S is subsequently obtained from the equation

$$n\frac{\partial S_2}{\partial y_1} - \nu\frac{\partial S_2}{\partial y_2} = \left\{\frac{3}{2}\frac{n}{x_1}\left(\frac{\partial S_1}{\partial y_1}\right)^2 + \frac{\partial F_1}{\partial x_1}\frac{\partial S_1}{\partial y_1} + \frac{\partial F_1}{\partial x_2}\frac{\partial S_1}{\partial y_2} - \frac{\partial F_1^*}{\partial y_2}\frac{\partial S_1}{\partial x_2}\right\}_{\mathrm{b}}$$

the subscript b indicating the part having y_1 in the arguments. The first part in the right-hand member contributes terms with new arguments of the form $jy_1 + 4y_2$ and with factors

$$(1 - 2\nu/jn)^{-1}(1 - 2\nu/kn)^{-1}$$

For $j \neq k$ it is easily seen that this product may be written as a sum

$$\frac{(1 - 2\nu/jn)^{-1}}{1 - j/k} + \frac{(1 - 2\nu/kn)^{-1}}{1 - k/j}$$

with

$$\tfrac{1}{2}(1 - 2\nu/jn)^{-1} + \tfrac{1}{2}(1 + 2\nu/jn)^{-1}$$

as special case for $k = -j$. For $j = k$ no such reduction is available.

In S_2, therefore, factors of this form as well as factors of the form

$$(1 - 2\nu/jn)^{-1}(1 - 2\nu/kn)^{-1}(1 - 4\nu/pn)^{-1}$$

will appear.

Obviously, the complexity of these factors increases with each order. On the IBM 650 and IBM 7090 the calculation of S_2 has been completed with the assistance of Mrs Lois Frampton and Dr G. Hori. The calculations for the higher approximations are in progress.

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4. PRESENT AND FUTURE REQUIREMENTS FOR PLANETARY EPHEMERIDES

R. L. Duncombe

The traditional uses of the planetary ephemerides have been twofold:

(*a*) to predict the places of the planets to facilitate their observation; and
(*b*) to form a standard against which to compare observations.

The long series of published national ephemerides have fulfilled these traditional requirements.

The introduction in astronomy of automatic computing techniques, and more recently of electronic calculators, has provided impetus in the evaluation of planetary co-ordinates over extended periods of time, the analysis of deficiencies in existing planetary theories, and the construction of new theories.

Examples are: the evaluation of Newcomb's tables of Venus, and of the Sun, from 1800–2000; the comparison of observations of Mercury and Venus over the past 200 years with Newcomb's tables of their motion; the simultaneous numerical integration of the equations of motion of the 5 outer planets, from 1653–2060; the comparison of Newcomb's theory of

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