Testing crevasse-depth models: a field study at Breiðamerkurjökull, Iceland

Ruth H. MOTTRAM,[1*] Douglas I. BENN[1,2]

[1]*School of Geography and Geosciences, University of St Andrews, St Andrews KY16 8ST, UK*
E-mail: rum@dmi.dk
[2]*The University Centre in Svalbard (UNIS), Box 156, NO-9171 Longyearbyen, Norway*

ABSTRACT. Interest in crevasses and associated ice-fracture processes has recently increased due to recognition of the importance of calving glaciers to the mass balance of the cryosphere, as well as the importance of fractures in glacier hydrology. Recently developed calving criteria make use of models which predict crevasse depth from surface strain rates, but these models have rarely been tested against observations. In this study, we present data on crevasse depth and surface strain rates, and compare the measured values with results of two crevasse-depth models: a simple function proposed by Nye and a linear elastic fracture mechanics (LEFM) model developed by Van der Veen. Our results indicate that both models predict crevasse depths within the correct order of magnitude. The LEFM model, incorporating measured values of crevasse spacing and tuned for fracture toughness, performs better in predicting crevasse depths, but where lack of input data precludes such tuning, the results are similar to Nye's model predictions. We conclude that both models may be used to calculate crevasse depths in calving models, although the Nye function is undoubtedly much simpler to implement within an ice-dynamics model.

INTRODUCTION

Crevasses are among the most distinctive and ubiquitous surface features of glaciers. They can be used as markers to measure ice velocity, and as indicators of basal topography and surface strain, and receive much attention from mountaineers and other glacier travellers because of the difficulties and dangers they pose. Crevasses can also be important meltwater pathways within glaciers (Fountain and others, 2005; Van der Veen, 2007; Das and others, 2008; Benn and others, 2009), potentially leading to temporarily accelerated flow through lubrication of the bed by meltwater (Zwally and others, 2002; Joughin and others, 2008; Van de Wal and others, 2008). However, crevasses are also among the least studied and understood of glacial phenomena, and they are rarely incorporated into glacier models. Recent renewed interest in the dynamics of calving glaciers has provided additional motivation for developing a detailed understanding of crevasse properties (Benn and others, 2007a,b).

Calving losses are important to the total mass balance of the cryosphere, but are currently poorly represented in ice-sheet models (Solomon and others, 2007, p. 17). Most models adopt either empirically derived functions relating calving rates to water depth, or height-above-buoyancy criteria which 'cut off' the glacier terminus if it approaches flotation (e.g. Vieli and others, 2002; Zweck and Huybrechts, 2003; Nick and Oerlemans, 2006). While these approaches allow models to exhibit some of the observed dynamic behaviour of calving glaciers, both have shortcomings that limit their usefulness as general 'calving laws'. Benn and others (2007a,b) proposed a new, physically based model in which calving is assumed to occur where surface crevasses penetrate the full thickness of the glacier. Water-filled surface crevasses can propagate downward without limit (Van der Veen, 1998a), so full-depth fracturing may occur if crevasses extend below sea or lake level (assuming a free connection exists between the crevasse field and the proglacial water body). The position of a calving margin can therefore be defined as the point at which crevasse depth equals the elevation of the glacier above water level, as crevasses of greater depth are assumed to propagate to the bed. Calving glaciers typically accelerate towards the terminus in response to reductions in basal or lateral drag and/or longitudinal stress gradients. Longitudinal stretching opens crevasses, which in turn promotes calving, so the model can explain how glacier dynamics can exert a physical control on calving-front position and its evolution through time.

Application of this and other similar models depends upon the ability to successfully predict crevasse depth from longitudinal strain rate or stress, and other variables. A number of methods for calculating crevasse depth have been proposed in the literature (e.g. Nye, 1955, 1957; Weertman, 1973; Smith, 1976; Van der Veen, 1998a,b), although few observations are available to allow testing of their predictions. Indeed, there have been only limited field studies of crevasses on glaciers (e.g. Meier, 1958; Ambach, 1968; Holdsworth, 1969), partly due to accessibility and safety issues. A good overview of the literature on crevassing is provided by Van der Veen (1999a), but surprisingly few measurements of crevasse geometry, including depths, have been published. In this paper, we present concurrent measurements of crevasse depth and surface strain rates made on Breiðamerkurjökull, Iceland. The results are then compared with the predictions of two alternative crevasse-depth models, for the case of closely spaced crevasses.

METHODS AND FIELD OBSERVATIONS

The field data were collected on and adjacent to the rapidly flowing central part of Breiðamerkurjökull, a southern outlet glacier from the Vatnajökull ice cap in Iceland (Fig. 1). This

*Present address: Danish Meteorological Institute, Lyngbyvej 100, DK-2100 Copenhagen, Denmark

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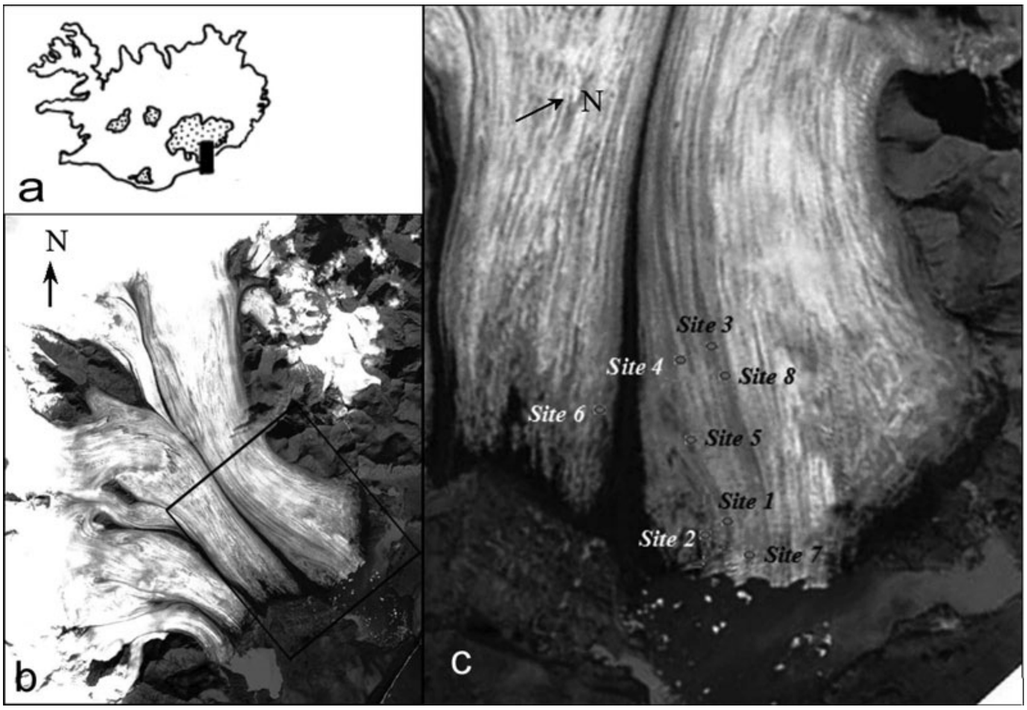

Fig. 1. Location of the field site. (a) Breiðamerkurjökull in Iceland. (b) An ASTER satellite image (NASA/Earth Observing System (EOS)) taken on 28 August 2004; the boxed area is magnified in (c). (c) The area immediately behind the calving terminus of Breiðamerkurjökull. The locations of the eight field sites are marked; the site numbers refer to the order in which they were set up. Sites 7 and 8 were surveyed in 2004; the other six were set up in 2005. The obvious black streaks on the satellite images are surface medial moraines and extensive tephra deposits.

glacier was primarily chosen for its ease of access and the wide range of data available for the glacier including ice thickness and bed topography (Björnsson and others, 1992, 2001; Björnsson 1996; Evans and Twigg, 2002). Fieldwork was carried out during two summer field seasons in 2004 and 2005. In total, eight field sites were selected for detailed observation, reflecting a variety of different crevasse patterns and strain regimes.

At each field site, surface strain rates were determined from repeat surveys of a network of marker stakes and converted to stress using the flow law for ice. The depths, lengths and orientations of crevasses within the network were also measured. Strain rates across crevasses were determined by repeat measurement of the positions of pairs of stakes within each network, using a Leica 1200 laser theodolite. Logarithmic strain rates were calculated following the method of Nye (1959b) and Hooke (2005):

$$\dot{\varepsilon} = \frac{1}{\Delta t} \ln \frac{l_2}{l_1}, \tag{1}$$

where l_1 and l_2 are the initial and final distances between stakes over the time interval, Δt.

To measure crevasse depths, a plumb-line system was used, comprising a 50 m length of strong twine attached to a small weight. This was lowered down the crevasse until the line went slack and was assumed to have reached the bottom; in most cases, this could be verified visually. The chief advantage of this technique is the simplicity of the set-up. The main problem is that the bottom was not always visible and, especially for the deeper crevasses, it was not always easy to determine with certainty that the weight had reached the bottom. There was some evidence of shallow water in some of the crevasses and this was another complicating factor in assessing depths. Crevasse depth was measured from the lower of the two sides of the crevasse. All measurements were made at least three times or until consistent depth values were achieved, and measurement error was $\pm$0.5 m. However, it is likely that some or all of the crevasses penetrated somewhat deeper than the measured depths since crevasses usually become too narrow to admit the weight at depth.

Figure 2 shows a frequency distribution of measured crevasse depths. Most of the crevasses were 5–6 m deep, although crevasses less than 2 m and up to 20 m in depth were also measured. Difficulties with gaining access to the most highly crevassed areas of the glacier make it likely that the depths of the sampled crevasses were at the lower end of the full range of crevasse depths found at Breiðamerkurjökull.

Out of the total of 44 crevasses studied in detail, the calculated strain rates across 19 were negative, indicating that the crevasse was closing due to compressive stress, and these were necessarily excluded from model experiments. In these cases, the crevasses appear to be relict features no longer in equilibrium with prevailing stresses, although it should be noted that all such crevasses were close to other fractures with a measured tensile stress, suggesting that the small-scale stress tensor is complex and the fracture response heterogeneous. As a finite amount of time is required to close a crevasse when strain rates become compressive, the crevasse patterns and depths within a field retain a memory of previous stresses, which means that deep crevasses can be measured in areas of negative strain rate. This effect, where crevasses occur in some places with very low strain rate and not in others with very high strain rate, is also reported in the literature (e.g. Nye, 1959a; Hambrey and Müller, 1978). These observations contradict the assumptions of equilibrium crevasse-depth models, and should be borne in mind when interpreting the results.

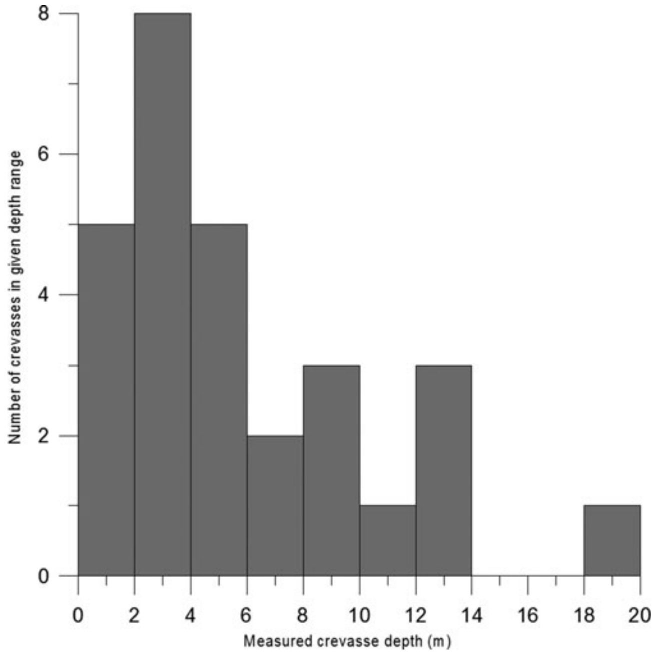

Fig. 2. Frequency distribution of crevasse depths measured at all field sites during fieldwork. Note that due to access difficulties, it is likely that shallower crevasses are over-represented in this analysis.

CREVASSE-DEPTH MODELS

The field data allow us to compare measured crevasse depths directly with depths predicted by theory, Two models were tested in this study: (1) a 'zero stress' model (Nye, 1955, 1957) and (2) a linear elastic fracture mechanics (LEFM) model (Van der Veen, 1998a). Values for the input parameters, such as the flow-law rate factors and yield stresses, were determined experimentally or chosen from the literature (Table 1).

Nye crevasse model

Nye (1955, 1957) proposed a simple crevasse-depth model based on the balance between the longitudinal tensile strain rate and creep closure due to ice overburden pressure:

$$d = \frac{2}{\rho_{\mathrm{i}} g}\left(\frac{\dot{\varepsilon}_{xx}}{A}\right)^{\frac{1}{n}}, \tag{2}$$

where d is crevasse depth, $\dot{\varepsilon}_{xx}$ is the longitudinal strain rate, A and n are the flow-law parameters, ρ_{i} is ice density and g is gravitational acceleration. Note that this version is given by Paterson (1994, p. 187), based on a more complete analysis of stresses by Nye (1957).

The Nye model is most appropriate for a field of closely spaced crevasses, where the stress concentration at the bottom of each fracture is blunted by the presence of other fractures nearby. Benn and others (2007a) modified the Nye crevasse-depth model to include a yield criterion for ice:

$$d = \frac{2}{\rho_{\mathrm{i}} g}\left(\frac{\dot{\varepsilon}_{*}}{A}\right)^{\frac{1}{n}}, \tag{3}$$

where $\dot{\varepsilon}_* = \dot{\varepsilon}_{xx}$ minus a 'yield strain rate', $\dot{\varepsilon}_{\mathrm{crit}}$. (Benn and others (2007a) also incorporated the effect of water depth in the Nye crevasse model, but this is not included here.) The idea of a yield strain rate is introduced as a heuristic device to fulfil the role of a critical-stress intensity factor required to overcome the fracture toughness of the ice. The inclusion of $\dot{\varepsilon}_{\mathrm{crit}}$ allows the Nye model to be tuned, to allow for the observation that crevasses only form when the applied stresses exceed some (variable) value (Vaughan, 1993; Van der Veen, 1999a). In this study, we experimented with alternative values of the yield strain rate to optimize the fit between observed and predicted crevasse depths.

LEFM model

LEFM was first applied to crevasse formation on glaciers by Smith (1976). More recently, Van der Veen (1998a,b, 1999a) developed a LEFM model to calculate crevasse depth, orientation and length. Although there are three linear fracture modes (simple opening, sliding and tearing fractures), the model used here only considers mode I, simple opening fractures. LEFM models account for the stress concentration at the tip of a crack, the stress intensity factor, K:

$$K = \beta\sigma\sqrt{\pi d}. \tag{4}$$

The magnitude of K is determined by the crack length d, the applied stress σ, and β, a dimensionless factor dependent on the geometry of the system (e.g. ratio of depth to ice thickness or fracture spacing). The equilibrium fracture depth for a given stress is defined as the point where the stress intensity factor is equal to the value of the fracture toughness of the material ($K = K_{\mathrm{IC}}$), so the chosen value for fracture toughness (K_{IC}) will affect the predicted depth. Compared with other parameters in the model, however, the effect on absolute depth is weak. The model also accounts for the effects of the following on crevasse depth: hydrostatic closure, fracture spacing in a field of crevasses and the variation in the density

Table 1. List of input parameters in crevasse-depth models. A number of sensitivity tests were run to determine the values of these parameters. The range of these experiments is shown in the third column. The values used to generate the results presented here are shown in the fourth column. In some cases, these were field measured values

Input variable	Model	Range of values tested	Values used in models
Strain rate, $\dot{\varepsilon}_{xx}$ (s^{-1})	Both	–	Measured from stake networks
Ice density, ρ_{i} ($kg\,m^{-3}$)	Both	600, 700, 800, 900	917 (Paterson, 1994)
Fracture spacing, W (m)	Van der Veen	10, 30, 50, 100	Measured in field; 30 m used in generalized model
Flow-law rate factor, A ($kPa^{-3}\,a^{-1}$)	Both	$(0.15–0.4) \times 10^{-6}$	3.484×10^{-7} (Paterson, 1994)
Flow-law rate factor, n	Both	–	3
Fracture toughness, K_{IC} ($kPa\,m^{-1/2}$)	Van der Veen	10, 30, 50, 100	30
Yield stress (kPa) (converted to yield strain rate in model)	Nye	10, 30, 50, 60, 100	60

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Table 2. A statistical comparison of the predicted depths with the measured depths, using the different forms of the two models. The second column shows the Pearson's correlation coefficient, and the third column the significance level of the correlation for each model in a one-tailed test, where *n*, the number of pairs of measured and modelled depths in each correlation, is 28 (Shaw and Wheeler, 1994). The last column gives the coefficient of determination, which compares the pattern of variability in the two models

Model	Correlation coefficient,**R*	Significance level ($n = 28$)	Coefficient of determination, R^2
Nye model	0.360	0.971	0.1297
Nye yield model	0.397	0.982	0.1575
LEFM model	0.448	0.992	0.2018
Generalized LEFM model	0.386	0.979	0.1487

*Pearson product-moment correlation coefficient.

of ice with increasing depth from the surface. A further term can be added to account for the presence of water within a crevasse, but this is not examined further here. The full details of the model are given by Van der Veen (1998a).

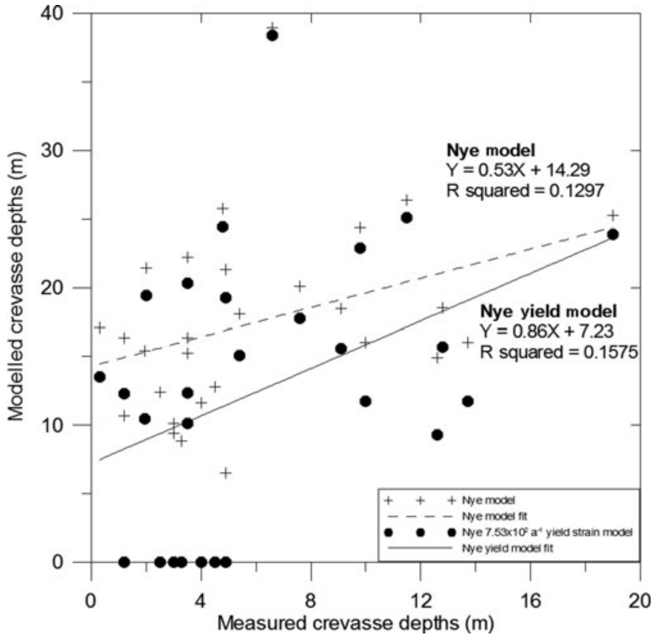

Fig. 3. Measured crevasse depths compared with depth calculated from the Nye model, using a yield strain rate of 7.53×10^2, equivalent to a yield stress of 60 kPa, and with a yield stress of zero.

COMPARISON BETWEEN MEASUREMENTS AND MODEL RESULTS

Figure 3 compares measured crevasse depths with values calculated from the Nye model using $\dot{\varepsilon}_{\text{crit}} = 0$ and $\dot{\varepsilon}_{\text{crit}} = 7.53 \times 10^{-2}\,\text{a}^{-1}$. The critical strain rate was determined experimentally and optimizes the fit between observations and model results. It is equivalent to a stress of 60 kPa, assuming a flow-law parameter $A = 3.48 \times 10^{-16}$ $\text{kPa}^{-3}\,\text{s}^{-1}$ (Paterson, 1994, p.97). The zero-yield model explains almost 13% of the variance in the data, with a significance level of 97% (Table 2). With the addition of the yield criterion, the Nye model explains an additional 2.8% of the variance in the data, and the significance level increases to 98%.

The measured crevasse depths are compared to the output of the LEFM model in Figure 4. The LEFM model performs best when the measured crevasse spacing at each field site is used as input. In this case, the r^2 value is 0.202, significant at the 99% level. However, detailed information regarding crevasse spacing and ice thickness is often not available, so experiments were carried out in which a single fracture spacing was specified. These experiments show that the accuracy of depth predictions is significantly reduced when average crevasse spacings are used, and the LEFM model performs no better than the simpler Nye model (Table 2).

DISCUSSION

Crevasse depths

Both the LEFM and Nye models tend to overestimate crevasse depths (Fig. 5). Given the difficulties of measuring crevasse depths, this overestimate may be due to a systematic under-measurement of crevasse depth rather than deficiencies in the models. As noted above, the measurements presented here are minimum depths because the plumb-bob was prevented from entering the deepest, narrowest parts of crevasses. Observations made from within crevasses suggest that very narrow cracks may well extend for at least tens of centimetres beyond the measured 'bottom' of the crevasse, and micro-cracks probably extend even further. Van der Veen (1998a) also suggests that the cracks may penetrate to deeper levels but without the fracture surfaces separating due to the overburden pressure at depth. Hence, the predicted crevasse depths in both the Nye and generalized LEFM models may actually be closer to the 'true' depth than the measured values suggest.

The largest uncertainty in both models is the difficulty in determining the flow-law rate factor *A*, which is a variable in the Nye model and is used to convert measured strain rates into stresses for input to the LEFM model. Very small differences in the chosen value can lead to dramatically different predicted crevasse depths. This is particularly problematic in temperate ice where the rate factor is not very well defined.

One of the most important outcomes of this study is that at least some of the crevasses are clearly not in equilibrium with the prevailing surface strain rates. As noted above, the local strain rate was compressive across 43% of the crevasses studied. Although these crevasses were excluded from the analysis, it appears likely that the remaining crevasses may also have been out of equilibrium with the measured strain rates to some degree. In all cases where compressive strain rates were measured across crevasses, they occurred within regions of dominantly tensile strain rates. Indeed, the strain patterns at all sites were remarkably inhomogeneous (Mottram, 2007). Local strain rates, therefore, may not be representative, and it may be more appropriate to use crevasse depths and strain rates averaged over a larger area when comparing observations with model predictions.

Given that a fundamental assumption of both models – that crevasse depths are in equilibrium with surface strain rates in the surrounding ice – is shown to be questionable, it is surprising that they perform as well as they do. It may be speculated that crevasses take most time to adjust their

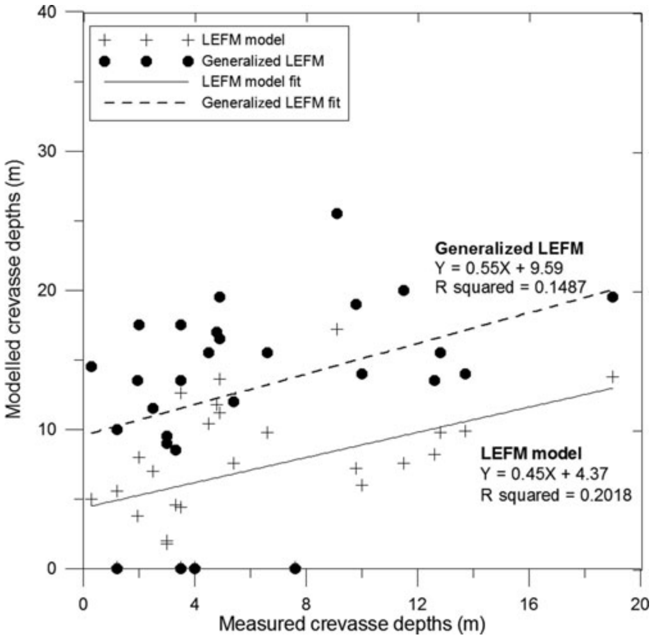

Fig. 4. LEFM model depths for precise values and generalized values, compared with measured crevasse depths.

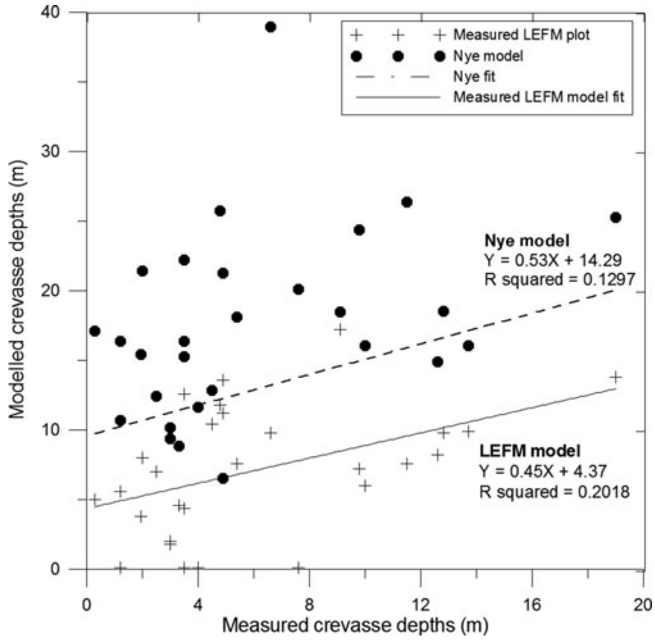

Fig. 5. Comparison of the Nye and LEFM models predicted depths with measured crevasse depths.

depths when advected into areas of compressive (or less extensional) stresses, but that their depth adjusts rapidly when stresses become increasingly tensile down-glacier. Further field data are required to test this idea.

Hydrostatic closure

In this study, it is assumed that bulk ice density is constant. However, the presence of crevasses reduces the bulk density of the ice, which in turn will reduce the effects of hydrostatic pressure on crevasse depth. Boundary element analysis by Sassolas and others (1996) indicates that this effect may extend over a horizontal distance up to six times the depth of each crevasse. The large number of voids reduces the rate of hydrostatic closure near the surface, and consequently crevasses in highly fractured glaciers are unlikely to close completely once opened, even under stress regimes that would ordinarily lead to closure. This effect is enhanced by ablation of the crevasse walls, which can be very rapid due to the effects of reflected radiation (Pfeffer and Bretherton, 1987). In glacier ablation zones, wall melting may be the main mechanism for maintaining open crevasses in areas where stresses have become compressive.

Fracture toughness and yield strength

Both the LEFM model and the modified Nye model (Equation (3)) require yield criteria to be defined, although changes in fracture toughness only weakly alter predicted crevasse depths. The results of experiments to determine a fracture criterion for ice in the laboratory are summarized by Petrovic (2003). Unfortunately, few of these studies have used glacier ice, and those that did (Fischer and others, 1995; Rist and others, 1996, 1999) used ice from deep cold Antarctic ice cores with limited applicability to other glacial areas. Vaughan's (1993) survey and reanalysis of field-measured strain rates concluded that realistic values for a fracture criterion lie between 90 and 320 kPa, when converted from strain rate to stress using the flow law. A numerical modelling study by Van der Veen (1998a) estimated that a stress of 30–80 kPa was needed for a single crevasse to form, with larger tensile stresses required for a field to open, depending on crevasse spacing. The reasons for the large variability in estimates are likely to include differences in temperature and spacing of crevasses in fields as well as systematic measurement biases.

The results presented here suggest that on temperate glaciers at least, the fracture toughness may be substantially smaller than previously published values. The best match in the LEFM model between predicted and measured depth used the lowest published fracture toughness value of 30 kPa $m^{-1/2}$. As fracture toughness varies substantially with temperature, much more work is necessary to define fracture criteria at a range of temperatures. Other factors (e.g. debris content, strain history) also influence fracture toughness, and in the case of Breiðamerkurjökull, it is likely that large quantities of tephra at the surface and entrained within the glacier also change the fracture toughness.

Given all these uncertainties, and the observed problems with the assumption of equilibrium in the models, it is clear that crevassing involves a complex suite of processes requiring a range of approaches, including field or remote-sensing measurements, to resolve them. Clearly much more work needs to be done to develop improved models of crevassing.

SUMMARY AND CONCLUSIONS

This paper presents the first detailed dataset on concurrent crevasse depths and surface strain rates. Measuring crevasse depths accurately is difficult, dangerous and time-consuming, and it is likely that the reported crevasse depths systematically underestimate the true depths.

For 19 of the 44 crevasses investigated, local surface strain rates across the crevasse were compressive, showing that crevasse depth is not in equilibrium with strain rate. This may be true of all the studied crevasses to a greater or lesser degree. Possible reasons for disequilibrium include: the time required for crevasses to adjust after being advected from one stress regime to another; the widening of crevasses by melting; and the effect of crevasses on ice bulk density and hydrostatic pressures.

Surface strain rates, and crevasse depth and spacing, were found to vary across each field site, even over small horizontal distances. This inhomogeneity within crevasse fields may also explain some of the observed mismatch between crevasse depths and surface strain rates.

Despite their simplifying assumptions, the Nye and LEFM crevasse-depth functions both perform remarkably well at predicting crevasse depths. In all cases, the correlation between modelled and observed crevasse depths was significant at the 97% level or better.

The LEFM model performed best when observed crevasse spacing was used as model input. However, in recognition of the fact that crevasse spacing cannot be specified in numerical ice-sheet models, a generalized form of the LEFM model was used, employing an average value for crevasse spacing. As expected, this model performed less well than the full model.

Two versions of the Nye crevasse-depth model were tested, using 'yield strain rates' of zero and $7.35 \times 10^{-2}\,a^{-1}$, equivalent to a yield stress of 60 kPa. The yield stress version of the model performed as well as the generalized form of the LEFM model. In most cases, the predicted crevasse depth is within a factor of two of the observed value. Both models perform sufficiently well to serve as crevasse-depth functions in calving models.

When detailed information characterizing an area is available, the LEFM model is the best choice. However, where crevasse spacing cannot be specified (i.e. in numerical ice-sheet models) the LEFM model performs no better than the Nye model.

The Nye model is therefore currently the best compromise between ease of use, availability of input data and accuracy of results.

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