Adaptation of Docobo's Method for the Calculation of Orbits of Spectroscopic-Interferometric Binaries with Mixed Data

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ABSTRACT: Based on Docobo's method for the calculation of orbits of visual binaries, an algorithm is presented for the calculation of short-period orbits when observational data comprise radial velocities and polar coordinates. The method is applied to some examples.

1. INTRODUCTION

The analytical method of Docobo (1985) for the calculation of orbits of visual double stars forms the basis of an algorithm to determine double star orbits when the data is either in the form (θ, ρ) or in the form of radial velocities (V). It is useful to recall that Docobo's method (program DOCO), applied in the usual way, calculates the orbit using three base points $(\theta, \rho; t)$, the required orbit being selected from the set of those generated which best fits certain control points (e.g., observations of greater weight) or even the remainder of the available observations.

In the present contribution we adapt this method to calculate orbits of binaries with very short periods and when the observational data available are mixed.

2. SEQUENCE OF CALCULATION

2.1. Determination of the period

We shall consider a binary with measurements $(\theta, \rho; t)$ and (V; t) corresponding to different revolutions. Let P_0 be an approximate value of the orbital period. The first step is to improve the value of P_0 by employing the procedure of Bopp $et\ al.\ (1970)$ using the (V;t) observations, although a similar method can also be used with just the observed position angles.

The latter consists of trying values of P close to P_0 and once all the position angles are ordered in one revolution, selecting that which minimizes the expression $\frac{S}{360^{\circ}}$, where

$$S = |\theta_1 - \theta_2| + |\theta_2 - \theta_3| + \ldots + |\theta_{N-1} - \theta_N| + 360^{\circ} - |\theta_N - \theta_1|$$

It is clearly the case that, in the absence of observational errors, the correct period would be 360°.

Improving the initial value of the period should be effected using the type of observation (radial velocities or position angles) which is available for the most revolutions.

2.2. Determination of a visual orbit

Having determined the improved value of the period P, three base points are chosen from the observations $(\theta, \rho; t)$, if possible, although it is not a prerequisite, such that the position of the principal star is a point inside the triangle whose vertices are the three positions of the secondary star.

The program DOCO generates a set of relative orbits whose apparent orbits pass through the chosen three normal points. Of these, the one selected is that whose orbital period coincides with the value previously calculated. In this way we have an orbit defined by the elements P, T, e, a'', i, Ω and ω , which can be improved using any standard method. If after this the differences betwen observation and calculation (O-C) are not satisfactory, the base points should be changed and the process repeated.

2.3. Determination of a spectroscopic orbit

Approximate values of $K = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}$ and V_0 are always possible to obtain using the radial velocity measurements; for example, as initial values we might take

$$K = \frac{V_{MAX} - V_{MIN}}{2}, \qquad V_0 = \frac{V_{MAX} + V_{MIN}}{2}.$$

Alternatively, if the spectroscopic binary is double lined, the value of V_0 can be chosen as that for which the velocities in both orbits coincide.

Using the values of P, T, e, and ω obtained from the visual orbit and the initial values of

$$a\sin i = \frac{KP\sqrt{1-e^2}}{2\pi},$$

the radial velocities

$$V = V_0 + (a \sin i) \frac{2\pi}{P\sqrt{1 - e^2}} [\cos \omega (e + \cos f) - \sin \omega \sin f]$$

are compared with the observed radial velocities. A standard method may be used to obtain new values of V_0 and $a \sin i$ which minimize the squares of the O-C defferences in the radial velocities. Should these not be satisfactory, it is possible at the same time to improve the element T or even ω .

2.4. Final determination orbit

Should any of the elements of the visual orbit have been modified in the previous section it will be necessary to recalculate the O-C differences. Improvement of these differences, if it is needed, should be effected by varying only the elements a'', i and Ω , that is, those which do not intervene in the spectroscopic orbit.

It is desirable to repeat the entire process with at least two values of the period which are close to the initial value. Lastly, the calculation is completed by stablishing the position of ascending node.

When the spectroscopic binary is double lined the *modus operandi* is analogous since we use the relative radial velocities.

3. APPLICATION

We have applied the method to the single-lined spectroscopic binary 94 Aquarii (ADS 16672 - WDS 23191S 1327). The observation used were taken from the studies of Sarma (1961) and McAlister & Hartkopf (1982).

On the basis of the observations we have taken an approximate period of 6.8 years and improved it using the radial velocities by means of Bopp's procedure, while the visual measurements only represent about two revolutions. After improvement, the period was 6.29 years.

Once all the observations were reduced to this period, we plotted the observations $(\theta, \rho; t)$ and chose the three base points:

1976.614 155°.9 0″.197 1979.770 310°.1 0″.148 1981.700 448°.1 0″.117

Applying the program DOCO gave the orbit which passes through the three base points as having period 6.29 years and elements

P = 6.290 years T = 1980.840 e = 0.1629 a = 0''.1823 i = 45°.988 $\Omega = 165°.156$ $\omega = 213°.774$

Taking the initial values $V_0=10.5~{\rm km\,s^{-1}}$ and $K=5.5~{\rm km\,s^{-1}}$ and using the values of $P,\,T,\,e,$ and ω previously calculated, we proceeded to improve V_0 and K by minimizing the squares of the O-C differences in the radial velocities. This yielded

 $V_0 = 10.931 \text{ km s}^{-1}, \qquad K = 5.374 \text{ km s}^{-1}.$

In view of the acceptability of the O-C differences in the velocities and the (θ, ρ) values, the final orbit is that defined by the elements of the visual binary, together with the improved values of V_0 and K.

4. REFERENCES

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