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Combinatorially regular Euler polytopes

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The main aim of this thesis is to classify the combinatorially regular Euler incidence polytopes. The classification is completed except for a few exceptional cases, where the subgroup structure of certain related Coxeter groups is not sufficiently well known. The topological structure of the objects is not considered.

The regular three dimensional (spherical) polytopes have been know since antiquity, and the higher dimensional ones since late last century. In recent times, various authors have attempted to abstract the concept of regular polytopes into purely combinatorial (non-geometric) settings. Currently, the most widely studied (but not the only) such abstraction is Egon Schulte's *regular incidence polytopes*, where a polytope is regular if and only if its automorphism group acts transitively on its set of flags. The key difference between this work and other work on incidence polytopes is that instead of defining regularity in the above terms, a combinatorial definition is used - an object being called *combinatorially regular* if its local structure remains the same all over the object (specifically, if corresponding sections are isomorphic). It is shown that any polytope that is regular in Schulte's sense is also regular in the combinatorial sense. This means that combinatorial regularity is a weaker condition than the earlier one, making the results obtained correspondingly stronger. The Eulerian condition is a similar condition to that satisfied by the classical geometric polytopes, and allows us to obtain an overview of the combinatorially regular incidence polytopes. While it is true that some authors have examined combinatorial 'polytopes' that are 'less regular' than usual, a literature search reveals that objects as 'weakly' regular as those examined in the thesis have usually been ignored. In particular, no-one has attempted a classification such as the one given.

The thesis contains a brief overview of some known results about geometric polytopes, and then a number of combinatorial results about the incidence polytopes are stated and proved. Some examples are given of particular polytopes, and then a strong link is established between the combinatorially regular incidence polytopes, and the

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theory of Coxeter groups. The main theorem of the thesis is a statement of this link, which is a classification of what might be termed "locally spherical" combinatorially regular incidence polytopes, the classification being in terms of certain subgroups of certain Coxeter groups. Although nowhere is the classification explicitly restricted to such 'locally spherical' polytopes, this result becomes the lynchpin of the classification theorems of the next-to-last chapter, which attempt to describe exactly what combinatorially regular Euler incidence polytopes exist with certain Schlafli symbols. Except for a few particular cases, the combinatorially regular Euler incidence polytopes are completely described.

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