CORRESPONDENCE.

MR. ACKLAND'S PAPER ON RATES OF MORTALITY AND WITHDRAWAL.

"DISCONTINUANCES."

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the course of the debate on the paper read before the Institute, last session, by Mr. Ackland, I ventured to suggest that discontinuances are due to a force essentially different in character from the force which causes deaths, that the former force would be more appropriately measured by rates of non-renewal than by rates or forces of withdrawal (as ordinarily understood), and that the facts of an experience of assured lives, as exhibited by the Nearest Duration Method are directly applicable to the calculation of rates of non-renewal. I now write, with the object of expressing my views more clearly.

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I suggest, in the first place, that discontinuances in ordinary business are attributable to a force essentially different in character from the force which operates to produce mortality. Death claims are caused by a force which operates *continuously* throughout the history of policies, whereas discontinuances are caused—so far as regards the large majority of policies—by a *discontinuous* force coming into operation at certain recurring epochs. A policyholder does not exercise his option of withdrawing *continuously* in the same sense that he is continuously subject to the risk of death; he exercises it periodically, in most cases on the occasions of his receiving a renewal notice, and having to decide whether to pay or not to pay. This applies, I think, to nearly all lapses, the only exceptions that occur to me being the few cases in which a policyholder pays a series of fines to extend the days of grace for successive short periods; and lapses constitute a very large proportion of the whole number of discontinuances. It applies, also, to all surrenders carried out at or about the renewal date. The proportion which these form of the entire body of surrenders, will no doubt vary in different classes of business; in a small experience of two years, I found that 55 out of a total of 94 surrenders took place during the days of grace, 11 took place within a month before the renewal date (probably on receipt of renewal notices), and the remaining 28 were scattered. Having regard to the small proportion that the number of scattered surrenders forms of the entire number of lapses and surrenders, I think it may fairly be stated as a general proposition, that discontinuances are mainly due to the exercise of a periodical option at or about the renewal date.

If this proposition be admitted, it follows that the force which causes discontinuances would be more appropriately measured by rates of non-renewal, than by rates or forces of withdrawal—in other words, by the ratios that the withdrawals at definite epochs bear to the exposed to risk of withdrawal at those epochs, than by the ratios that the withdrawals in given periods bear to certain numbers supposed to be continuously exposed to the risk of withdrawal throughout those periods. As applied to a collected experience, this second proposition pre-supposes a policy-year tabulation of the observed facts. A tabulation by calendar-years, or years of life, will, of course, have the effect of spreading the discontinuances over the years of observation, and will thus exhibit something of the nature of a continuous force of discontinuance. If it be admitted that such a force has no real existence, the fact that it is artificially created by any method of tabulation other than one that follows the years of assurance may be considered another argument in favour of the Policy-year Method.

I proceed now to consider the applicability of the facts of an experience, as exhibited by the Nearest Duration Method, to the calculation of rates of non-renewal. For the first two years of assurance, during which the exposed to risk of death and the rate of mortality are rapidly changing—the former by discontinuances, and the latter by the lapse of time since selection—it may, perhaps, be considered desirable to exhibit a large experience, such as that in course of compilation by the Institute, by quarters of a year. The

discontinuances during that period will consist almost entirely of lapses. Even assuming that the instructions given by the New Experience Committee have achieved the difficult task of securing absolute uniformity of treatment by the contributing offices in regard to such matters as dating-back and days of grace, the effect of an attempt to classify these discontinuances *in* the quarters—with the object of calculating rates of withdrawal on the analogy of rates of mortality—will be to locate in, and to represent as spread over, say, the second quarter, a large number of lapses which occurred by nonpayment of the second quarterly premium; the effect of a Nearest Duration tabulation, on the other hand, will be to convey what seems to me to be the real meaning of these discontinuances by representing them as having occurred at the *end* of the first quarter. If absolute uniformity of treatment has *not* been secured, the first-mentioned method of tabulation will stereotype the divergencies by locating some of the discontinuances in the first quarter and some in the second, whereas the Nearest Duration Method will neutralize them. Passing on to the general effect of the Nearest Duration Method, as applied to a tabulation by integral years, I cannot do better than refer to the result of Mr. Ackland's investigation of the average error resulting from the operation of the method (vol. xxxiii, pp. 144-5). Upon the assumption that yearly, half-yearly, and quarterly cases enter into a general experience in the proportion of $62\frac{1}{2}$, $32\frac{1}{2}$, and 5 per-cent respectively, and that surrenders take place *one month* on the average before the renewal date, Mr. Ackland shows that the Nearest Duration Method will dislocate the average date of surrender by only about $1\frac{1}{2}$ days. The conclusion to which this result leads is enforced by the considerations that the proportion of yearly cases persisting long enough to be entitled to a surrender-value is relatively larger than the proportion coming on the books, and that the majority of surrenders take place (according to my observation), not before the renewal date, but during the currency of the days of grace. A fact of importance, moreover, is that the Nearest Duration Method not only locates the whole body of surrenders correctly *on the average*, but also locates the majority of them *exactly*. Taking Mr. Ackland's proportions of yearly, half-yearly, and quarterly cases, and assuming that all are equally likely to discontinue, 80 per-cent of the renewal-date discontinuances (*i.e.*, lapses and such surrenders as occur at the renewal date), represented by the Nearest Duration Method as having occurred at the end of the year, will have actually occurred there; the remaining 20 per-cent will have been about equally drawn from the preceding and succeeding quarters or half-year. Hence, as applied at annual intervals, the Nearest Duration Method accurately represents the rates of non-renewal for a majority of the cases under observation, and makes an approximate allowance for the rates experienced at adjacent quarters. Objection may be taken to the dislocation of the discontinuances occurring at the ends of half-years—one-half of which would have to be collected at the end of the year, and the other half at the end of the preceding year—but this objection does not appear so serious as the objection that may be urged to tabulating as having occurred *during* the years the far

larger number of discontinuances occurring exactly at the *end* of the year.

The general conclusions at which I arrive are that the Nearest Duration Method locates the discontinuances fairly, and, to a large extent, exactly, for the purpose of an experience compiled by *years*, and that, if the experience for the first year or two be compiled by quarters, the Method will, during that period, represent the discontinuances with absolute accuracy. I venture to suggest, moreover, not only that the Nearest Duration Method collects the discontinuances in such a way as to enable the force which causes them to be appropriately measured, but also that the curtate duration method collects them in such a way as to lead to a misrepresentation of the force, and that the Exact Duration Method offers no advantages over the Nearest Duration Method for the calculation of rates of non-renewal.

Although it is my immediate object to show that the way in which discontinuances are dealt with by the Nearest Duration Method is appropriate to the accurate measurement of the force which causes discontinuances and does not, therefore, constitute any objection to the employment of that method, it may be of interest to consider the application of the resulting functions to one or two of the problems that arise in connection with the question of withdrawal. The most important of these problems relates to the influence of withdrawals on the rate of mortality, and, for its solution, rates of non-renewal—exhibiting the force of discontinuance as operating at the ends of years—appear likely to be more useful than rates of withdrawal, the effects of which have to be looked for partly in the year to which they relate and partly in subsequent years. If on comparison of two experiences a materially higher rate of non-renewal is exhibited by one, at the end, say, of the third year of assurance, than by the other, a more confident conclusion could, I think, be drawn from the relative progression of the rates of mortality from the third to the fourth year than would be possible from a comparison of rates of withdrawal. In this connection I may refer to Mr. Frank Sanderson's recently-published exhibition of the Canada Life Experience, the facts of which are tabulated by the Nearest Duration Method; the following is an extract from a section relating to the rate of discontinuance in Mr. Sanderson's Report:

"In obtaining the rates of mortality by years of assurance, we have seen that it is a necessary condition of the policy-year method that the deaths should be allocated to the policy-year in which death takes place.

"If it were thought necessary to obtain with equal precision the rate of discontinuance, it would have been necessary to tabulate the discontinuances in a manner similar to the deaths, *i.e.*, in the exact policy-year of discontinuance. But, in view of the fact that the rate of discontinuance is less regular than that of mortality, differing according to different companies' different plans of assurance, and other circumstances, it was thought that for the present purpose, at least, the tabulation of the withdrawals, according to the Nearest Duration Method, would give results sufficiently approximate for all practical purposes.

"From the explanation given on page 10, it will be remembered that the withdrawals are made to pass from observation at the end of the policy-year. In consequence of this, the rate of discontinuance is determined as at the end of the year, and not in the year. The function tabulated, therefore, is not exactly the same as in some other experiences."

It will be seen that the function Mr. Sanderson has tabulated is the rate of non-renewal, and this rate appears to me to exhibit the experience in regard to discontinuances with greater precision than would have been obtained by tabulating the discontinuances in a manner similar to the deaths. With the object of seeing whether any evidence as to the influence of the discontinuances upon the rate of mortality could be obtained by comparison of different sections of the experience, I grouped the figures for ages 22, 24, 26, 27 and 31 at entry (for each of which the total discontinuances in the first five years were over 28 per cent. of the entrants), and the figures for ages 21, 23, 25, 28, 29, 30 and 32 (for each of which the total discontinuances in the first five years were under 28 per cent. of the entrants), with the following results:

Ages at Entry 22, 24, 26, 27, 31.

Year of Assurance	Exposed to Risk of Death	d	q	Exposed to Risk of Non-Renewal	w	Rate of Non-Renewal
1	8,016	30	·0037	7,986	1,288	·161
2	6,313	36	·0057	6,277	476	·076
3	5,535	25	·0045	5,510	297	·054
4	4,945	29	·0059	4,916	149	·030
5	4,404	25	·0057	4,379	176	·040
6	3,958	15	·0038	3,943	95	·025
Total	33,171	160	·0048	33,011	2,481	·075

Ages at Entry 21, 23, 25, 28, 29, 30, 32.

Year of Assurance	Exposed to Risk of Death	d	q	Exposed to Risk of Non-Renewal	w	Rate of Non-Renewal
1	11,053	22	·0020	11,031	1,649	·150
2	8,845	56	·0063	8,789	557	·063
3	7,869	47	·0060	7,822	379	·048
4	7,085	32	·0045	7,053	257	·036
5	6,281	28	·0045	6,253	233	·037
6	5,695	35	·0062	5,660	165	·029
Total	46,828	220	·0047	46,608	3,240	·070

The deaths are not sufficiently numerous, nor is there a sufficient divergence between the rates of non-renewal, to admit of any conclusion being drawn from the figures; but the results may, perhaps, be of interest as an example of the concurrent calculation of the probabilities of dying and the rates of non-renewal in successive

years of insurance,* and as an illustration of a method of comparison which in a larger experience might afford some indication as to the effect of withdrawals upon the average vitality of a body of assured lives.

The problem of the calculation of annual premiums for term assurances may be briefly referred to. If it is a correct assumption that a body of assured lives is deteriorated by withdrawals, it is obvious that select tables compiled from an experience chiefly consisting of whole-term assurances are not directly applicable to the calculations of annual premiums for more numerously discontinued assurances. It is possible, however, to provide for more numerous withdrawals of healthy lives than are covered by the tables by using the formula

$$P^1_{\overline{xn}|} = \frac{M_{[x]} - M_{[x]+n} - p(M_{[x+1]} - M_{[x+1]+n-1}) - q(1-p)(M_{[x+2]} - M_{[x+2]+n-2}) - \&c.}{\mathbb{N}_{[x]} - \mathbb{N}_{[x]+n} - p(\mathbb{N}_{[x+1]} - \mathbb{N}_{[x+1]+n-1}) - q(1-p)(\mathbb{N}_{[x+2]} - \mathbb{N}_{[x+2]+n-2}) - \&c.}$$

where, p, q, &c., represent the extra proportions of healthy lives withdrawing at the end of the first, second, &c., years.

Now it is questionable whether the term-assurance experience of life offices is sufficient to admit of the construction of a special set of select tables for the calculation of premiums, and whether, in any case, it would be worth while to construct such tables, but the experience will, of course, readily lend itself to the calculation of rates of non-renewal. The excess of these rates over the rates of non-renewal shown by the whole-term table, would then afford an indication of the values to be given in the above formula to the quantities p, q, &c.

In conclusion, I may just mention one other problem to the solution of which rates of non-renewal appear to be directly applicable —the determination of a superior limit to the amount allowable in commutation of an annual commission.

I am, Sir,
Your obedient Servant,
R. TODHUNTER.

39, *King Street, Cheapside, E.C.*,
20 *February* 1897.

MR. KING'S INTER-VALUATION FORMULA FOR "EXPOSED TO RISK."

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In reading Mr. Ackland's interesting paper on "Methods for deducing the Rates of Mortality and Withdrawal", I notice that, in referring to Mr. King's Inter-Valuation Method, Mr. Ackland states that there is a possible error of twelve months in the age at

* To obtain the exposed to risk of non-renewal, I have followed Mr. Sanderson in deducting the deaths from the exposed to risk of death. It is assumed that lives which were assured up to the close of the observations, but withdrew immediately afterwards, are included in the "withdrawals", and not in the "existing."