compact surfaces, to name just a few. As the title of the book suggests, the emphasis is mostly on the analysis aspect of the theory, but not exclusively so. For example, in the chapter on Chern–Simons systems there is quite a lot of work on Lie algebras. For the enthusiastic, there are some open problems to think about at the end of each chapter.

Solitons in field theory and nonlinear analysis is a solid research monograph with a lot of information in it and should certainly stimulate those who already have some background in field theory and analysis. Although it has a hefty 553 pages, I feel that for a full understanding one would need to delve into some of the recommended books as well. Reading this book is quite challenging and is not for the faint hearted.

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de la Harpe, P. *Topics in geometric group theory* (University of Chicago Press, 2000), vi+310 pp., 0 226 31721 8 (paperback), £13.00, 0 226 31719 6 (hardback), £28.50.

Geometric group theory is a relatively new subject with old roots. The deep connections between groups and geometry have been recognized since the nineteenth century, with Klein's Erlangen Program highlighting the importance of the symmetry group of a geometry, and Poincaré's work on topological spaces, manifolds and fundamental groups.

The use of topological and geometric methods as a tool to prove algebraic theorems about groups can be traced back to the early twentieth century work of Dehn on the fundamental groups of surfaces. This led to the development of the subject known for much of the twentieth century as *combinatorial group theory*, often defined as the study of groups expressed in terms of generators and defining relations (*presentations*).

For those of a topological bent, a presentation of a group G is (essentially) a two-dimensional cell complex whose fundamental group is isomorphic to G. Given such an object, one can apply the machinery of topology (covering spaces, continuous maps) to prove things about G. For the more algebraically minded, one can instead work directly with the presentation, using combinatorial operations on words in the generators in place of topological tricks. Combinatorial group theory appears in both guises (and a spectrum of compromise forms) throughout its history. However (at least in the reviewer's opinion), the topology is always present, either explicitly or implicitly.

The topological version of combinatorial group theory is for the most part purely topological rather than geometric, in the sense that no underlying metric plays any role in the arguments (although the spaces involved are all metrizable as nicely embedded subspaces of Euclidean space). The notable exception to this is what is known as *small cancellation theory*. Again, this can trace its roots back to the work of Dehn on surface groups. In modern parlance, Dehn produced an algorithm to solve the word problem for the fundamental group of an orientable surface of genus at least 2. His arguments effectively use the negative curvature of such surfaces, and the resulting algorithm works in linear time (as a function of the length of the input word). Small cancellation theory is a combinatorial way of applying curvature arguments to more-general group presentations.

Geometric group theory emerged in the 1980s, in work of Gromov. The distinction from combinatorial group theory is that an explicit metric is always present: namely, the word metric on the group G itself with respect to a given (finite) generating set for G. This metric is of course dependent on the choice of generating set, but many of the coarse properties of the metric turn out to be independent of this choice, which makes it a useful tool. It is the idea of viewing a group as a metric space that is inherently new here, and leads to further new concepts such as the 'boundary' of a group. Gromov initially used this idea in his classification of groups of polynomial growth [**8**]. He then developed the concept of *hyperbolic group*: roughly speaking, a group which as a metric space appears to have a kind of negative curvature property. One of the many equivalent definitions of hyperbolic group is that Dehn's (linear-time) algorithm solves its word problem. These have been objects of intensive study over the last two decades.

Nowadays, the terms combinatorial and geometric group theory are often used interchangeably. Certainly there is a large area of intersection between the two fields, and there are very few practitioners who can truly be said to operate only in one or the other of them.

There is a well-established literature in combinatorial group theory, including the eponymous books of Magnus, Karrass and Solitar [**13**] and of Lyndon and Schupp [**12**], from which many of us learnt our trade. Also worthy of mention are the earlier work of Coxeter and Moser [**4**], the more recent book of Baumslag [**1**], and the very accessible student text of Johnson [**11**]. These and other textbooks combine to provide for a range of needs, from the beginner's textbook to the professional's reference source.

As a younger subject, geometric group theory is as-yet less well endowed with good source literature. There are excellent books on closely related material, such as [**2**, **5**]. There are a number of book-length articles or research monographs by Gromov [**8**–**10**] in which the subject is developed, and there are some very good expository texts on hyperbolic groups, such as [**3**, **6**]

The appearance of this, probably the first textbook exposition of geometric group theory, is very welcome. There is much good material in the book, making it a valuable addition to the literature.

A warning to the reader: this is not a gentle introduction to the subject. The reader is expected to work through the text, rather than leaf through it leisurely. Much of the exposition takes the form of exercises, of varying levels of difficulty. For anyone not already at least partly familiar with the subject, it will be hard going, because a number of technical terms are left undefined.

Thus it is not a book from which to learn the subject from scratch; in particular, it is not a suitable sourcebook for a first course. I think the book will be most useful to those who already have some working knowledge of geometric group theory and are anxious to learn more. The exercise-based style will make it particularly useful for graduate students specializing in the subject.

From the point of view of content, the book does not set out to be encyclopaedic. I think it can best be described as a survey of selected areas of the subject. The selection is, not unnaturally, coloured by the author's own analytical background and special interests.

The first half of the book (apart from a short opening chapter on random walks) is devoted to fundamental topics such as free groups and free products, finite generation, the Cayley graph, and the notion of quasi-isometry (the appropriate equivalence relation on metric spaces or groups in this 'fuzzy' set-up). There is a lot of very useful material here, some of it at quite an advanced level. As mentioned earlier, the uninitiated reader will find it quite hard going, but perseverance with it will lead to a good grounding in the subject.

The second half of the book is devoted to the notion of *growth* of a group G generated by a finite set S, by which is meant the asymptotic behaviour of the function

$$f(t) = |\{\gamma \in G;\ \ell_S(\gamma) \leqslant t\}|$$

as $t \to \infty$, where $\ell_S(\gamma)$ denotes the least length of any word in the generating set S that represents the element γ. Equivalently, $f(t)$ is the volume of the ball of radius t around the identity vertex in the Cayley graph for G (with respect to S).

This is quite fitting, given the roots of the subject in Gromov's work on Milnor's problem of polynomial growth [**8**]. A 'randomly chosen' finitely generated group has, with high probability, *exponential growth*, in the sense that $f(t) \geqslant a^t$ for some $a > 1$ and for $t \gg 0$. On the other hand, nilpotent groups have *polynomial growth* in the sense that $f(t) \leqslant t^n$ for $t \gg 0$, for some n. Although the growth function f depends on the choice of finite generating set S, these asymptotic properties are independent of the choice of S, and moreover they are closed with respect to passage to subgroups of finite index. Gromov's big contribution was the converse of the above statement on nilpotent groups: G has polynomial growth if and only if it has a nilpotent subgroup of finite index.

A second problem of Milnor's—whether every group has either exponential or polynomial growth—was solved in the negative by Grigorchuk in the 1980s [**7**]. Grigorchuk's example of a group of intermediate growth and other similar examples later constructed by Gupta and Sidki are often expressed as groups of automorphisms of rooted trees. All these topics, and more, are covered in the final three hefty chapters of the book.

I do not see this book being usable as a course textbook, for example, because of the combination of style and choice of material. Nevertheless, anyone working seriously in the area of geometric group theory will want to have access to it as a handy reference manual, and its appearance is greatly to be welcomed.

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Palmer, Theodore W. *Banach algebras and the general theory of ∗-algebras*, vol. II, *∗-Algebras* (Cambridge University Press, 2001), 834 pp., 0 521 36638 0 (hardback) £90.00.

This book is the second, and final, volume of a great work on Banach ∗-algebras. The first volume [**4**], which was published in 1994, was devoted to the general theory of algebras and Banach algebras. This volume was reviewed by me in [**1**]. This second volume is devoted to Banach algebras with an involution. It has already been reviewed by George Willis in [**7**].