2. NORMAL STARS

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RADIATION HYDRODYNAMICS IN PULSATING STARS

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1. INTRODUCTION

The topic of this review encompasses all aspects of pulsation theory, for the radiation field is never negligible in stellar stability problems, on the contrary, it is usually the primary destabilizing factor through its thermal effects, and modifies the envelope structure and stability through its dynamic effects. The impossibility of a general review of such a broad topic is apparent, and I will concentrate in this talk on the most striking aspect of pulsating stars: nonlinear effects in the outer layers. To focus the discussion, I will address primarily two problems of current interest: shock development driven by the pulsating velocity field, and time dependent turbulence in the ionization zones. The emphasis will be on methodology rather than specific problems and developments.

2. OVERVIEW

We will consider the problem of radial pulsation in low modes, with special attention to the various effects of the radiation field. Figure 1 depicts in a pictorial fashion some of these effects and their location in the star. They may be categorized as follows:

1. Radial pulsational instabilities, generally driven in the ionization zones.

2. Thermal (secular) instabilities associated with the nuclear burning region in the core.

3. Non-radial instabilities, including convection, affecting the surface layers or the core.

4. Radiation pressure effects on the static structure, stability properties, and flow in the atmosphere.

5. Wave behavior at the photosphere and in the atmosphere, especially shock development.

All of these processes are dependent either directly or indirectly on properties of the radiation field, and show marked changes in their characteristics as radiation increases in importance.

The set of equations that describe the general stellar oscillation problem have been discussed recently by B. Mihalas (1984), based on earlier work by Castor (1972), and Buchler (1979). The main equations are those of conservation of momentum and energy for both the gas and the radiation field. These equations require some type of closure assumption, such as the Eddington approximation, to relate P to E. Usually, more drastic assumptions are made, in order to eliminate the integral terms in these equations, the most popular is the diffusion approximation, valid at large optical depth. As a step in this direction, we may retain the distinction between the mean intensity, J, and the equiblibrium value, B, to obtain the "nonequilibrium diffusion" approximation (Mihalas, 1984).

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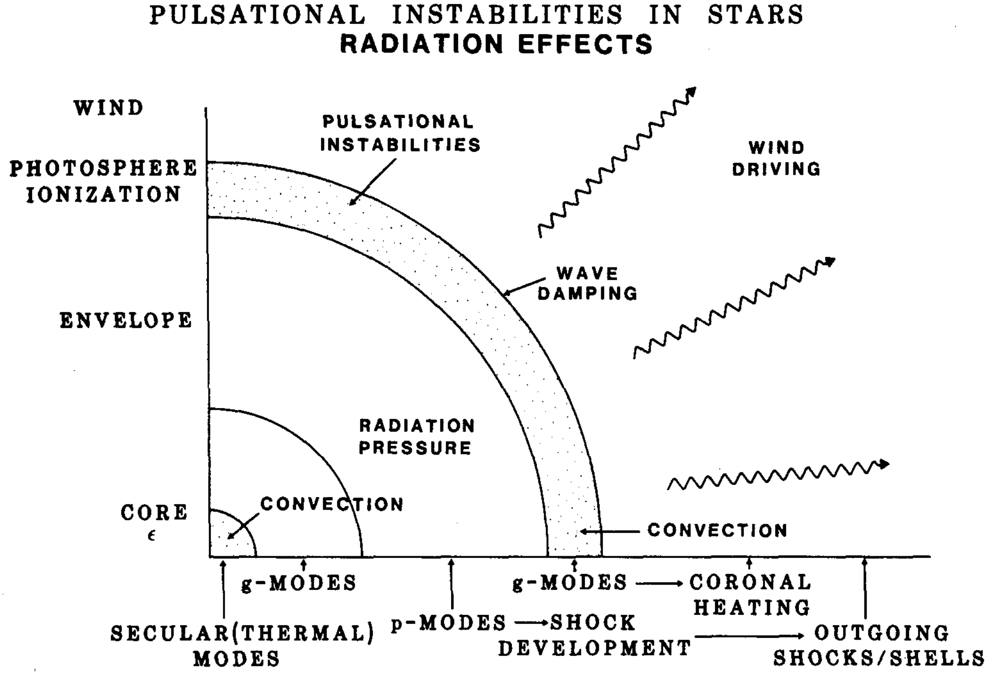

Fig. 1. Schematic representation of a star, showing the structural components on the left, the principal instabilities along the bottom, and various processes that tend to be strongly dependent on radiation effects.

Taking the process one step further, we obtain the complete equilibrium diffusion treatment (J=B). This treatment is inapplicable at small optical depth, but is often used in this case nonetheless owing to its simplicity: radiation quantities appear directly as correction terms in the gas dynamic equations, and they are trivial to compute. The only physical input needed is the Rosseland mean opacity. The primary effect of the radiation field enters through the flux divergence term, which represents a "non-adiabatic" energy exchange with gas. The magnitude of this "non-adiabaticity" is given by the ratio of the thermal timescale (time needed to radiate away all of the thermal energy in a layer) to the dynamic timescale (free-fall time). This is the "thermal" effect of the radiation field responsible for pulsational instability in most classes of variable stars. The second effect of the radiation is the direct contribution to the pressure and internal energy, affecting not only the work done during a pulsational cycle, but the static structure of the star as well.

Table 1 lists some of the work in this field according to the type of model and the radiation treatment employed. This is not intended to be a complete list, nor is it a full description of the contribution of the workers listed. It does show that the models fall into two general categories: envelope models and atmosphere models -- to my knowledge no model of the entire system has ever been attempted. We also see that the majority of the results in this field have been obtained with the equilibrium diffusion treatment, although a few more accurate models have been computed.

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Table 1

TREATMENT OF PHOTOSPHERE AND RADIATION
IN NONLINEAR PULSATION MODELS
(not a complete list)

I...Envelope Models (Photosphere = outer boundary)		
-> Equilibrium Diffusion		
Christy	1966	RR Lyrae
A. Cox, et. al.	1966	<all>
Stobie	1969	Cepheids
Stellingwerf	1974	RR Lyrae, Cepheids
von Sengbusch	1974	RR Lyrae
Stothers	1981	RR Lyrae, Cepheids
Fadeyev	1981	Supergiants
Aikawa	1984	Supergiants
-> Non-local Transfer Equation		
Castor	1967	RR Lyrae
-> Non-equilibrium Diffusion		
Spangenberg	1975	RR Lyrae
-> Multigroup Radiation		
Keller	1969	RR Lyrae
Bendt & Davis	1971	Cepheids
Karp	1974	Cepheids
II...Atmosphere Models (Photosphere = inner boundary)		
-> Non-equilibrium Diffusion		
Hill	1972	RR Lyrae, Cepheids, Miras
-> Radiation Pressure only		
Wood	1979	Miras

As an example, Figure 2, from Stellingwerf (1974), shows the variation of the radial velocity of the outer zones of a nonlinear RR Lyrae model (every other zone shown, shifted), as well as the variation of the bolometric magnitude. The outgoing shock at phase 0.7-0.8 and the ingoing shock visible at phases 0.9-1.2 show the movement of the hydrogen ionization zone in mass. Castor (1967) has studied the behavior of this ionization front and finds that it is a weak D type during the outgoing phase, and weak R type during the inward movement. The strong shock at phase 0.75 therefore precedes the ionization front into the neutral material, and undoubtedly continues to propagate into the atmosphere. The inward moving shock follows the front.

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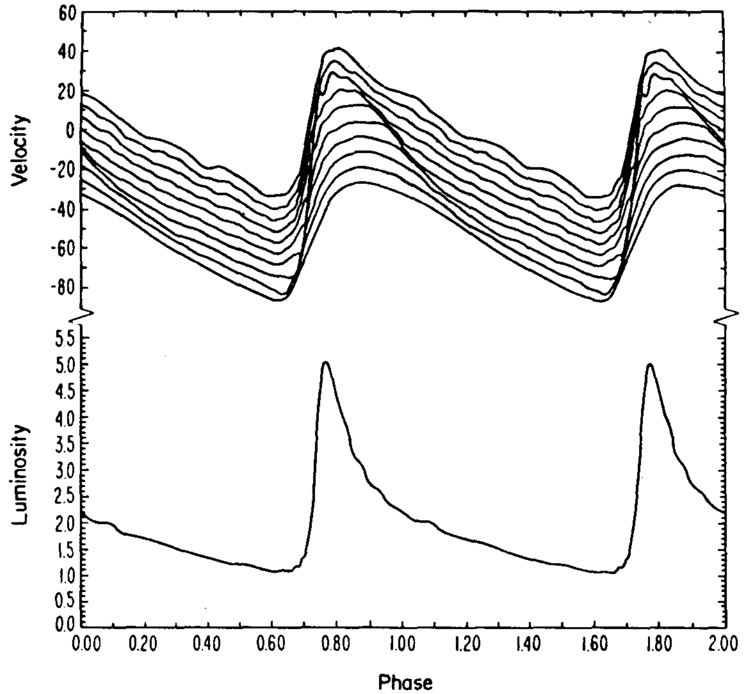

Fig. 2. Variation of the velocity of the outer zones and the luminosity of an RR Lyrae model envelope using the equilibrium diffusion treatment of the radiation field.

This same model was computed by Spangenberg (1975) using the non-equilibrium diffusion treatment of radiation. Variation of the outer zone temperature, opacity, and intensity are shown in Figure 3. The luminosity variations in the two models are remarkably similar, both in shape and in amplitude. The temperature variation in the first model, however, is tied to the fourth root of the luminosity, whereas, the non-equilibrium model shows much more structure in the temperature variation, as well as a large dip near the phase of rising light. Although the reality of these features is not clear, this certainly indicates that a careful numerical treatment is necessary to accurately obtain the variation of the temperature in optically thin regions.

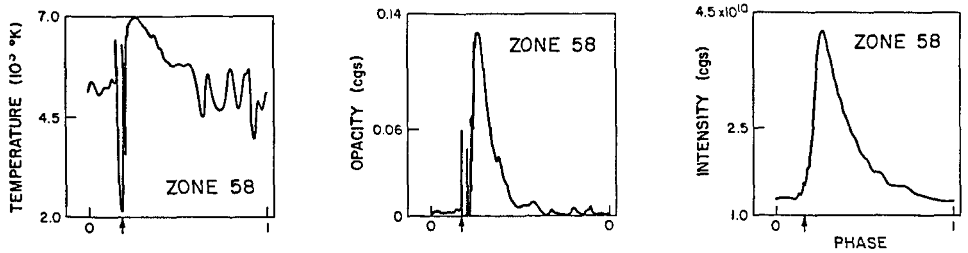

Fig. 3. Variation of the outer temperature, opacity, and luminosity in a non-linear RR lyrae using the equilibrium diffusion treatment of the radiation field.

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An interesting example of ionization front movement is shown in Figure 4, from Stellingwerf (1975), showing the variation of various quantities in a model showing a mixture of modes. Note also the smooth variation of the outer temperature. All of these models completely ignore the effects of convection, and this last case is unrealistic in that it lies near the red edge of the instability strip, where convective effects would be strong.

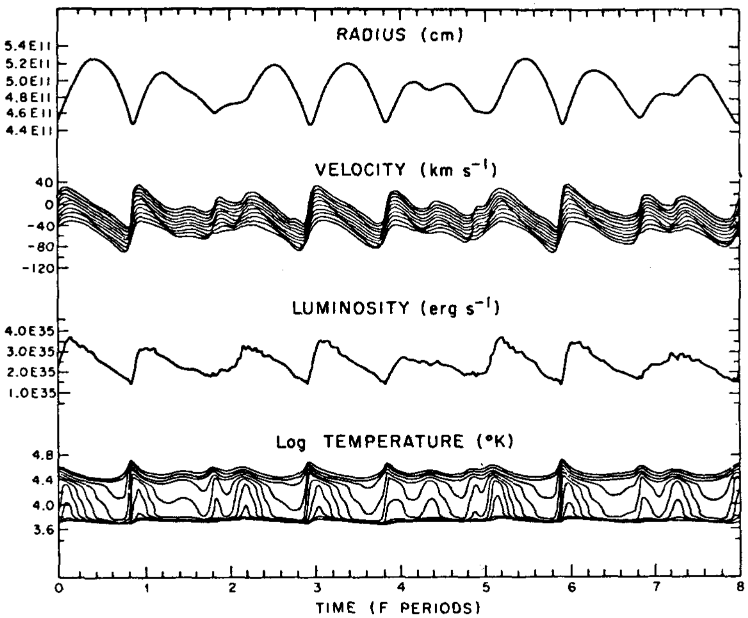

Fig. 4. Variation of outer zone parameters in a mixed-mode RR Lyrae model.

To see the effect of the radiation pressure on the static structure of Cepheids, in Figure 5 (from Cox and Stellingwerf, 1979) shows the run of an adiabatic "gamma", the specific heat C_V, and the radiation pressure in a Cepheid model, and in "b" the same model structure computed without radiation pressure. These quantities are important contributors to the pulsational stability of the star, and it is clear that the radiation pressure, which attains a value of about 25% of the total pressure over a limited layer in the star, does affect these quantities strongly, and in a destabilizing fashion. For comparison, Figure 6 shows the same information for a model differing only in effective temperature from the previous case: this is a model of a Beta Cepheid. Clearly, the radiation pressure is more important in this case, reaching a value of about 30% of the temperature over wider range of temperatures. Its destabilizing effect is also more pronounced, allowing a driving region to appear at $\log T = 5.1$. The radiation pressure can also be shown to cause the instability strip to slant toward higher effective temperatures at higher luminosities, as observed for Beta Cepheids, rather than the opposite case as in the instability strip. The destabilization due to this mechanism seems, alas, to be too weak to overcome the envelope damping (Stellingwerf, 1978), but certainly could contribute to the instability of these stars.

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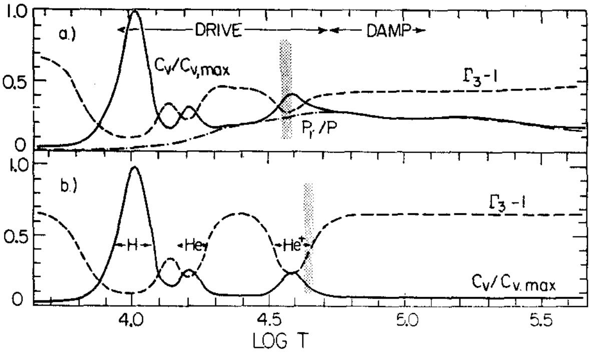

Fig. 5. Structure of an envelope of the Cepheid model: (a) including radiation pressure, and (b) with radiation pressure suppressed. Plotted are: specific heat (solid line), adiabatic exponent (dashed line), and radiation pressure (dot-dashed line).

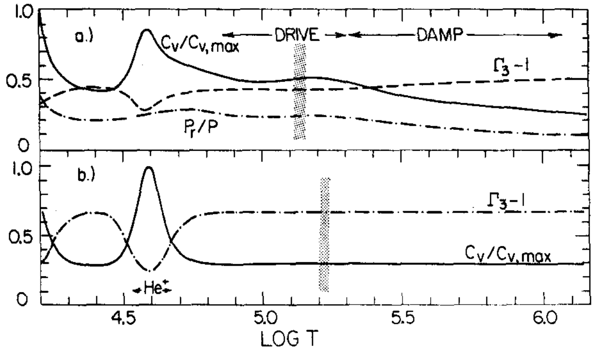

Fig. 6. Same as figure 5, but for a hotter model, near the Beta Cepheid region of the HR diagram.

3. ATMOSPHERIC EFFECTS: SHOCK DEVELOPMENT

It is well known that any sort of turbulence near the photosphere may launch outgoing radiative/acoustic waves that grow in amplitude due to the exponential density gradient, and may become shocks. The large scale periodic motions present in pulsating stars is an extreme example of this phenomenon: in many types of oscillating stars very strong shock waves are generated each period and can profoundly change the atmosphere's properties. This process probably represents a dissipative energy loss to the pulsation, and this energy is carried into the atmosphere to raise its temperature, modify the density structure, and in some cases perhaps drive a stellar wind.

The increase in shock strength with radius does not continue indefinitely. Figure 7 shows the modulus and phase of an eigenfunction obtained by linear analysis of an extended isothermal atmosphere (from Stellingwerf and Buff, 1982), compared with the (high frequency) WKB result. Two effects are apparent in this simple case: a rapid growth in the amplitude at small radii, caused by the stratification, and a subsequent decline, due to spherical effects. Peak amplitude occurs at the sonic radius $R_s = GM/2c$, where c is the isothermal sound speed. The mode depicted is constrained to be a standing wave by boundary conditions, but in fact a consequence of the weakening gravity at large radii is

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a drop in the acoustic cutoff frequency, and a gradual conversion of the evanescent pulsation motion into traveling waves as the radius increases. These traveling waves show the same amplitude variation as seen in Figure 7 in the linear (small amplitude) limit.

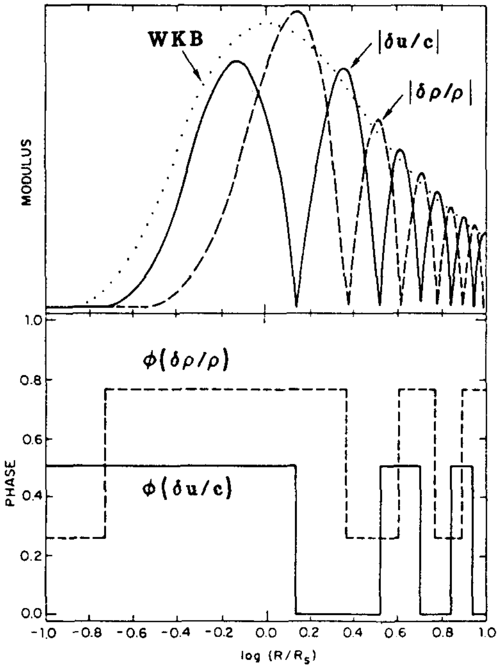

Fig. 7. Oscillation eigenfunction for an extended isothermal atmosphere, showing the modulus and phase of the velocity and density variations.

The development of the outgoing shock in an RR Lyrae model can be seen in Figure 2. Hill (1972) studied the dynamics of the radiating shock in some detail as it develops in the atmoshere. Figure 8 is taken from Hill (1975), and shows the movement of mass shells in the (Lagrangian) model of the Cepheid Beta Dor. The shock development in this model is rather complicated, and certainly this is caused in part by the rigidly prescribed piston motion at the base of the atmosphere. This is not a periodic solution, and, indeed, periodicity may be impossible to attain in a Lagrangian model due to long period motion of the outer layers, and possible ejection of the outer zones (Wilson and Hill, 1976).

One striking feature of such models is their strong deviation from the linear results. Figure 9 shows the results of calculations by Wood (1979) of Mira atmospheres driven by large amplitude pulsations. The lines in these diagrams depict the motion of shocks as a function of time in the two cases of an adiabatic atmosphere (with specified temperature structure), and an isothermal atmosphere. In both cases the first shock is unique, subsequent shocks see a very different structure and move with changed velocity. In the isothermal case subsequent shocks move into material falling inward in the wake of the first shock and tend to merge, building a shell that will eventually be ejected. In the adiabatic case, the pulsation drives a steady wind. Radiative effects are important in all aspects of this process.

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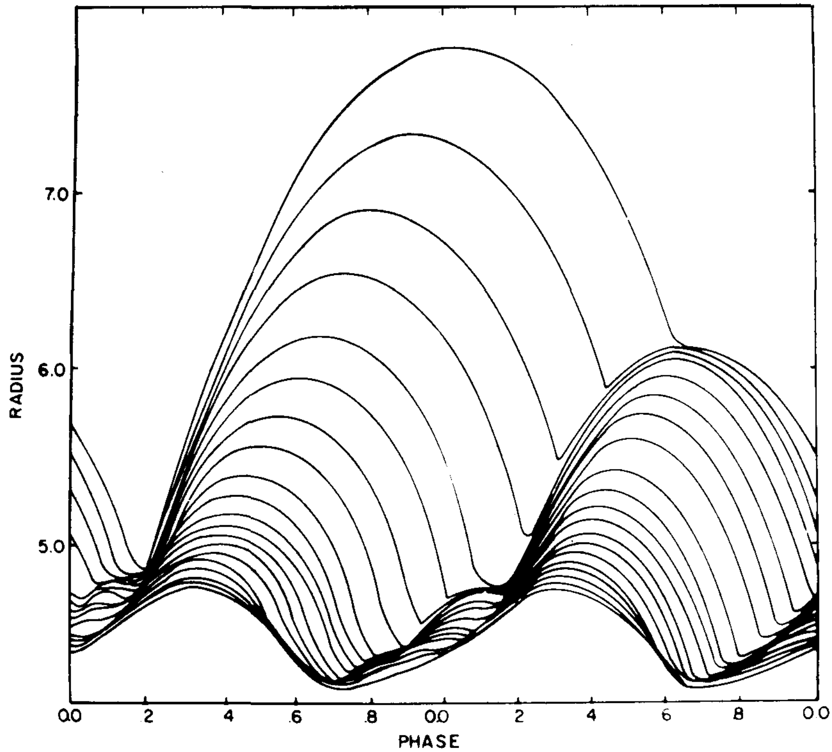

Fig. 8. Shock wave development in a Cepheid atmosphere.

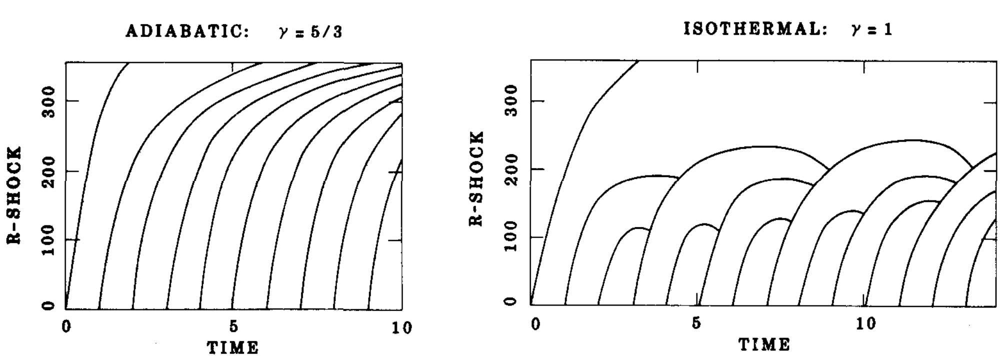

Fig. 9. Shock paths in adiabatic and isothermal atmospheres, shock radius versus time.

Models such as these can be used to estimate the dissipation of energy in the atmosphere, but very little is known concerning the energetics of the complete system. From the point of view of the envelope motion, simple modification of

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the outer boundary condition has been proposed, but is probably inadequate to model such a complicated system. A surprising result obtained by Aikawa (1984) is a destabilization caused by a running-wave outer boundary condition for supergiant oscillations, casting some doubt on the usual assumptions concerning the energy budget. What is needed to address these questions is a full model of the envelope/atmosphere system, using a Lagrangian grid inside the photosphere, and gradually changing to an Eulerian grid as the character of the motion changes to running waves. Such a computation, including a reasonable radiation treatment, is probably feasible with current computers.

4. CONVECTION IN PULSATING STARS

Throughout the development of pulsation theory during the past twenty years undoubtedly the most serious problem confronting the theory has been the question of convection in the ionization regions of the stellar envelope. The temperature gradient in these zones is strongly superadiabatic over very thin shells (thinner than a pressure scale height in some cases). The density is too low to allow effective convective transport. The resulting picture of thin, highly turbulent regions with possibly near-sonic fluid velocities, and substantial overshooting into stable layers, but still carrying only a small fraction of the energy flux, is not a pleasant one computationally. One is strongly tempted to simply ignore the problem. On the other hand, these ionization zones are precisely the seat of the pulsational instability itself, and certainly deserve careful treatment.

Convection has two effects on pulsating stars: 1) modification of the static structure of the star (reducing the temperature gradient in the outer layers), and 2) modification of the time dependence (phase) of the flux in the outer layers. Baker and Kippenhahn (1965) included convective effects of the first type in models of Cepheids and found that the structural changes in the stars caused a neutralization of the pulsational instability in very cool models, but this effect occurred far to the red of the observed red edge of the instability strip, as shown in Figure 10 where the observed strip is labeled "OBS", and the Baker/Kippenhahn linear growth rates for the fundamental and first overtone modes are plotted versus effective temperature.

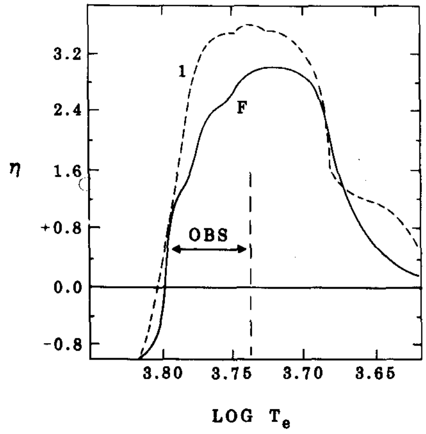

Fig. 10. Linear non-adiabatic growth rates of the fundamental and first overtone modes of Cepheid models as computed by Baker and Kippenhahn in 1966. Marked "OBS" is the observed width of the instability strip.

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The early Cepheid models computed by Cox, et. al. (1966) included convection via a "phase lag" equation, in which the rate of change in the convective flux is limited to the eddy circulation time, and the limiting value is taken to be that of the mixing-length theory of Bohm-Vitense. Later work by Baker and Gough (1979) and Gonczi and Osaki (1980) showed that such a scheme is subject to an instability that causes unphysical fluctuations in the convective flux as a function of radius.

The first computational models to actually demonstrate the quenching effect of convection at the red edge were the two-dimensional computations of Deupree (1977a-d). Later, Xiong (1980) using a more detailed local theory and Stellingwerf (1982a,b, 1984a-c) using a spherical model with nonlocal convection included via a diffusion term obtained similar results. These models also predict a convective effect near the blue edge, but it is a destabilizing effect for these hotter models. It seems that the effects of convection in pulsating stars can be of either sign, rather than a purely stabilizing influence.

The growth rates of the first two modes with convection included are shown in Figure 11, adopted from Stellingwerf (1984a). The width of the instability strip is in good agreement with observations of cluster variables. The dashed curves represent possibilities for the growth rates of one mode toward the other, and determine the mode of pulsation (not yet computed). If the two dashed lines cross below the neutral stability line (as shown) a small hysteresis region is expected, if they cross above neutral, then mixed mode behavior, as seen in M15 and M3 will result.

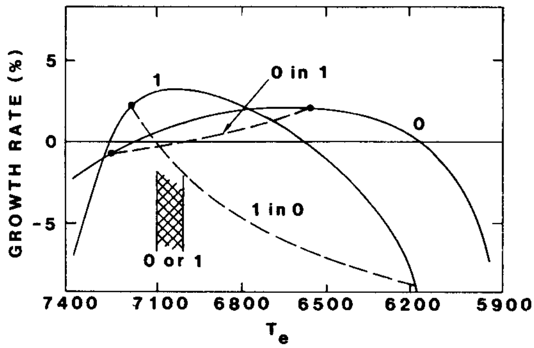

Fig. 11. Linear non-adiabatic growth rates of RR Lyrae models showing estimated switching rates between modes.

The equations used in these models are given below. Equations 1-3 are the usual conservation laws of mass, momentum, and energy, with the addition of convective quantities: P_t = turbulent pressure, P_{tv} = eddy viscosity, E_t = turbulent kinetic energy, F_c = turbulent thermal flux, and F_t = turbulent kinetic energy flux. Equation 4 is the equation for the convective energy, including the effects of overshooting ("diffusion"), superadiabatic destabilization ("driving"), and compressional effects ("pulsation interaction") -- see Stellingwerf (1982a) for details. In these equations, the equilibrium radiation pressure and energy are included in P and E. The convective terms are highly nonlinear, and dynamic as well as thermal effects need to be included. In this treatment, the system of equations is closed by taking the fluctuating temperature T' in the driving term, E_{t0}, to be the mixing-length value.

Continuity:

$$\frac{D\rho}{Dt} + \rho\nabla.\langle u\rangle = 0 \quad , \tag{1}$$

Momentum:

$$\frac{D\langle u\rangle}{Dt} = \frac{1}{\rho}\nabla(P+P_t+P_{tv}) - \nabla\phi \quad ,$$

$$P_t = \rho\langle(u')^2\rangle \quad , \tag{2}$$

$$P_{tv} = -u'\ell\,\partial\langle u\rangle/\partial r \quad ,$$

Thermal Energy:

$$\frac{D}{Dt}(E+E_t)+(P+P_t+P_{tv})\frac{DV}{Dt} = -\frac{1}{\rho}\nabla.(F_r+F_c+F_t) \quad , \tag{3}$$

$$E_t = 1/2\langle(u')^2\rangle \quad ,$$

$$F_c = \rho C_p\langle u'\ T'\rangle \quad ,$$

$$F = 1/2\ \rho\langle(u')^2u' \quad ,$$

Convective Energy:

$$\frac{D}{Dt}(E_t) = \underbrace{\frac{\partial}{\partial r}\left(\ell\sqrt{2E_t}\,\frac{\partial E_t}{\partial r}\right)}_{\text{Eddy Diffusion}} + \underbrace{\frac{\sqrt{2E_t}}{\ell}(E_{to} - E_t)}_{\text{Driving}}\ \underbrace{-2E_t\frac{\partial\langle u\rangle}{\partial r}}_{\text{Pulsation Interaction}} \quad , \tag{4}$$

ℓ = Diffusion Scale Length,

$$E_{te} = -\ell\left(\frac{\partial P}{\partial T_p}\right)\nabla P\ \langle u'T'\rangle/\langle(u')^2\rangle^{1/2},$$

$$E_t = 1/2\ \langle(u')^2\rangle \ .$$

Since the convective zone lies just at the photosphere, the model results should be subject to observational verification. Figure 12 shows observations of Benz and Mayor (1982) of the Cepheids SV Vul and X Cyg using the CORAVEL instrument. Radial velocity is at the right, velocity dispersion (width of the correlation function, essentially an average line width) shown at right as crosses, and the dispersion expected from a non-turbulent atmosphere shown as boxes. In both cases we note a discrepancy between the two dispersions of about 5 km/sec near the phase of minimun radius (phase 0.5) and a very rapid disappearance of this discrepancy soon thereafter. A similar behavior has been seen by Benz in RR Lyrae. (Benz and Stellingwerf, 1985).

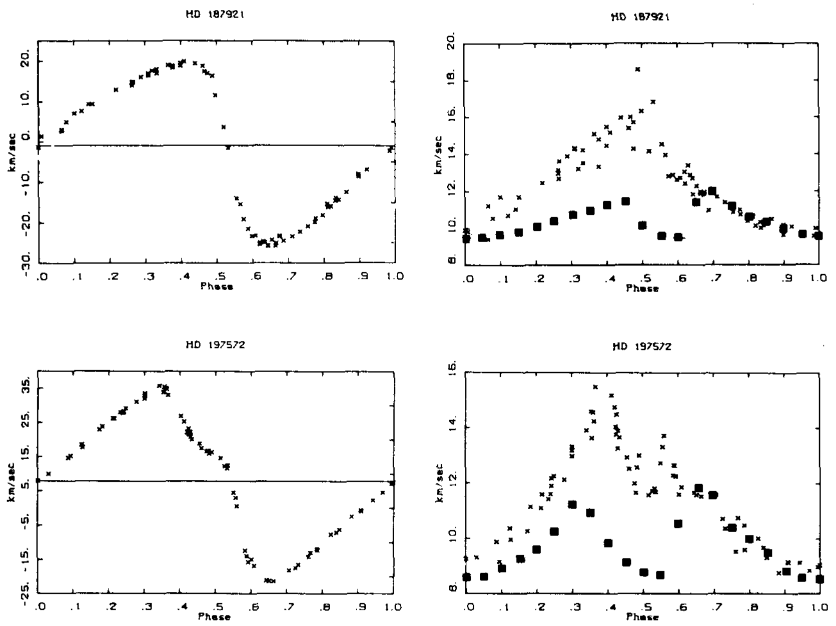

Fig. 12. Observed variation of the radial velocity and the width of the velocity correlation function (measure of turbulence) for the two Cepheids SV Vul and X Cyg as given by Benz and Mayor (1982). The boxes on the right represent the expected variation due to projection effects, while the x's give the observed variation.

Figure 13 shows snapshots of the convective quantities in a nonlinear RR Lyrae model (Stellingwerf, 1984a): solid line is the rms convective velocity, dashed is the convective flux, stellar surface at the right. The phases of max and min radius are indicated. Notice the strong increase in the convection near minimum radius (phases 0.275-0.325) and the remarkably sudden drop to near zero convection at phase 0.4, exaclty as seen in the Cepheid observations. Maximum velocity in this model at the photosphere is 6 km/sec. Although one does not expect too much from such a simple model of a complex phenomenon, it does appear that at least the gross aspects of the convective motions are being handled correctly.

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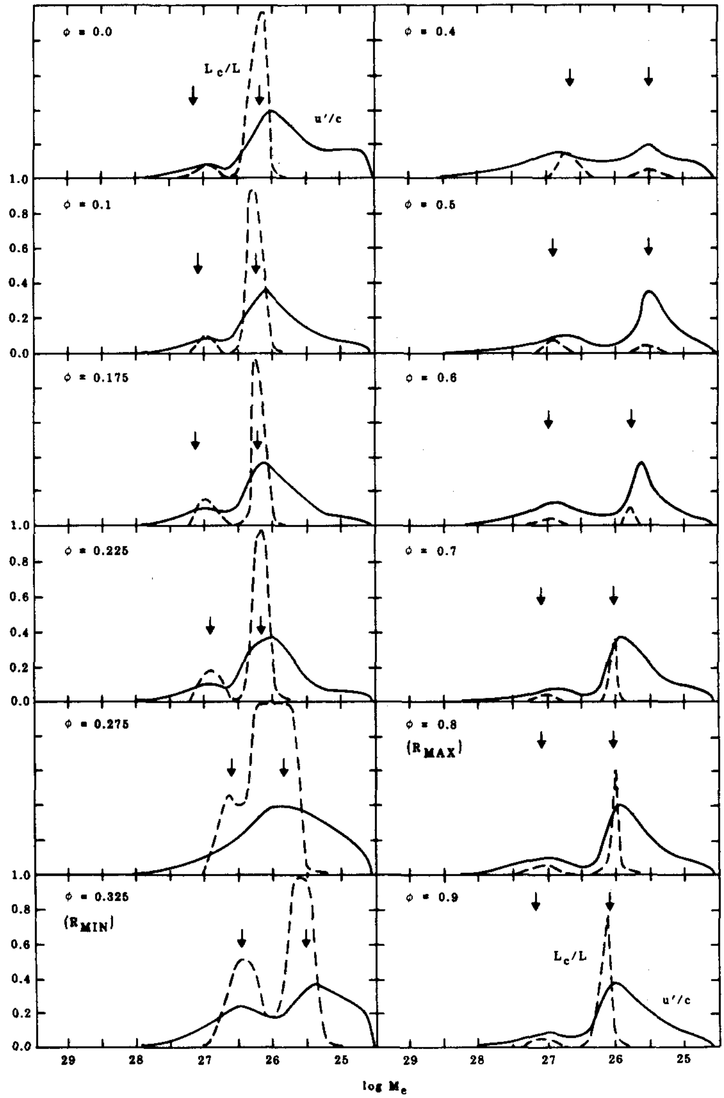

Fig. 13. Snapshots of a convective envelope model of an RR Lyrae star at various phases. Note the strong increase near minimum radius, and the subsequent rapid decline of the convection.

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DISCUSSION

SYLVIE VAUCLAIR: There is a class of pulsating stars that you did not mention in your talk and which is interesting from a physical point of view: The rapidly oscillating Ap stars which have been observed by D. W. Kurtz since 1980. (Series of papers in MNRAS). Ten of these stars are now known. They lie on the main sequence within a narrow range of effective temperature (around 7000 K). They have magnetic fields of ≈ 500 - 1000 G which vary with time (this is explained in the oblique rotator model). They oscillate with periods between 4 and 15 min, and the amplitudes vary in phase with the magnetic field. These observatons suggest that the pulsations are preferentially amplified along the magnetic axis and damped at the magnetic equator. N. Dolez, D. Gough and myself have shown (still unpublished) that these observations may be accounted for if these stars suffer a magnetically controlled wind of $\simeq 10^{-13}$ Ω yr^{-1}. In such a wind helium is left behind due to gravitational settling in the same way as in helium rich stars (Vauclair 1975). In these cooler stars the helium accumulation region lies deeper and doesn't appear at the surface, but we show that it can trigger the pulsations as observed by κ-mechanism.

STELLINGWERF: It seems likely that even weak magnetic fields can influence pulsation amplitudes appreciably, since this may be the cause of the Blazko effect in RR Lyrae Stars.

M. GEHMEYR: You have talked a lot on Cepheids and RR Lyrae stars. Can you give some idea low to explain Miras, esp. by convection/puslation coupling?

STELLINGWERF: I have not computed Mira models. As I mentioned, there is some evidence that convection may destabilze for very cool envelopes, but I have not seen this effect in RR Lyrae models.

KARL-HEINZ A. WINKLER: How does one treat stellar winds in stellar pulsation theory?

STELLINGWERF: They are usually ignored. Probably, no simple model will suffice. A full model with an extended atmosphere is needed.

KARL-HEINZ A. WINKLER: Is the convective energy equation all terms are proportional to E_t. That means that E_t will always be 0 if it was on 0 everywhere. Is this physical deficiency serious for the applications?

STELLINGWERF: In this theory perturbations are needed to start the convection in unstable zones. In stable regions, the convective velocity decays, but remains nonzero due to the diffusion term.

MIGUEL H. IBAUEZ S.: 1. In your equations I didn't see anywhere the corresponding equation for the degree of ionization of Hydrogen but you are working in just the range of temperature of $T \simeq 10^3 - 10^4$ where the effects of hydrogen recombination are crucial. Have you any comment about this point?

2. I have worked, for some time, the problem of thermochemical instabilities in hydrogen plasma, i. e., instabilities related with the hydrogen recombination process, and I have found that the plasma is highly unstable, at $T \simeq 10^3 - 10^4$ against formation of small elements (clumps). Do you take into account in your models such inhomogenities?

STELLINGWERF: 1. In the detailed models, ionization is obtained via solution of the (SAHA) equation (LTE). For the simple models, ionization effects are lumped into the value of Γ_1.

2. No.

J. CHRISTENSEN-DALSGAARD: Concerning the observations of line broadening at minimum radius: This may partly be caused by velocity gradients in the stellar atmosphere, which would evidently give rise to some broadening. Have your taken this into account?

STELLINGWERF: Velocity gradients above the photosphere contribute about 1-2 km/sec to the broading, whereas, the turbulent contribution is 5-10 km/sec.

J. CHRISTENSEN-DALSGAARD: I have some comments on your discussion of convection. First, a few philosophical remarks. We should remember that the problems of convection are orders of magnitude more difficult than the problems of radiation hydrodynamics, with which the Colloquium is mainly concerned. With radiation hydrodynamics we know the equations, and the difficulty is how to

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solve them, under circumstances that are sufficiently simple to allow a solution. With convection, on the other hand, we do not have a consistent treatment based on fundamental principles. We have a number of simple models which appear to work well under some circumstances (often because they contain an adequate set of free parameters), but we have no confidence that the models are correct. Also it should be remembered that convection is only one form of three-dimensional instability affecting hydrodynamical structures and flows. There is little doubt that other three-dimensional instabilites, leading to equally complex and ill understood pheonomena, affect may of the other ojects discussed at this meeting (jets, supernovae, . . .); this should be at the back of our minds when considering the results of the simple models now available. To solve these problems will undoubtedly keep astrophysical hydrodynamicists busy well into the next century.

Then a few comments on your discussion of convection in pulsating stars. It is probably fortuitous that Deupree obtained a reasonable red edge. The stability of a star is predominantly decided near the surface, where the scale of convection is probably of the order of a scale height, i.e., a very small fraction of the radius of the star; in contrast Deupree modeled the pulsation and convection with a very small number of meshpoints, thus totally failing to resolve the convection. Even with the expansion of computing power foreseen by Dimitri, a direct numerical attack on the coupled problem of convection and pulsation seems impractical. The time dependent mixing length theories by Unno and by Gough were derived consistently from a simple physical description of convection, and in that sense are internally consistent, even though their relation with reality may be somewhat tenuous. The oscillations found in the interior of the convection zone are not, as you implied, a numerical problem, but rather a mathematical property of the equations which Baker & Gough (although possibly not Gonzci & Osaki) resolved fully numerically. However, they are almost certainly unphysical, and in any case not consistent with the local approach used in the mixing length theory. On the other hand, they occur deep in the model, where the oscillations are nearly adiabatic, and so may not be very important for the stability at the red edge of the instability strip, when convection was taken into account in this manner.

Finally, I should be interested in some remarks as to how your equation for the turbulent energy was derived, and how it relates to the real world.

STELLINGWERF: The derivation of the model is given in the reference: Stellingwerf (1982a).

STARRFIELD: 1. In pulsating central stars of Planetary Nebulae, the red edge of the instability strip is caused by the growing thickness of the surface convection zone.

2. The cause of the ZZ Ceti pulsations may be directly due to pulsation convection interactions.

M. GEHMEYR: You showed that one needs some resolved theory for the convection/pulsation coupling. To do the convection to radiation right you have to use proper values for the opacity. Especially, within ionization fronts $\langle K(T)\rangle = K(\langle T\rangle)$. How did you take care of that fact?

STELLINGWERF: Yes, the opacity average is quite likely the strongest source of error in this theory. Evaluation of the proper average of opacity and thermodynamic quantities is not straightforward, but may be implemented using a beam-scheme. I am currently studying this problem.

VINCENT ICKE: (In reply to Mihalas) There is definite evidence (circumstellar shells, OH/IR stars, etc.), that long period variability is connected with mass loss. But I don't know if that extends to the shorter periods discussed here.

REUBEN OPHER: The probable model for the ejection of matter is fairly complicated. An upcoming shock lifts matter to a given height which then falls back and is lifted up to even greater heights by a subsequent shock. This process continues, each time the matter is lifed up to greater heights by subsequent shocks, until it is ejected. Recent water maser work partially substantiates this model.