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CORRIGENDUM

Towards a finite-time singularity of the Navier–Stokes equations. Part 2. Vortex reconnection and singularity evasion – CORRIGENDUM

H. K. Moffatt and Yoshifumi Kimura

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Results of numerical integration (using Mathematica) of the following dynamical system were presented in Moffatt & Kimura (2019*b*, hereafter MK19b):

$$\frac{\mathrm{d}s}{\mathrm{d}\tau} = -\frac{\gamma\,\kappa\cos\alpha}{4\pi}\left[\log\left(\frac{s}{\delta}\right) + 0.4417\right], \tag{1}$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}\tau} = \frac{\gamma\,\kappa\cos\alpha\sin\alpha}{4\pi s^2}, \tag{2}$$

$$\frac{\mathrm{d}\,\delta^2}{\mathrm{d}\tau} = \epsilon - \frac{\gamma\,\kappa\cos\alpha}{4\pi s}\,\delta^2, \tag{3}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\tau} = -\epsilon\,\frac{s\,\gamma}{2\sqrt{\pi}\,\delta^3}\exp[-s^2/4\delta^2], \tag{4}$$

with $\alpha = \pi/4$. Unfortunately, the exponential factor $\exp[-s^2/4\delta^2]$ in (4) was consistently misrepresented as $\exp[-s^2\delta^2/4]$ in the Mathematica code, leading to errors in the numerical results and in figures 4–8, although the conclusions concerning the approach to a 'near singularity' of the system (1)–(4) remain unchanged.

Figure 4 of MK19b should be replaced by figure 1 below, and figure 5 by figure 2, which exhibit the same qualitative behaviour in each case. The difference, however, is that the spike of vorticity now occurs for significantly larger ϵ (1/45 in figure 1, and 1/200 in figure 2), and at smaller 'critical time' t_c (0.7773 in figure 1, and 0.3574 in figure 2). (When $\epsilon = 0$, with the same initial conditions, the singularity occurs at time $t_c \approx 0.2434$; when $\epsilon = 1/4000$, it occurs at the slightly greater time $t_c \approx 0.2522$. As ϵ increases, t_c continues to increase.)

Figures 6–8 and the detailed figures of tables 1 and 2 of MK19b are similarly incorrect. Indeed, when $\epsilon = 1/4000$, the minimum value of δ^2 ($\lesssim 10^{-42}$) and the maximum value of $\omega(\tau)/\omega(0)$ can no longer be accurately determined.

The slanted vortex-ring configuration considered by MK19b has been investigated by direct numerical simulation (DNS) by Yao & Hussain (2020), and it has become apparent through this work that the model of MK19b fails to represent the true nature of the reconnection process. This model assumes that the vortex cores remain compact during reconnection, and that the reconnected vortex strands have negligible

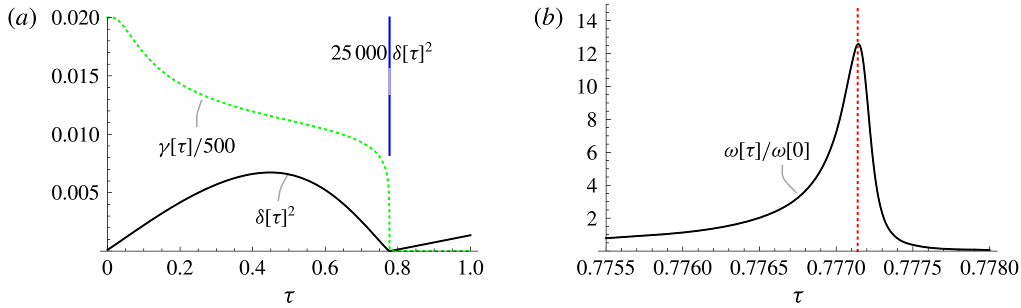

FIGURE 1. (Replacing figure 4 of MK19b) Evolution governed by (1.1)–(1.4), with $\epsilon = 1/45$ and initial conditions $s(0) = 0.1$, $\delta(0) = 0.01$, $\kappa(0) = \gamma(0) = 1$.

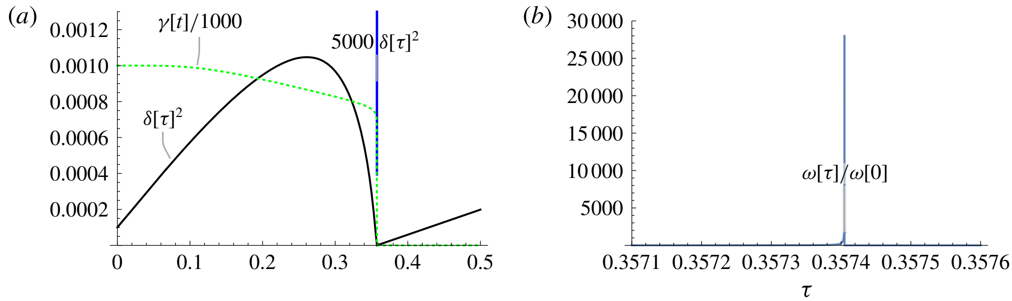

FIGURE 2. (Replacing figure 5 of MK19b) Same as figure 1, but with $\epsilon = 1/200$.

effect on the intensification of vorticity in the incident vortices. The DNS indicates that these assumptions are untenable. Nevertheless the MK19b model captures key elements of a possible approach to a singularity, and it is for this reason that the dynamical system (1)–(4) merits critical investigation.

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