statement, the identity of expressions, and the occurrence of one expression in another. Rules of procedure are adopted which yield both logical and syntactical theorems.

Natural numbers are introduced by explaining "0", "1", "2", ··· as abbreviations of "*cc", "*00", "*11", ··· . The statement that E is a natural number, in this sense, is itself expressed within the formal system, in terms of substitution; in effect, E is a natural number if *EE results from substituting 0 for 1 in **EE*EE.

Quantification is introduced much in the manner of substitution and alternative denial: certain arbitrary complexes of "c" and "x" are interpreted as variables, and certain further complexes containing these variables are interpreted as quantifiers. A distinction is maintained between logical and so-called semantic variables, and also between variables of different logical types.

The authors restrict quantification in a manner similar to Poincaré's repudiation of "impredicative" definitions, or to Russell's ramified theory of types without the axiom of reducibility. They introduce a hierarchy of "elementary systems," in each of which an upper limit is imposed upon the admissible types of variables; and they allow quantification only within one or another definite elementary system, specified numerically in the quantifier. But some relief is gained through metamathematical channels; it is possible within the formal notation to deal with successive elementary systems, and to state that such and such is or is not a theorem of each.

Under the head of applications, an elementary arithmetic is outlined. The definitions of sum, product, and power are essentially the traditional recursive ones, transformed into direct formal definitions. But Frege's method of transforming recursive definitions into direct ones, through the medium of hereditary classes (or relations), is not followed here; Frege's use of the class variable is avoided in favor of the expression variable. Where Frege's construction refers to membership in all infinite classes of a certain sort, the present authors accomplish the same purpose by speaking rather of peculiarly environed occurrence within all finite expressions of a certain sort. (For a fuller account of the essential procedure, see the reviewer's *On derivability* (III 53(1)), and *Definition of substitution* (I 116)).

Likewise under the head of applications, the authors sketch the beginnings of a theory of classes, a theory of relations, and a metamathematical calculus. Classes are arbitrarily identified with universally quantified statements; roughly, the class $\hat{x}(\ldots)$ is identified with the statement "$(x)(\ldots)$". Huntington's postulates for Boolean algebra are derived. In the metamathematical calculus, the principal notion constructed is that of the arithmetized syntax corresponding to a given elementary system.

Constructions are obscured by failure to distinguish clearly between use and mention of expressions. This difficulty pervades the paper; by way of a sample instance, however, it will suffice to consider the explanation of "$(EFGH)[c]$". In this explanation, as observed earlier, the authors use the idiom ". . . is the result of substituting . . . for . . . in . . . ." Now grammar would direct us to fill the blanks here with nouns—names of the things talked about. But the things talked about here are expressions in turn. The blanks should thus be filled by names of the expressions with which the substitution deals; not by the expressions themselves. (Illustration: substitution of alpha for eta in zeta yields zeta, not zalpha.) It would appear, then, that the complexes of "c" and "*" which supplant the four capitals, in any specific instance of "$(EFGH)[c]$", are *names of* the expressions involved in the denoted substitution. But the use made of the form "$(EFGH)[c]$" in subsequent constructions denies this; we find that the intended objects of substitution are the expressions which actually appear in the statement of substitution. (Cf. the definition of "E is a natural number," mentioned above.) Does this mean that all complexes of "c" and "*" are to be construed as names of themselves? But then how can the authors give those same complexes other meanings in addition—meanings of alternative denial, substitution, etc.? It is difficult to determine how much of the essential theory of the paper could be reconciled with a strict distinction between use and mention of expressions. (Cf. Church's review of Chwistek, this JOURNAL, vol. 2 (1937), p. 170.) W. V. QUINE

JIRÔ HIRANO. *Einige Bermerkungen zum v. Neumannschen Axiomensystem der Mengenlehre.* ***Proceedings of the Physico-mathematical Society of Japan,*** 3 s. vol. 19 (1937), pp. 1027–1045.

This is an attempt to improve upon von Neumann's systematization of set theory. Entities are classified as *I-Dinge* and *II-Dinge*, as with von Neumann, and von Neumann's primitive constants A and B are retained; but his two primitive operations, "$[f, x]$" (application of a one-place function) and "$<x, y>$" (formation of the ordered pair), are dropped in favor of one: "$[f; x, y]$" (application of a two-place function). $[f, x]$ is then defined as $[f; x, x]$. The gain in economy is questionable, for in order to derive $<x, y>$ the author is compelled to assume another primitive, namely a constant p such that $[p; x, y] = <x, y>$.

Of von Neumann's nineteen postulates, thirteen are carried over either unchanged or directly extended to deal with "$[f; x, y]$" instead of "$[f, x]$". The remaining six are dropped in favor of four new ones. But the total is brought back up to nineteen by a stylistic change: principles governing the identity sign are lifted from the level of intuitive logic and given explicit formulation.

Among Hirano's postulates there is one (V.1) to the strange effect that every I-Ding is an ordered pair of I-Dinge. No intuitive justification is undertaken. The author states, indeed, that this postulate is included only to simplify certain projected constructions; but surely many readers would prefer the alternative complexities.

The theorems proved here are intended as lemmas for a future paper. The chief ones are these. Let ϕ be a formula (statement form) built up of the primitive devices mentioned above together with truth-function signs, quantifiers, the identity sign, names of any specific elements, and I-Ding variables "x_1", "x_2", $\cdots$ "x_n" Let ϕ' be ϕ with "y" substituted for "x_1". Then

(1) $(\exists f)(x_1)(x_2)\cdots(x_n) \,.\, \phi \equiv [f, <x_1, < x_2, \cdots <x_{n-1}, x_n>\cdots>>] = A,$

(2) $(\exists f)(x_1)(x_2)\cdots(x_n) :. (y) \,.\, \phi \,.\, \phi' \,.\supset x_1 = y : \equiv x_1 = [f, <x_2, \cdots <x_{n-1}, x_n>\cdots>].$

Minor errors: the verbal rendering of postulate IV.3 diverges from the symbolic rendering in two essential respects; also some mention of the identity sign seems to be wanted in the statement of Hilfssätze 2 and 4. W. V. Quine

Myśl katolicka wobec logiki współczesnej (Catholic thought and modern logic). Studia Gniesnensia no. 15. 1937, 196 pp.—Therein:

C. Michalski, *Wstęp* (Introduction), pp. 7-11.

J. Łukasiewicz, *W obronie logistyki* (In defense of logistic), pp. 12-26.

I. M. Bocheński, *Tradycja myśli katolickiej a ścisłość* (The tradition of Catholic thought and exactness), pp. 27-34.

J. Salamucha, *Zestawienie scholastycznych narzędzi logicznych z narzędziami logistycznymi* (Comparison of Scholastic logical techniques with logistic techniques), pp. 35–49.

J. F. Drewnowski, *Neoscholastyka wobec nowoczesnych wymagań nauki* (Neoscholasticism and the contemporary requirements of science), pp. 49-57.

Dyskusja (Discussion): J. Chechelski, pp. 61–64; P. Chojnacki, pp. 65–74; J. Pastuszka, pp. 75-77; J. Stepa, pp. 78-83.

Odpowiedzi (Replies): I. M. Bocheński, *O "relatywiźmie" logistycznym* (On logistic "relativism"), pp. 87-111; J. Salamucha, *O "mechanizacji" myślenia* (On the "mechanization" of thought), pp. 112-121; J. Salamucha, *O możliwościach ścisłego formalizowania dziedziny pojęć analogicznych* (On possibilities of precise formalization in the domain of analogical concepts), pp. 122–155.

La pensée catholique et la logique moderne, pp. 155-196. (A summary in French of the preceding material)

See previous reviews in this Journal of Łukasiewicz's paper (III 43), and of the French summary, which was separately published by the theological faculty of the University of Kraków (III 44).

Morgan Ward. *Structure residuation.* ***Annals of mathematics,*** 2 s. vol. 39 (1938), pp 558–568.

Saunders MacLane. *A lattice formulation for transcendence degrees and p-bases.* ***Duke mathematical journal,*** vol. 4 (1938), pp. 455–468.