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Metacyclic groups of odd order

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Recently, Short [1] produced a computer library of all finite soluble primitive permutation groups of degree less than 256. The work on which we report here is the first step towards producing libraries of such groups chosen by restricting the structure of the point stabilisers rather than the degree. The restriction we impose is that the point stabilisers be metacyclic. A classical result of Galois reduces the problem to one concerning metacyclic irreducible linear groups over finite prime fields. Our contribution is a theoretical approach to the latter problem for groups of odd order.

The first part of the thesis is devoted to determining the abstract isomoprhism types of metacyclic groups of odd order. For the case of prime-power order, the basic questions have been answered in the literature. In our context further technical questions concerning automorphisms become important, and we develop the required answers. We then turn to presentations (in terms of generators and relations) of metacyclic groups of (general) odd order, and focus attention on presentations of a special kind which we call standard presentations. The main result is that each group of this kind has at least one standard presentation, and there is straightforward test for deciding whether two standard presentations define isomorphic groups.

For a sketch of how these presentations arise, let G be a metacyclic group of odd order and N the smallest normal Hall subgroup of G with G/N nilpotent. Then G is a semidirect product of N with direct product H of metacyclic groups of prime power order. As indicated above, for the latter the solution of the corresponding problem is well known. It is in comparing the possible actions of H on N that we use the information developed earlier on automorphisms of metacyclic groups of prime-power order. What makes this approach succeed is that N itself is the semidirect product of two cyclic groups, and so is much easier to describe than metacyclic groups in general.

The second half of the thesis begins with a study of faithful irreducible representations of metacyclic groups of odd order over fields of positive characteristic. A natural correspondence is shown to exist between such representations of the group and certain

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representations of the centre of the Fitting subgroup. This provides the key to solving the linear isomorphism problem for abstractly isomorphic irreducible linear groups of our kind.

Primitive linear metacyclic groups of odd order may also be approached from another direction, namely as (conjugates of) subgroups of the normaliser of a Singer cycle. We exploit this in the odd prime-power degree case, extending results obtained by Short[1] for the prime degree case.

The thesis concludes with a tentative discussion of future work towards practical implementations.

References

[1] M.W. Short, *The primitive soluble permutation groups of degree less than 256*, Lecture Notes in Math. **1519** (Springer-Verlag, Berlin, Heidelberg, New York, 1992).

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