EMANUELE FRITTAION, ***Reverse Mathematics and Partial Orders***, University of Udine, Italy, 2014. Supervised by Alberto Marcone. MSC: 03F35, 03B30, 06A07. Keywords: reverse mathematics, partial orders.

Abstract

The thesis investigates the reverse mathematics of several theorems about partial orders with emphasis on scattered (no copy of the rationals) and FAC (no infinite antichains) partial orders. The thesis consists of 7 chapters.

Chapter 1 introduces some reverse mathematics and recursion theoretic results that are used throughout the thesis.

Chapter 2 investigates a theorem due to Bonnet which characterizes FAC partial orders in terms of initial intervals. Although the classical proof of Bonnet's theorem is based on a result by Erdös and Tarski on strong antichains, it is shown that one direction of Bonnet's theorem has indeed the same reverse mathematics strength of Erdös and Tarski theorem: in fact they are both equivalent to ACA_0 over RCA_0. The other direction of Bonnet's theorem is shown to lie below WKL_0 and strictly above RCA_0.

Chapter 3 studies four classically equivalent definitions of scattered FAC partial orders and provides a reverse mathematics analysis similar to that for well-partial orders given by Cholak, Marcone, and Solomon, *Reverse mathematics and the equivalence of definitions for well and better quasi-orders*, J. Symbolic Logic, 69(3):683–712, 2004. The analysis leads to consider a partition theorem on the rationals due to Erdös and Rado, which turns out to be of reverse mathematics strength between RT^2_2 and ACA_0.

Chapter 4 analyzes another characterization theorem for scattered FAC partial orders due to Bonnet. This theorem asserts that a countable partial order is scattered and FAC if and only if there are countably many initial intervals. It is shown that one direction (left to right) is equivalent to ATR_0 over ACA_0, while the other is provable in WKL_0 but not in RCA_0.

Chapters 5 and 6 investigate several results about scattered partial orders. The well-known Hausdorff's classification theorem for scattered linear orders is shown to be equivalent to ATR_0 over ACA_0. Two generalizations of Hausdorff's theorem, for scattered FAC partial orders and for countable FAC partial orders, are studied and shown to be provable in Π^1_2-CA_0.

Chapter 7 studies the notion of linearizability by showing that for certain order types τ (namely, $\omega, \omega^*, \omega + \omega^*$ and ζ) and for a certain notion of τ-likeness, the statement "every τ-like partial order has a τ-like linear extension" is equivalent to either $\mathsf{B}\Sigma^0_2$ or to ACA_0 over RCA_0.

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JAMES FIROZE APPLEBY, ***Choice Sequences and Knowledge States: Extending the Notion of Finite Information to Produce a Clearer Foundation for Intuitionistic Analysis***, Keele University, UK, 2017. Supervised by Peter Fletcher. MSC: 00A30, 03A05, 03F55, 03B60. Keywords: intuitionism, intuitionistic logic, choice sequences, lawless sequences, lawlike sequences, foundations of analysis, foundations of mathematics, foundations of intuitionistic analysis.

Abstract

There are currently four major formal foundational systems for intuitionistic analysis: *LS*, *CS* (both in [1]), *FIM* (given in [2]), and the derivable *FIRM-INT* (given in [3] but not named). All these systems rely on different universes of choice sequences and different conceptions of what a choice sequence is. There is a strong common ground between these systems as they use the same very restrictive notion of finite information when dealing with these choice sequences—the notion of restricting ourselves to initial segments. This text