FLARE MODEL WITH FORCE-FREE FIELDS AND HELICAL SYMMETRY

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ABSTRACT.

Physical arguments are given indicating that solar flare magnetic energy storage may happen through force-free fields with helical symmetry ($\partial_z + \lambda(r)\partial_\varphi = o$). The mathematical results turn out simple for helical fields whether general, in equilibrium or force-free. A preliminary stability analysis points to appropriate properties.

1. PHYSICAL ARGUMENTS.

1.1 Force-free magnetic fields.

The gas density and pressure are fairly low above the solar surface. Hence the great energy storage which becomes apparent in solar flares points strongly to force-free magnetic fields (curl $\bar{H}=\alpha H, \alpha(\bar{r},t)$ being the current-to-field ratio and $\bar{H}.\nabla\alpha=o$) or at least to fields that are very nearly force-free. (Callebaut, 1976) The author has often advocated that the difference between force-free and nearly force-free may be relevant since a minor difference may have a strong influence on the stability and the energy release and even on the structure and the evolution. Nevertheless, here the restriction is made to pure force-free fields. However α is not restricted to be constant and a nice example will even turn up in which the energy storage corresponds to non-constant α.

1.2 Helical symmetry.

If one considers a cylindrical tube, with axis parallel to the solar surface (considered as flat) and partially or wholly above it, one is at first inclined to look for cylindrical symmetry ($\partial_z=o$) or even symmetry of revolution $\partial_\Theta=o$. Much less restrictive is a combination of both ($\lambda\partial_\varphi+\partial_z=o$, λ will be taken to be a constant or a function of r (the distance to the axis) only), which means that no quantity varies

M. Dryer and E. Tandberg-Hanssen (eds.), Solar and Interplanetary Dynamics, 279-282.

https://doi.org/10.1017/S0074180900067711 Published online by Cambridge University Press

along certain helices lying on circular cylinders. It seems plausible indeed to have some kind of symmetry on general expectation grounds. Moreover when the field emerges from the solar surface (Cfr. the Kuperus and the Kuperus-van Tend models, 1978) it has to adapt itself to become a force-free field. This adaptation process may impress the same pitch on the field. Obviously there are some restrictions on this helical symmetry: (a) the situation during the emerging phase changes a bit; (b) the small pressure above the solar surface decays with height; (c) the tube is not infinitely long but finite and curved into the solar surface.

The idea is that a field may emerge from the solar surface and with the axis parallel to it and lifting up the potential or force-free field of the solar atmosphere. How much of the cylinder is above the surface is not specified here, but a probable choice is to consider half a cylinder. Furthermore of this half only a shell with thickness say 1/3 of the radius is pervaded by the force-free field with helical symmetry. The inner part may again contain a potential field. This arcade configuration may be related to the two ribbon flare, the axis and the "feet" being parallel to the ribbons.

2. MATHEMATICAL RESULTS FOR FIELDS WITH HELICAL SYMMETRY.

The helical symmetry reduces the situation to a 2-dimensional one in which the variables are r and $\xi = \Theta - \lambda(r)z$.

2.1 General magnetic fields (Callebaut and Raadu, 1976)

2.1.1. λ= constant. Then there is a streamfunction so that:

$$rH_r = \partial_\xi \Psi \quad \text{and} \quad H_\Theta = \lambda r H_z - \partial_r \Psi$$

2.1.2. $\lambda(r) \neq$constant. Then the solution is

$$H_r = C/r \quad \text{and} \quad H_\Theta = \lambda r H_z + A(r)$$

with C a constant (usually zero, see 3.1), H_r an arbitrary function of r and ξ and A(r) an arbitrary function of r.
Pressure balanced fields were studied by Callebaut (1979)

2.2 Force-free fields.

2.2.1. λ = constant. A. $\alpha \neq$cst, $\Psi \neq$cst. Then one can show that $r\lambda H_\Theta + H_z = f(\Psi)$ with $\Psi = \Psi(\alpha)$ and $d(r\lambda H_\Theta + H_z)/d\Psi = \alpha$. Spicer (1976) also studied this case. In the very probable case (see below) that $rH_r = \partial_\xi \Psi = o$ one obtains in fact a one dimensional problem: $\alpha(r), H_\Theta(r), H_z(r)$.
B. Ψ=cst. Then one obtains the force-free field of constant pitch studied by Piet van der Laan (1968) for the pressureless region surrounding the plasma in the screw pinch in Jutphaas (Utrecht, the Netherlands):

$$H_r = o, H_z = C(\lambda^2 r^2 + 1)^{-1} \text{ , } \quad H_\Theta = \lambda r C(\lambda^2 r^2 + 1)^{-1} \text{ , } \quad \alpha = 2\lambda(\lambda^2 r^2 + 1)^{-1}.$$

C. α =cst. This case is well-known.

2.2.2. $\lambda \neq$constant. For H_r=o one obtains again that α, H_Θ and H_z are functions of r only. E.g. if α is constant then only the Lundqvist field (or Bessel function field) $\bar{H}=H_o(o,J_1(\alpha r),J_o(\alpha r))$ is possible.

3. FURTHER CONSIDERATIONS.

3.1 Boundary conditions.

H_r=o can not always be inferred from the singularity at the axis, because the axis may be excluded, e.g. when only a cylindrical shell (arch) is considered or when the axis is still under the solar surface. However, if the external region is pervaded by a potential field it can be shown that H_r has to vanish at the boundary. Then H_r vanishes everywhere if $\lambda(r) \neq$cst.

3.2 Stability.

The linear stability of these fields is not yet fully analyzed. The van der Laan field showed theoretically and experimentally a fair stability without being stable under all circumstances. However in the plasma experiment (torus) there was the stabilizing influence from the wall and the finite length. In the flare storage one may expect some stabilization from the anchoring in the solar surface. It may be expected that the field is quite a time stable, and then, by having evolved further to a less stable configuration or by some strong trigger, becomes unstable, releases energy and becomes after flaring a potential field or a force-free field. In this connection it has to be stressed that, for constant α, the lowest α compatible with the geometry is stable for fixed boundaries and also sometimes for non-fixed boundaries.

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DISCUSSION

Low: Gene Parker has pointed out from physical considerations that a magnetic field in static equilibrium must possess "suitable" invariance in its field pattern. I have recently derived an expansion to express these required invariances in terms of the Euler potentials defining the magnetic field. (Paper to appear in *Solar Phys.*) In principle, then, one can classify equilibrium fields according to their types of invariances. The problem is nonlinear and very difficult. Your interesting example of vertical invariance seems like an ideal class of equilibrium field to start classifying the field.

Callebaut: One of the aims of presenting this paper was precisely to bring to the attention: (a) some intuitive feeling for some underlying symmetry or invariance; (b) the possibility of extensively using helical symmetry with variable pitch: it is a wide class of fields and yet the solutions can be handled with care.

It is very pleasing to hear that you have already worked on the first feature and that you may be able to use the second one. I am very interested in this work.

Kuperus: Soloviev proved that a non-constant α force-free-field relaxes to a constant α force-free-field thus releasing the excess energy. The condition is that the Alfvén crossing time is much smaller than any photospheric perturbation time. How does this relate to your analysis?

Callebaut: That non-constant α force-free fields are unstable even when confined in rigid boundaries was already published in a book of Belgium Academy of Sciences by one of my students (T. Krüger) in 1967 or 1968. See also Kruger in *Journal of Plasma Physics* (1976).

This (weak) instability is precisely a desirable feature in my view. E.g., the van der Laan field is "fairly" stable, i.e., it has some weak instability for some perturbations which is useful to make the flare. This is again related to "nearly force-free" fields. I suppose that the original field has some small pressure and is stable in the beginning. Then, it evolves slowly to the van der Laan field (e.g.) by losing some matter, by cooling, by raising, by resistive evolution, etc., and thus becomes unstable. The MHD instability has (at least in the beginning) to be fairly weak, because the low pressure field and the force-free field (or a very low pressure field) may not be far from each other in these considerations. I am presently elaborating this process in detail.