CORRESPONDENCE

(*To the Editors of the* Journal of the Institute of Actuaries)

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DEAR SIRS,

In his *Note on the Gompertz Table*, Mr Fraser regrets not being able to explain in a brief and simple way why the 'abacus' could be used to calculate the coefficients $A_0, A_1, A_2, \ldots$.

The following explanation I found when I put to myself the question: 'How did Mr Fraser get the inspiration to try and "alternate" the "abacus"?'

I began by systematically writing down l_x and its first derivatives, so that the terms with the same powers of μ_x came into the same column.

	1	2	3
$l_x = l_x$			
$Dl_x =$	$-l_x\mu_x$		
$D^2l_x =$	$-l_x\mu_x(\lambda c)$	$+\ l_x\mu_x^2$	
$D^3l_x =$	$-l_x\mu_x(\lambda c)^2$	$+3l_x\mu_x^2(\lambda c)$	$-l_x\mu_x^3$
$D^4l_x =$	....	....	....

For one familiar with the 'abacus' the resemblance is not difficult to spot. To explain it we must bear in mind that:

(1) Applying the operator D to l_x is the same as multiplying l_x by $-\mu_x$; in the scheme above this means that the term is transferred to the next row and the next column.

(2) Applying the operator D to μ_x is the same as multiplying μ_x by λc; this means that the term is transferred to the next row but stays in the same column.

(3) The number of the column is the same as the power of μ_x; applying the operator D to the power of μ_x gives this number as an extra factor.

Let us call the term in the mth row and the nth column $T_{m,n}$.

It is clear that

$$T_{m+1,n} = -\mu_x T_{m,n-1} + n\lambda c T_{m,n}.$$

If we put

$$T_{m,n} = C_{m,n} l_x \mu_x^n (\lambda c)^{m-1},$$

we find that

$$C_{m+1,n} = -C_{m,n-1} + nC_{m,n},$$

which is the law of formation of the 'alternating abacus'.

Yours faithfully,

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