3.4   THE MERIDIANS OF REFERENCE OF INDIAN ASTRONOMICAL CANONS

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ABSTRACT   The canons of Sanskrit astronomy depend on mean
motions which are normally postulated to refer to the
central meridian of Ujjain. The present work is a stati-
stical analysis of these mean motions designed to discover
the optimum position of the meridian, by comparison with
modern mean motions. This follows earlier work done by
Billard in determining the optimum year. The results
confirm that from the time of Āryabhaṭa all the canons were
referred to meridians lying well within India, and in many
cases clearly identifiable with Ujjain within the
statistical bounds.

1.   INTRODUCTION
        The general idea underlying the research which is summarised
in the present paper is hardly original, and indeed is a direct develop-
ment of that employed so successfully in Roger Billard's l'Astronomie
Indienne (Billard (1971)). The medieval mean longitudes of Sun, Moon
and planets are compared directly with the corresponding modern means,
and the differences are treated by the method of least squares in order
to determine values of the year and of the meridian to which the
medieval longitudes are referred.

In Billard's researches the meridian was always assumed to be that
passing through Ujjain, the central meridian repeatedly referred to in
Sanskrit sources. In this way he was able to fix, within narrow limits
as a rule, the year when the Indian mean longitudes best agreed with
the true configuration.

In the present work not only the year, but also the meridian are allowed
to vary in the search for the best fit. There are also improvements in
the modern parameters, taking advantage of quite recent developments,
and besides the statistical control follows more closely the formal
method of least squares.

Apart from a natural confirmation of Billard's results for the optimum
year, we have a series of results for the optimum meridian showing that
it always lies in India, indeed generally near to Ujjain (longitude
75;46 East of Greenwich). This reinforces very well Billard's general
conclusion that with the Sanskrit canons we are presented with a
continuous millenium of observational astronomy in India.

All of the canons cited by Billard have been analysed in this way, as
well as a number of others, but here there is room only to present a
few particularly important ones.

2. MEAN LONGITUDES AND DEVIATIONS

Let the modern mean longitudes be denoted $L_i(t)$, where the suffix runs from 1 to 9:

1. Sun	5. Mercury
2. Moon	6. Venus
3. Lunar apogee	7. Mars
4. Lunar node	8. Jupiter
	9. Saturn

All these are tropical longitudes. The precise numerical expressions are those now to be employed in the national ephemerides, following an IAU recommendation (Francou, e.a., 1983). These expressions are functions of Terrestrial Dynamical Time (TDT) which differs from Universal Time by the quantity ΔT, which allows for the changes in the rate of rotation of the earth; this includes both a steady rate of decrease, and fluctuations. A satisfactory expression for the steady part of ΔT depends necessarily on ancient eclipse records, and this calculation is in no way as accurate as that determining L_i. For many years a formula for ΔT which was determined by Spencer Jones (1939) has been used, although there are theoretical objections (van der Waerden (1961)) to its derivation. Many new formulae have been derived, but they do not agree especially well among themselves, nor do they meet the objections brought by van der Waerden against the older formula. Naturally the uncertainty in the value of ΔT appears directly in the meridian which we determine, but it is unlikely that further revisions of ΔT would lead to alterations in the meridian of more than a few minutes of arc.

In this paper the expressions for $L_i(t)$ include terms as far as t^3 (as taken from references listed by Francou, e.a., (1983)), and in addition a large number of trigonometrical terms expressing the various perturbations (1). These are included for Venus, the Sun, Mars, Jupiter and Saturn, but are effectively important only for the last two, which are affected by the well known resonance.

The medieval mean longitudes $\lambda_i(t)$, where t is simply the Universal Time, are linear expressions. In most cases, the canons which we analyse are already included in Billard's survey, and there one will find the numerical details. As a general rule the Sanskrit canons define sidereal longitudes.

3. THE METHOD OF LEAST SQUARES

We now change the notation slightly, so that t denotes the Universal Time, and TDT as required for L_i will be $t + \Delta t$, with Δt provided by the Spencer Jones formula, *faute de mieux*. The deviation between the modern and the medieval mean longitudes is defined as

$$D_i(t,\phi) = \lambda_i(t-\phi/360) - L_i(t+\Delta t)$$

where ϕ is the longitude of the meridian East of Greenwich.

The method of least squares will be used to determine jointly the optimum estimates t_0 and ϕ_0. For this purpose we must postulate 'true' values of $D_i(t,\phi)$, which may be done in two ways. If λ_i are tropical longitudes, such as we may calculate from the Sanskrit canon when we are also given a model of precession, then the 'true' value of D_i is zero, and then we would seek the minimum of the sum

$$Q = \sum_I D_i^2$$

where I is a selection of values of i. In this case we say that the deviations are 'absolute'. On the other hand, if λ_i is sidereal, the 'true' value of D_i would depend on the rate of precession together with an unknown constant. In this case therefore, which we refer to as 'relative', we can only take the mean of the set D_i as the true value, and calculate the minimum of

$$Q = \sum_I (D_i - (\sum_I D_i)/N)^2$$

where N is the number of deviations included. When the deviations are relative, the number of random variables is then $N - 1$. Let I be indicated by a sequence of 1's or 0's, indicating whether a particular value of i is included or not: thus (1101 00000) would indicate that the Sun, Moon and lunar node only are included. Further the set I will be augmented by an initial 1 or 0 to indicate whether we are concerned with absolute or relative deviations, respectively. With each determination of the year and the meridian, then, we associate a statistic I, such as (0 1101 0000). This is the same symbol as used by Billard.

The method of least squares has been explained in a suitably general and clear way by van der Waerden (1967). In the following only the briefest summary of the application is possible.

If we write in the neighbourhood of the minimum

$$Q = h_{11}(t-t_0)^2 + 2h_{12}(t-t_0)(\phi-\phi_0) + h_{22}(\phi-\phi_0)^2 + Q_0,$$

then the estimate of the variances of t_0 and ϕ_0 are s_t^2 and s_ϕ^2:

$$s_t^2 = \frac{Q_0}{n-r} \frac{h_{22}}{h_{11}h_{22}-h_{12}^2}$$

$$s_\phi^2 = \frac{Q_0}{n-r} \frac{h_{11}}{h_{11}h_{22}-h_{12}^2}$$

where n, the number of random variables, is one less than the number of 1's in the symbol I. Moreover if $\tilde{t}$ and $\tilde{\phi}$ and are the true

values, then

$$(t_0-\tilde{t})/s_t, \quad (\phi_0-\tilde{\phi})/s_\phi$$

follow the Student t-distribution with n-r degrees of freedom. We may thereby assign confidence limits to the estimates t_0 and ϕ_0, as obtained from the minimum value Q_0. If n-r were large, then t_0 would have a probability of 0.682 of lying within one standard deviation of the true value. We use the Student distribution in this way to find the equivalent range for a probability of 68%.

If σ is the standard deviation of individual deviations D_i, then $s^2 = Q_0/(n-r)$ is an unbiased estimate of σ^2. Moreover Q_0/σ^2 has a χ^2 distribution with n-r degrees of freedom, so that one may find limits of confidence for s^2/σ^2 at a given level, in our case chosen to be 95%.

There are canons, such as that known from Lalla's Śiṣyadhīvṛddhidatantra, in which the year cannot be effectively determined. In that case it is more practical to fix in advance the value of t, and to apply the present methods to determine ϕ_0 alone, in which case r = 1, in the above expressions.

4. DISCUSSION OF THE RESULTS
4.1 General remarks
For each of the canons, and each of the statistics I, the results are given in the table below, Section 5.

These results for the determination of the meridian are satisfying in that they lie in every case in India. Nevertheless, they differ to a varying extent from the meridian of Ujjain. There are two obvious ways in which this may occur, for we may have either an error on the part of the author of the canon, or it may have been the case that the observations were established for some other meridian, without any attempt to reduce them to that of Ujjain. No doubt both these reasons have some degree of application.

A third possibility is that the discrepancy might arise from the equation of time. I have indicated elsewhere, however, (Mercier 1985), that while difficulties would arise because of the way in which the equation is defined in Greek, Arabic and Latin usage, in Sanskrit usage the equation takes positive and negative values symmetrically, so that there the Mean Solar Time would be the same as that used now. This would seem to be the case from the beginning of Sanskrit astronomy, since Varāhamihira, in his discussion of the Sūryasiddhānta (Pañcasiddhāntikā IX,9) indicates an approximate rule for the equation.

4.2 The Romaka Siddhānta
The earliest of the canons in this survey is the Romaka

Siddhānta, based on parameters given by Varāhamihira in the
Pañcasiddhāntikā (VIII, 1-8). The deviations are limited in use, since
the lunar apogee and node are not in good agreement with the Sun and
Moon. Therefore only the statistic (1 1100 00000) is available, and we
can only find a year and meridian such that $Q_0 = 0$, since $n = r$.
Therefore no statistical bounds can be assigned to the year and
meridian. Nevertheless the results are very interesting, especially in
regard to the meridian 86;40. This is considerably too large for one
to suppose that Indian observations were responsible for the canon, but
if we recollect the longitude difference of 56;30 between Alexandria
and Ujjain, according to Ptolemy's Geography (IV,5,9; VII,1,63)

        Alexandria   60:30
        Ujjain   117;0,

we see that the Romaka canon might very well have been transferred to
Ujjain, by means of such presumed positions, from a Greek source at a
meridian around 30;0, which is indeed the correct longitude of
Alexandria. The name Romaka, and the use of the year 365.24666...,
indicate clearly enough the dependence on a Greek source. The longi-
tude difference 60;0 between Yavanapura (Alexandria) and Ujjain is
entailed by a remark concerning two astronomers of the time, Lāṭācārya
and Siṃhācārya (Pañcasiddhāntikā, XV, 17-20) (2).

4.3 Āryabhaṭa
        In the list of results in Section 5 one naturally groups
together the next three canons, those of the Āryabhaṭīya, and the two
from Varāhamihira's Pañcasiddhāntikā: the Sūrya Siddhānta, and the
emendation to the planetary parameters of that work defined in XVI,
10-11. We know that only the Āryabhaṭīya is accurate in the case of
Jupiter, so that D_8 must be omitted from the statistic for the other
two. That much is clear from Billard's analysis of the deviations,
and the use of the most recent modern ephemerides does not alter the
position. The use of the new ephemerides has however an extremely
interesting consequence, for the optimum year is now very nearly equal
to 499 A.D., incomplete. That year is singled out by Āryabhaṭa
(Āryabhaṭīya III,10) although he contents himself with merely telling
us that he was then aged 23, so that one has never been altogether
clear as to its astronomical significance. This point is however
distant 3600 years, quite precisely, from the Kaliyuga, so that it can
hardly be doubted that it had, for Indian astronomy, some decisive
importance. When the older ephemerides of Newcomb, Brown, etc., are
used to obtain the year and the meridian, we obtain

        (0 1111 01111) 507.6 ± 4.05 79;8 ± 1;24.

It is now apparent that although with the new parameters, no great
change results, nevertheless the results point more clearly than before
to the year A.D. 499 as that year to which Āryabhaṭa referred observa-
tional results which were collected around that time. It is not clear
why exactly that year was chosen, although it may have been believed by

Āryabhaṭa that the precessional correction vanished then. In any case,
how can one avoid the conclusion that the Kaliyuga, 3600 years earlier
was defined and fixed as a direct consequence? The two versions of that
epoch, Sunrise and Midnight, are fixed exactly, so that in terms of the
related canons, Āryabhaṭīya, and the Sūryasiddhānta, respectively, 3600
sidereal years separate the epoch from Noon 499 March 21. The enduring
use of the Kaliyuga in all the following centuries testifies to the
decisive importance of the work of Āryabhaṭa in the history of Indian
astronomy (3).

The statistical results for the emendation to the Sūryasiddhānta show
remarkably that when some later astronomers established it, the effect
was to improve the accuracy at the epoch year A.D. 499. We now have
498.1 for the Sun and Moon alone, and 499.1 with the planets included.
It is as if those responsible had access to the observational data, and
were able to make better use of it so as to obtain even more accurate
longitudes for the selected epoch year. Is it not wonderful in any
case, that the most modern and accurate parameters help us to discover
and appreciate better than ever before the very accurate Indian
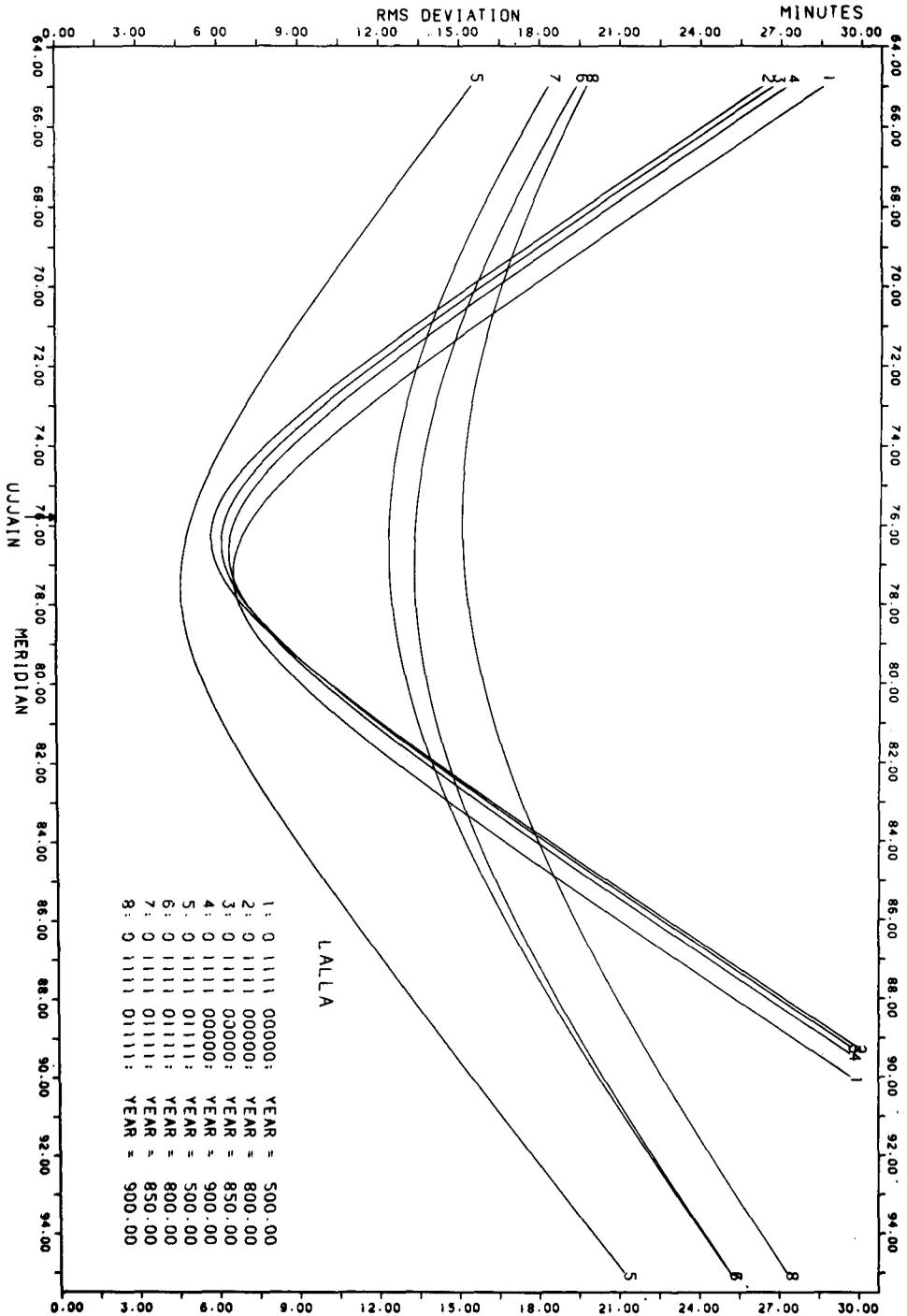observations made nearly 1500 years ago?

4.4 The canon of Lalla
            The canon taken from a set of emendations given to us by
Lalla is singularly interesting, because unlike most others, indeed all
others preceding it, it is in good agreement with observations over a
long period. This means that one cannot determine the year by the
method of least squares, although other considerations such as the form
of the emendations, lead one to associate it with the late ninth
century. The determination of the meridian, however, is secure, and
indeed safer than in some other cases, precisely because the optimum
meridian is so insensitive to the choice of year. One is struck in
this case by the close approximation of the optimum meridian to that of
Ujjain, reinforcing the view that the canon was the product of out-
standingly careful work. In the diagram there are shown the graphs of
σ against φ for the selection of statistics given in Section 5.

4.5 The Dṛggaṇita
            This work was composed in Śaka 1353 (A.D. 1431-2), by
Parameśvara, an astronomer of Kerala, and constitutes the exposition of
the Dṛk ('Observational') system (Sarma (1963)). It is produced by a
set of emendations applied to that version of the Sūryasiddhānta on
which Parameśvara wrote his commentary (Shukla (1957)), or equivalently,
to the Karaṇa Tilaka of Vijayanandin, ca. 950 (Rizvi (1963)). The
emendations of the solar and lunar parameters were derived from
eclipses of Sun and Moon observed by Parameśvara in the period 1398 –
1432 inclusive, from which he determined mean longitudes referred to
the date with ahargaṇa 1651700 (essentially 4522 sidereal years), at
Sunrise, which is A.D. 1421 March 29. The text in which the dates and
circumstantial details of the eclipses are given has been edited by
Sarma (1966). For that date he found the following: Sun 0,13;0, Moon
10,4;6, Apogee 3,9;57, Node 4,23;55. These figures are most interesting.

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in that the longitude of the Sun is tropical, while the others are sidereal; that is the night-time observations are sidereal, the day-time tropical. Should we not see here an illustration of the paradigm of observation and tabular correction such as had been used throughout Indian astronomy, beginning with Āryabhaṭa?

The present methods applied to the eclipse deviations confirm well enough the documented circumstances, the year near 1421, and a meridian appropriate to Kerala (76;30) or perhaps Ujjain, for we cannot be certain whether Parameśvara reduced his parameters to the normal meridian.

4.6 Sphuṭanirṇayatantra

This work, by the Keralite astronomer Acyuta (1550-1620) was edited by Sarma (1974), and is of interest here because it is the one example known to me in Indian texts of a change of meridian. For, there is a short tract in another Malayalam MS, published as Appendix 5 by Sarma, in which the mean longitudes differ only by a shift of meridian, exactly 6.804 degrees Westward. Thus the respective meridians are approximately 80.5 and 74, neither of which however is suited to southern India. Moreover the accuracy is generally best in the thirteenth century, so one might infer that Acyuta obtained the canon from an earlier astronomer who lived further north.

5. TABLE OF RESULTS

The quantities given after the statistic are respectively, t_0, ϕ_0, with their standard deviations; σ, in minutes; the mean deviation at the point (t_0, ϕ_0); and the year and standard deviation for the meridian of Ujjain. In the case of those canons for which the year cannot be determined, the statistics concerning the meridian are given for certain preassigned values of the year. In all cases the year given is 'complete'.

Romaka Siddhānta
1 1100 00000
400.0   86;40

Sūrya Siddhānta
0 1111 00000
498.1±31.67   77;19±6;25   10.67   0;14   493.8±14.7
0 1111 01101
502.0± 5.41   77;28±1;54   5.00   0;11   501.3±5.23

Āryabhaṭa
0 1111 00000
498.1±31.67   77;20±6;01   10.67   0;14   494.3±15.82
0 1111 01111
502.1±5.10   77;38±1;45   4.70   0;12   501.7±5.13

Pañcasiddhāntikā
0 1111 00000
498.1±31.67   77;19±6;25   10.67   0;14   493.8±14.7
0 1111 01101
499.1±6.84   77;16±1.49   4.72   0;14   497.8±6.36

Lalla
0 1111 00000
500   77;11±2;40   6.6   0;5
800   76;11±2;20   5.8   -4;47
850   76;23±2;30   6.1   -5;35
900   76;35±2;36   6.4   -6;24
0 1111 01111
500   77;23±1;43   4.6   0;8
800   77;00±4;58   13.33   -4;44
850   76;23±4;37   12.38   -5;34
900   75;47±5;38   15.1   -6;24

Karaṇa Tilaka
0 1101 01011
951.6±8.51   81;26±3;35   9.4   -6;59   950.7±10.43
1 1101 01011
955.4±17.45   87;22±6;34   19.73   0;8   955.5±20.58

Dṛggaṇita
0 1111 00000
1424.5±27.19   77;19±6;6   9.9   -14;40   1420.5±12.30

Sphuṭanirṇayatantra
0 1111 00110
1250   80;47±1;45   4.64   -12;3
1350   81;35±1;53   5.00   -13;39
1550   81;47±5;39   14.99   -16;55

    6. SELECTED CANONS
        There are three canons which are not given by Billard, and
which are not readily available elsewhere.

        6.1 Romaka Siddhānta
Epoch A.D. 505 March 21 Sunrise (1905588.75)

Sun   (150t-65)/54787 revs
Moon   (38100t-1984)/1040953 revs
Lunar argument   (110t+664)/3031 revs
Lunar node   -(24t+56278)/163111 revs

The time t is measured in days from the epoch. These results are
taken from unpublished work on R. Billard.

6.2 Sphuṭanirṇayatantra (a)
   Sphuṭanirṇaya tulyagrahamadhyamānayanam (b)
Epoch 588465.75 (Kaliyuga Sunrise)
Period (Kalpa) 1577917517019 days

	radix (a)	radix (b)	yugabhagaṇa
Sun	0;0	0;1,7,4	4320000000
Moon	4;21,21,36	4;36,18,7	57753321009
Apogee	118;50,9,36	118;50,17,11	488123229
Node	201;18,43,12	201;18,39,36*	-232297832
Mercury	350;12,28,48	350;17,7,14	17937072112
Venus	35;6	35;7,49,1	7022270775
Mars	348;9,21,36	348;9,57,15	2296862959
Jupiter	15;15,50,24	15;15,56,3	364172296
Saturn	340;22,48	340;22,50,17	146626695

*The text as edited gives 204;18,39,36, which must be emended to
201;18,39,36.

The radices (a) are equal to 0.4569 bhagaṇa, which recalls the
construction used in the Brāhmasphuṭasiddhānta, which has 0.4567
bhagana as a general formula for its radices.

Both the texts (a) and (b) were edited by Sarma (1974), who gives (b)
in Appendix 5.

6.3 Karaṇa Tilaka
   The parameters are identical to those of the modern Sūrya
Siddhānta in the version given by Billard, except for two yugabhagaṇa:

Lunar apogee 488211 in place of 488203
Lunar node   -232234 in place of -232238
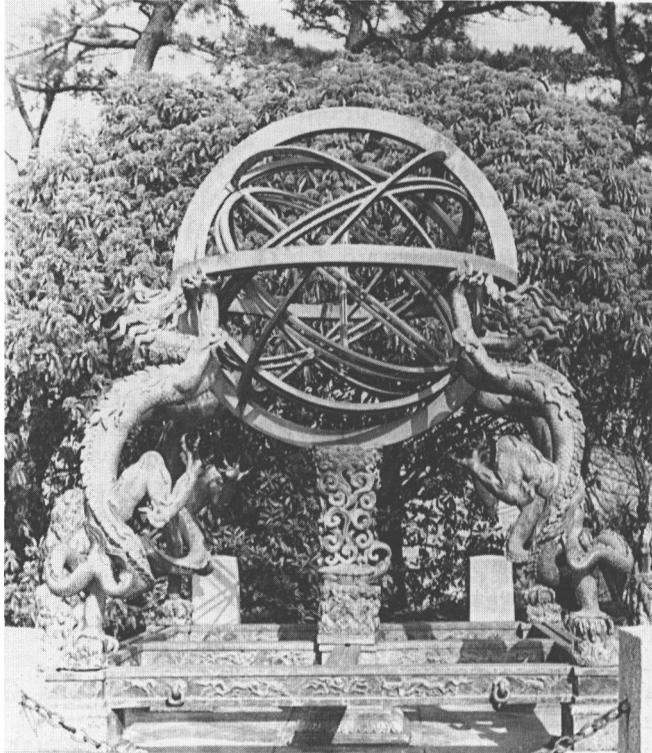
See Rizvi (1963).

NOTES
(1) The trigonometrical terms are available from the Bureau
des Longitudes, Paris.

(2) I am indebted to some important unpublished work on this
canon by Roger Billard, who proposed the interpretation of the
meridian 86;40.

(3) This proposal, that the Kaliyuga originates strictly with
Āryabhaṭa, contradicts van der Waerden's conclusions (1978 and 1980).
His argument begins by observing that Abū Maᶜshar made use of the
Kaliyuga as the date of the Deluge, and that he also made use of
'Persian' tables. If this only meant the Zīj-i Shāh in the mid-
sixth century, there would be no problem, but van der Waerden argues
for a Hellenistic dependence, via an earlier Persian system.

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Armilary Sphere. It is an ancient China's cast-bronze astronomical instrument, engraved with excellent artistry. It has an observing tube, which is used mainly for determining the equatorial coordinates and the longitudes and horizontal coordinates of celestial bodies. It was made in 1437, the second year of the Zheng-Tong Reign of the Ming Dynasty.