A Theorem on Alternants

By D. E. Littlewood (University College, Swansea).

Communicated by H. W. Turnbull.

(*Received* 17*th January*, 1936. *Read* 7*th February*, 1936.)

The following is a direct proof of a theorem by Zia-ud-Din[1].

Let $\{\nu\} \equiv \{\nu_1, \nu_2, \ldots, \nu_p\}$ be any S-function[2] of weight $r + s$ such that

$$\{\nu\} \Delta (x_1, x_2, \ldots, x_p) = \Sigma \pm x_1^{\nu_1 + p - 1} x_2^{\nu_2 + p - 2} \ldots x_p^{\nu_p}$$

in the alternant denoted in the theorem by $A(\alpha\beta\gamma\ldots)$. Let $\{\mu\}$ be an S-function of weight s, equal to $h\binom{0pq\ldots}{012\ldots}$, and let $\{\lambda\}$ be an S-function of weight r, such that $\{\lambda\}\Delta(x_1, \ldots, x_p)$ is obtained with coefficient $g_{\lambda\mu\nu}$ by diminishing the indices in the alternant $\{\nu\}\Delta(x_1, \ldots, x_p)$ according to the theorem.

Obviously, by direct multiplication, $g_{\lambda\mu\nu}$ is the coefficient of $\{\nu\}\Delta(x_1, \ldots, x_p)$ in the product of $\{\lambda\}\Delta(x_1, \ldots, x_p)$ and $\{\mu\}$, *i.e.*, the coefficient of $\{\nu\}$ in the product $\{\lambda\}\{\mu\}$.

If we now proceed to the associated S-functions, denoting these by a bar, clearly $g_{\lambda\mu\nu}$ is also the coefficient of $\{\nu\}$ in $\{\lambda\}\{\mu\}$. This proves the theorem.

---

[1] *Proc. Edinburgh Math. Soc.*, **4** (1934), 51.

[2] D. E. Littlewood and A. R. Richardson, *Phil. Trans. Roy. Soc.* (A), **233** (1934), 99.